Physics 207, Lecture 5, Sept. 20

Agenda

- Chapter 4
  - ❖ Kinematics in 2 or 3 dimensions
  - ❖ Independence of *x*, *y* and/or *z* components
  - ❖ Circular motion
  - ❖ Curved paths and projectile motion
  - ❖ Frames of reference
  - ❖ Radial and tangential acceration

Assignment: For Monday read Chapter 5 and look at Chapter 6

- WebAssign Problem Set 2 due Tuesday next week (start ASAP)

See text: 4-1

Chapter 4: Motion in 2 (and 3) dimensions
3-D Kinematics

- The position, velocity, and acceleration of a particle in 3-dimensions can be expressed as:

$$\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$$
$$\mathbf{v} = v_x\,\mathbf{i} + v_y\,\mathbf{j} + v_z\,\mathbf{k} \qquad (\mathbf{i}, \mathbf{j}, \mathbf{k} \text{ unit vectors})$$
$$\mathbf{a} = a_x\,\mathbf{i} + a_y\,\mathbf{j} + a_z\,\mathbf{k}$$

$$x = x(t) \qquad y = y(t) \qquad z = z(t)$$
$$v_x = \frac{dx}{dt} \qquad v_y = \frac{dy}{dt} \qquad v_z = \frac{dz}{dt}$$
$$a_x = \frac{d^2x}{dt^2} \qquad a_y = \frac{d^2y}{dt^2} \qquad a_z = \frac{d^2z}{dt^2}$$

- Which can be combined into the vector equations:

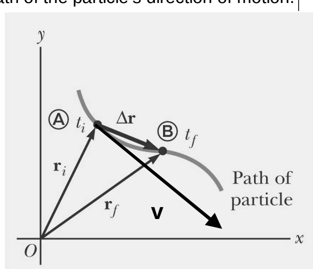
$$\mathbf{r} = \mathbf{r}(t) \qquad \mathbf{v} = d\mathbf{r}/dt \qquad \mathbf{a} = d^2\mathbf{r}/dt^2$$

Instantaneous Velocity

- The **instantaneous velocity** is the limit of the average velocity as Δt approaches zero
- The direction of the **instantaneous velocity** is **along** a line that is **tangent** to the path of the particle's direction of motion.

$$\mathbf{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt}$$

- The magnitude of the instantaneous velocity vector is the speed. (The speed is a scalar quantity)

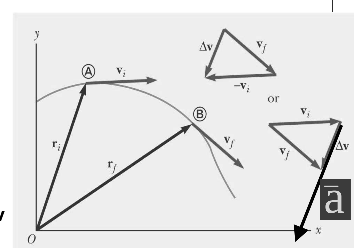


Average Acceleration

- The average acceleration of a particle as it moves is defined as the change in the instantaneous velocity vector divided by the time interval during which that change occurs.

$$\bar{\mathbf{a}} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t_f - t_i} = \frac{\Delta \mathbf{v}}{\Delta t}$$

- The average acceleration is a vector quantity directed along $\Delta\mathbf{v}$



Instantaneous Acceleration

- The instantaneous acceleration is the limit of the average acceleration as $\Delta\mathbf{v}/\Delta t$ approaches zero

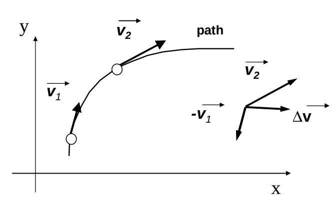
$$\mathbf{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}$$

- The instantaneous acceleration is a vector with components parallel (tangential) and/or perpendicular (radial) to the tangent of the path
- <u>Changes in a particle's path</u> may produce an <u>acceleration</u>
  - ❖ The **magnitude** of the <u>velocity vector</u> may change
  - ❖ The **direction** of the <u>velocity vector</u> may change (Even if the magnitude remains constant)
  - ❖ Both may change simultaneously (depends: path *vs* time)

Motion along a path
(displacement, velocity, acceleration)

- 3-D Kinematics : **vector** equations:

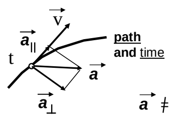
$$\vec{\mathbf{r}} = \vec{\mathbf{r}}(t) \qquad \vec{\mathbf{v}} = d\vec{\mathbf{r}}/dt \qquad \vec{\mathbf{a}} = d^2\vec{\mathbf{r}}/dt^2$$



