Physics 207 – Lecture 4

Physics 207, Lecture 4, Sept. 18

Agenda

- Chapter 3, Chapter 4.1, 4.2
  - ❖ Coordinate systems
  - ❖ Vectors (position, displacement, velocity, acceleration)
  - ❖ Vector addition
  - ❖ Kinematics in 2 or 3 dimensions
  - ❖ Independence of *x*, *y* and/or *z* components

Assignment: Finish reading Ch. 4, begin Chapter 5 (5.1 and 5.2)
- WebAssign Problem Set 1 due tomorrow (should be done)
- WebAssign Problem Set 2 due Tuesday next week (start today)

(Slightly modified from original syllabus)

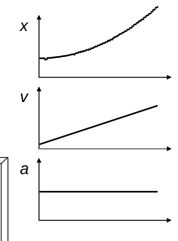
Chapter 2 recap: Two "perspectives" to motion in one-dimension

Starting with *x(t)*

$$x = x(t)$$
$$v = \frac{dx}{dt}$$
$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Starting with *a*, v_0 and x_0

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$v = v_0 + at$$
$$a = const$$



Rearranging terms gives two other relationships

- For constant acceleration:

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$v = v_0 + at$$
$$a = const$$

- From which we can show (caveat: **constant acceleration**):

$$v^2 - v_0^2 = 2a(x - x_0)$$
$$\bar{v}_{avg} = \frac{1}{2}(v_0 + v)$$

See text: 3-1

Coordinate Systems / Chapter 3

- In 1 dimension, only 1 kind of system,
  - ❖ Linear Coordinates (*x*) +/-
- In 2 dimensions there are two commonly used systems,
  - ❖ Cartesian Coordinates (*x*,*y*)
  - ❖ Polar Coordinates (*r*,θ)
- In 3 dimensions there are three commonly used systems,
  - ❖ Cartesian Coordinates (*x*,*y*,*z*)
  - ❖ Cylindrical Coordinates (*r*,θ,*z*)
  - ❖ Spherical Coordinates (*r*,θ,φ)

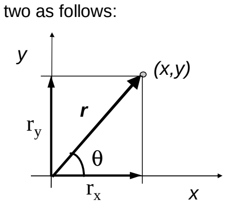
See text: 3-1

Converting Coordinate Systems

- In ***polar*** coordinates the vector ***R*** = (r,θ)
- In Cartesian the vector ***R*** = (r_x, r_y) = (x,y)
- We can convert between the two as follows:

$$r_x = x = r\cos\theta$$
$$r_y = y = r\sin\theta$$
$$\mathbf{R} = x\,\mathbf{i} + y\,\mathbf{j}$$

$$r = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1}(y / x)$$



- In 3D cylindrical coordinates (*r*,θ,*z*), *r* is the same as the magnitude of the vector in the *x*-*y* plane [*sqrt*(x^2 + y^2)]

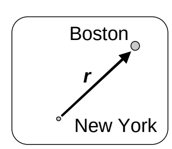
See text: 3-2

Vectors

- In 1 dimension, we can specify direction with a + or - sign.
- In 2 or 3 dimensions, we need more than a sign to specify the direction of something:
- To illustrate this, consider the ***position vector r*** in 2 dimensions.

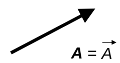
Example: Where is Boston?
- ❖ Choose **origin** at New York
- ❖ Choose coordinate system

Boston is 212 miles northeast of New York [in (r,θ)] OR
Boston is 150 miles north and 150 miles east of New York [in (x,y)]



Vectors...

- There are two common ways of indicating that something is a vector quantity:
  - ❖Boldface notation: ***A***



$\vec{A}$

  - ❖"Arrow" notation:

Vectors have rigorous definitions

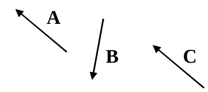
- A vector is composed of a magnitude and a direction
  - ❖ Examples: displacement, velocity, acceleration
  - ❖ Magnitude of **A** is designated |**A**|
  - ❖ Usually vectors include units (m, m/s, m/s^2)
- A vector has no particular position

(Note: the position vector reflects displacement from the origin)

- Two vectors are <u>equal</u> if their <u>directions</u>, <u>magnitudes</u> and <u>units</u> match.

A = **C**

A ≠ **B**, **B** ≠ **C**



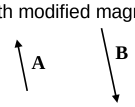
Comparing Vectors and Scalars

- A scalar is an ordinary number.
  - ❖ A magnitude **without** a direction
  - ❖ May have units (kg) or be just a number
  - ❖ Usually indicated by a regular letter, no bold face and no arrow on top.

  Note: the lack of specific designation of a scalar can lead to confusion

- The **<u>product</u>** of a vector and a scalar is another vector in the same "direction" but with modified magnitude.