Velocity :
$$\vec{v}_{av} = \Delta\vec{r} \,/\, \Delta t$$
$$\vec{v} = d\vec{r} \,/\, dt$$

Acceleration :
$$\vec{a}_{av} = \Delta\vec{v} \,/\, \Delta t$$
$$\vec{a} = d\vec{v} \,/\, dt$$

General 3-D motion with non-zero acceleration:



$\vec{a} = \vec{a}_{\parallel} + \vec{a}_{\perp}$

$\vec{a} \neq 0$

Two possible options:

Change in the magnitude of $\vec{v}$ $\quad \vec{a}_{\parallel} \neq 0$

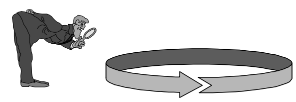
Change in the direction of $\vec{v}$ $\quad \vec{a}_{\perp} \neq 0$

Animation

- Uniform Circular Motion is one specific case:

See text: 4-4

Uniform Circular Motion



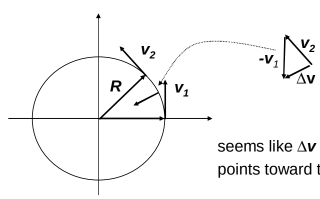
- What does it mean ?
- How do we describe it ?
- What can we learn about it ?

See text: 4-4

Average acceleration in UCM:

- Even though the *speed* is constant, velocity is ***not*** constant since the direction is changing: must be some acceleration !
  - ❖ Consider average acceleration in time Δt

⇨ $a_{av} = \Delta \mathbf{v} / \Delta t$

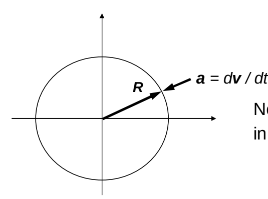


seems like $\Delta \mathbf{v}$ (hence $\Delta \mathbf{v}/\Delta t$) points toward the origin !

See text: 4-4

Instantaneous acceleration in UCM:

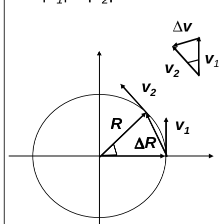
- Again: Even though the *speed* is constant, velocity is ***not*** constant since the direction is changing.
  - ❖ As Δt goes to zero in $\Delta \mathbf{v} / \Delta t$ ⇨ $d\mathbf{v} / dt = \mathbf{a}$



Now ***a*** points in the - ***R*** direction.

Acceleration in UCM:

- This is called Centripetal Acceleration.
- Calculating the magnitude:
- $|\mathbf{v}_1| = |\mathbf{v}_2| = v$



Similar triangles: $\frac{\Delta v}{v} = \frac{\Delta R}{R}$

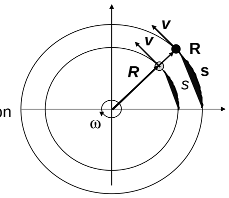
But $\Delta R = v\Delta t$ for small Δt

So: $\frac{\Delta v}{v} = \frac{v\Delta t}{R}$ ⇨ $\frac{\Delta v}{\Delta t} = \frac{v^2}{R}$

$$a = \frac{v^2}{R}$$

Period and Frequency

- Recall that 1 revolution = 2π radians
  - ❖ Period (T) = seconds / revolution = distance / speed = $2\pi R / v$
  - ❖ Frequency (f) = revolutions / second = 1/T (a)
  - ❖ Angular velocity (ω) = radians / second (b)
- By combining (a) and (b)
  - ❖ $\omega = 2\pi f$
- Realize that:
  - ❖ Period (T) = seconds / revolution
  - ❖ So $T = 1 / f = 2\pi/\omega$

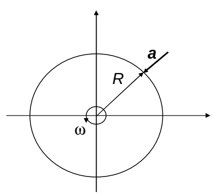


$$\omega = 2\pi / T = 2\pi f$$

See text: 4-4

Recap: Centripetal Acceleration

- UCM results in acceleration:
  - ❖Magnitude: $a = v^2 / R = \omega^2 R$
  - ❖Direction: $- \boldsymbol{r}$ (toward center of circle)



Lecture 5, *Exercise 1*
Uniform Circular Motion

- A fighter pilot flying in a circular turn will pass out if the centripetal acceleration he experiences is more than about *9* times the acceleration of gravity *g*. If his F18 is moving with a speed of *300 m/s*, what is the approximate radius of the tightest turn this pilot can make and survive to tell about it ? (Let g = 10 m/s²)

UCM (recall)

Magnitude: $a = v^2 / R$

Direction: $-\hat{r}$ (toward center of circle)

(a) *10 m*

(b) *100 m*

(c) *1000 m*

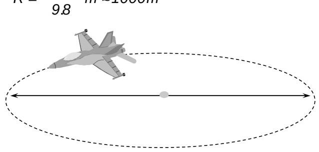
(d) *10,000 m*

Lecture 5, *Exercise 1*
Solution

$$a = \frac{v^2}{R} = 9g$$

$$R = \frac{v^2}{a} = \frac{v^2}{9g} = \frac{(300\,m/s)^2}{9 \times 9.8\,m/s^2}$$

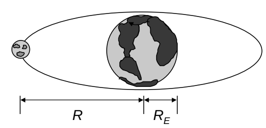
$$R = \frac{10000}{9.8} m \approx 1000 m$$



Answer (c)

Example: Newton & the Moon

- What is the acceleration of the Moon due to its motion around the earth?
  - ❖T = 27.3 days = 2.36 x 10^6 s (period ~ 1 month)
  - ❖R = 3.84 x 10^8 m (distance to moon)
  - ❖R_E = 6.35 x 10^6 m (radius of earth)



Moon...

- Calculate angular frequency:

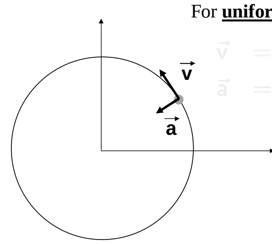
$$\frac{1}{27.3}\frac{\text{rot}}{\text{day}} \text{ x } \frac{1}{86400}\frac{\text{day}}{\text{s}} \text{ x } 2\pi\frac{\text{rad}}{\text{rot}} = 2.66 \times 10^{-6} \text{ rad s}^{-1}$$

- So $\omega = 2.66 \times 10^{-6}$ rad s^{-1}.
- Now calculate the acceleration.
  - ❖ $a = \omega^2 R = 0.00272$ m/s² = .000278 g
  - ❖direction of ***a*** is toward center of earth (-**R**).

Radial and Tangential Quantities

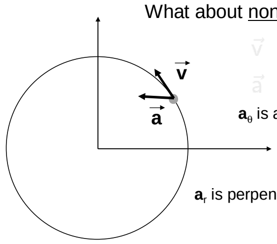
For **uniform** circular motion

$$\vec{v} = 0\hat{r} + v\hat{\theta}$$
$$\vec{a} = a\hat{r} + 0\hat{\theta}$$



Radial and Tangential Quantities

What about <u>non-uniform</u> circular motion ?



$$\vec{v} = 0\hat{r} + v\hat{\theta}$$

$$\vec{a} = a_r\hat{r} + a_\theta\hat{\theta}$$

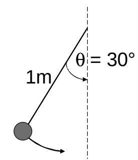
$\mathbf{a}_\theta$ is along the direction of motion

$$a_\theta = \frac{d|v|}{dt}$$

$\mathbf{a}_r$ is perpendicular to the direction of motion

$$a_r = \frac{v^2}{r}$$

Lecture 5, Exercise 2
The Pendulum



Which statement best describes the motion of the pendulum bob at the instant of time drawn ?

i. the bob is at the **top** of its swing.

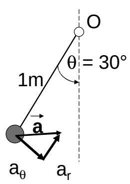
ii. which quantities are non-zero ?

A) $v_r = 0$ $a_r = 0$ $v_\theta \neq 0$ $a_\theta \neq 0$

B) $V_r = 0$ $a_r \neq 0$ $v_\theta \neq 0$ $a_\theta = 0$

C) $v_r = 0$ $a_r \neq 0$ $v_\theta = 0$ $a_\theta \neq 0$

Lecture 5, Exercise 2
The Pendulum
Solution



NOT uniform circular motion :
is circular motion so must be a_r not zero,
Speed is increasing so a_θ not zero

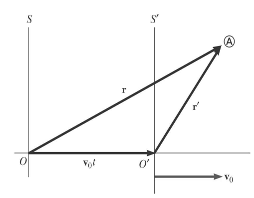
At the top of the swing, the bob temporarily stops, so $\mathbf{v} = 0$.