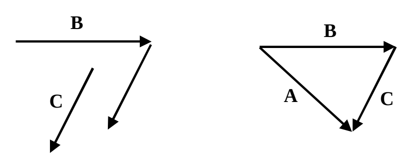
A = -0.75 **B**



Vector addition

- The sum of two vectors is another vector.

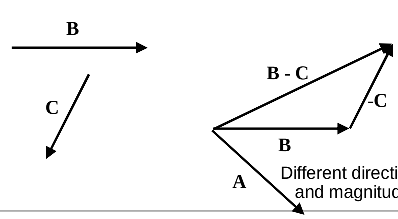
A = **B** + **C**



Vector subtraction

- Vector subtraction can be defined in terms of addition.

B - **C** = **B** + (-1)**C**

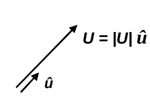


Different direction and magnitude !

See text: 3-4

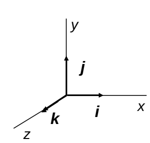
Unit Vectors

- A **Unit Vector** is a vector having length 1 and no units
- It is used to specify a direction.
- Unit vector ***u*** points in the direction of ***U***
  - ❖Often denoted with a "hat": ***u = û***



- Useful examples are the cartesian unit vectors [***i, j, k***]
  - ❖ Point in the direction of the *x*, *y* and *z* axes.

    R = r_x **i** + r_y **j** + r_z **k**



See text: 3-4

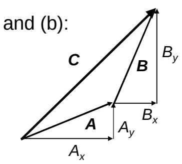
Vector addition using components:

- Consider $\mathbf{C} = \mathbf{A} + \mathbf{B}$.

(a) $C = (A_x\,\mathbf{i} + A_y\,\mathbf{j}) + (B_x\,\mathbf{i} + B_y\,\mathbf{j}) = (A_x + B_x)\mathbf{i} + (A_y + B_y)$

(b) $C = (C_x\,\mathbf{i} + C_y\,\mathbf{j})$

- Comparing components of (a) and (b):
  - ❖ $C_x = A_x + B_x$
  - ❖ $C_y = A_y + B_y$



Lecture 4, Example 4
Vector addition

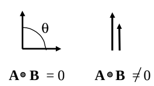
An experimental aircraft can fly at full throttle in still air at 200 m/s. The pilot has the nose of the plane pointed west (at full throttle) but, unknown to the pilot, the plane is actually flying through a strong wind blowing from the northwest at 140 m/s. Just then the engine fails and the plane starts to fall at 5 m/s^2.

What is the magnitude and directions of the resulting velocity (relative to the ground) the instant the engine fails?

Multiplication of vectors

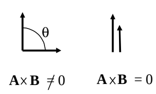
- There are two common ways to multiply vectors
  - ❖ "Scalar or dot product": Result is a scalar

$$\mathbf{A} \circ \mathbf{B} = |\mathbf{A}|\,|\mathbf{B}|\,\cos(\theta)$$



❖"Vector or cross product": Result is a vector (not now…)

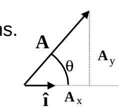
$$|\mathbf{A}\times\mathbf{B}| = |\mathbf{A}|\,|\mathbf{B}|\,\sin(\theta)$$



Scalar product

- Useful for performing projections.

$$\mathbf{A} \circ \hat{\mathbf{i}} = A_x$$



- Calculation is simple in terms of components.

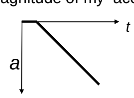
$$\mathbf{A} \circ \mathbf{B} = (A_x)(B_x) + (A_y)(B_y)$$

Calculation is easy in terms of magnitudes and relative angles.

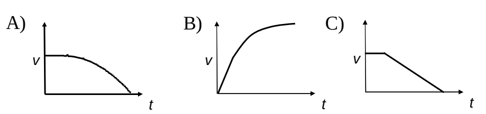
$$\mathbf{A} \bullet \mathbf{B} = |A|\,|B|\cos\theta$$

Lecture 4, Example
Review Motion *vs.* Time Graphs

- In driving from Madison to Chicago, Initially my speed is at a constant 65 mph. After some time, I see an accident ahead of me on I-90 and must stop quickly so I decelerate increasingly fast until I stop. The magnitude of my acceleration *vs* time is given by,



- My velocity *vs* time graph looks like which of the following ?