C) $v_r = 0$ $a_r \neq 0$ $v_\theta = 0$ $a_\theta \neq g$

In the next lecture we will learn about forces and how to calculate just what **a** is.

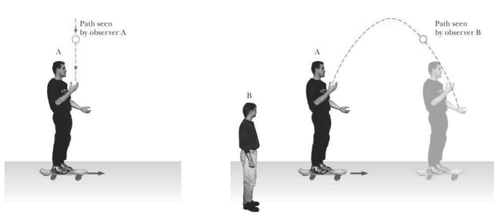
Relative motion and frames of reference

- Reference frame *S* is stationary
- Reference frame *S'* is moving at $\mathbf{v}_o$
  - ❖ This also means that *S* moves at $-\mathbf{v}_o$ relative to *S'*
- Define time $t = 0$ as that time when the origins coincide



Relative Velocity

- Two observers moving relative to each other generally do not agree on the outcome of an experiment
- For example, observers A and B below see different paths for the ball



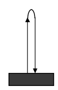
Relative Velocity, equations

- The positions as seen from the two reference frames are related through the velocity
  - ❖ $\mathbf{r}' = \mathbf{r} - \mathbf{v}_o\, t$
- The derivative of the position equation will give the velocity equation
  - ❖ $\mathbf{v}' = \mathbf{v} - \mathbf{v}_o$
- These are called the **Galilean transformation equations**

Central concept for problem solving: "*x*" and "*y*" components of motion treated independently.

- Again: man on the cart tosses a ball straight up in the air.
- You can view the trajectory from two reference frames:

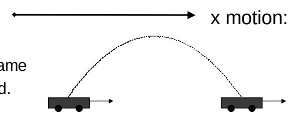
Reference frame on the moving train.



y(t) motion governed by
1) ***a*** = -g ***y***
2) $v_y = v_{0y} - g\,t$
3) $y = y_0 + v_{0y} - g\,t^2/2$

x motion: $x = v_x t$

Reference frame on the ground.



Net motion: **R** = *x(t)* **i** + *y(t)* **j** (vector)

Acceleration in Different Frames of Reference

- The derivative of the velocity equation will give the acceleration equation
  - ❖ $\mathbf{v}' = \mathbf{v} - \mathbf{v}_o$
  - ❖ $\mathbf{a}' = \mathbf{a}$
- The acceleration of the particle measured by an observer in one frame of reference is the same as that measured by any other observer moving at a *constant velocity* relative to the first frame.

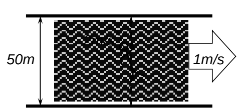
Lecture 5, *Exercise 3*
Relative Motion

- You are swimming across a *50* m wide river in which the current moves at *1* m/s with respect to the shore. Your swimming speed is *2* m/s with respect to the water.

You swim across in such a way that your path is a straight perpendicular line across the river.

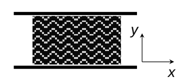
- How many seconds does it take you to get across?

a) $50/2 = 25$ s
b) $50/1 = 50$ s
c) $50/\sqrt{3} = 29$ s
d) $50/\sqrt{2} = 35$ s

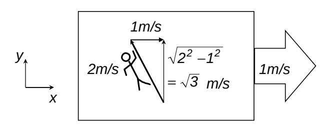


Lecture 5, *Exercise 3*
Solution

Choose *x* axis along riverbank and *y* axis across river

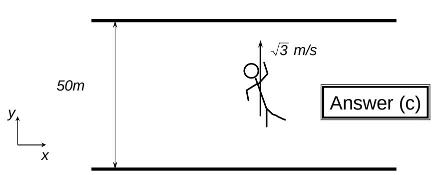


- The time taken to swim straight across is *(distance across) / (v_y)*
- Since you swim straight across, you must be tilted in the water so that your *x* component of velocity with respect to the water exactly cancels the velocity of the water in the *x* direction:



Lecture 5, *Exercise 3*
Solution

- The *y* component of your velocity with respect to the water is $\sqrt{3}$ m/s
  - The time to get across is $\frac{50m}{\sqrt{3}\,m/s} = 29s$



Recap

First mid-term exam in just two weeks, Thursday Oct. 5

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  - ❖ Kinematics in 2 or 3 dimensions
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  - ❖ Curved paths and projectile motion
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