See text: 4-1

Chapter 4: Motion in 2 (and 3) dimensions
3-D Kinematics

- The position, velocity, and acceleration of a particle in 3-dimensions can be expressed as:

$$\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$$
$$\mathbf{v} = v_x\,\mathbf{i} + v_y\,\mathbf{j} + v_z\,\mathbf{k} \qquad (\mathbf{i}, \mathbf{j}, \mathbf{k} \text{ unit vectors})$$
$$\mathbf{a} = a_x\,\mathbf{i} + a_y\,\mathbf{j} + a_z\,\mathbf{k}$$

❖ We have already seen the 1-D kinematics equations.

$$x = x(t) \qquad v = \frac{dx}{dt} \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

See text: 4-2 and 4-3

3-D Kinematics

- For 3-D, we simply apply the 1-D equations to each of the component equations.

$$x = x(t) \qquad y = y(t) \qquad z = z(t)$$

$$v_x = \frac{dx}{dt} \qquad v_y = \frac{dy}{dt} \qquad v_z = \frac{dz}{dt}$$

$$a_x = \frac{d^2x}{dt^2} \qquad a_y = \frac{d^2y}{dt^2} \qquad a_z = \frac{d^2z}{dt^2}$$

- Which can be combined into the vector equations:

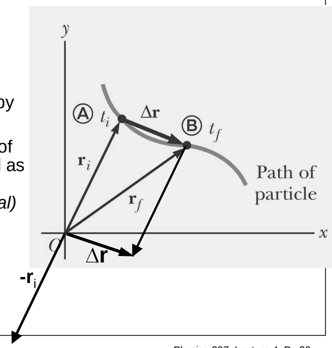
$$\mathbf{r} = \mathbf{r}(t) \qquad \mathbf{v} = d\mathbf{r} / dt \qquad \mathbf{a} = d^2\mathbf{r} / dt^2$$

This compact notation hides the actual complexity

Thinking about motion in 2 Dimensions

- The <u>position</u> of an object is described by its position vector, **r**
- The **<u>displacement</u>** of the object is defined as the ***change in its position*** *(final –initial)*

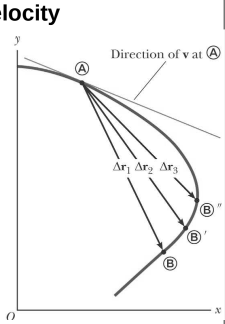
❖ $\Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$



Average Velocity

- The **average velocity** is the ratio of the displacement to the time interval for the displacement
- The **direction** of the **average velocity** is in the direction of the displacement vector, $\Delta\mathbf{r}$

$$\bar{\mathbf{v}} = \frac{\Delta\mathbf{r}}{\Delta t}$$



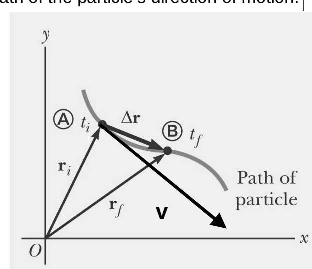
- The **average velocity** between points is ***independent of the path taken***

Instantaneous Velocity

- The **instantaneous velocity** is the limit of the average velocity as Δt approaches zero
- The direction of the **instantaneous velocity** is **along** a line that is **tangent** to the path of the particle's direction of motion.

$$\mathbf{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt}$$

- The magnitude of the instantaneous velocity vector is the speed. (The speed is a scalar quantity)

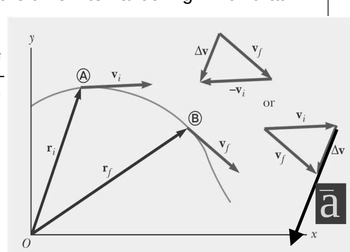


Average Acceleration

- The average acceleration of a particle as it moves is defined as the change in the instantaneous velocity vector divided by the time interval during which that change occurs.

$$\bar{\mathbf{a}} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t_f - t_i} = \frac{\Delta\mathbf{v}}{\Delta t}$$

- The average acceleration is a vector quantity directed along $\Delta\mathbf{v}$



Instantaneous Acceleration

- The instantaneous acceleration is the limit of the average acceleration as $\Delta\mathbf{v}/\Delta t$ approaches zero

$$\mathbf{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}$$

- The instantaneous acceleration is a vector with components parallel (tangential) and/or perpendicular (radial) to the tangent of the path

(see Chapter 5)

Producing an Acceleration

- Various changes in a particle's motion may produce an acceleration
  - ❖ The **magnitude** of the velocity vector may change
  - ❖ The **direction** of the velocity vector may change
    (Even if the magnitude remains constant)
  - ❖ Both may change simultaneously

Recap of today's lecture

- Chapter 3, Chapter 4.1, 4.2
  - ❖ Coordinate systems
  - ❖ Vectors (position, displacement, velocity, acceleration)
  - ❖ Vector addition and the scalar product
  - ❖ Kinematics in 2 or 3 dimensions
  - ❖ Independence of *x*, *y* and/or *z* components

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