Physics 207 – Lecture 5

Agenda
- Chapter 4
  - Kinematics in 2 or 3 dimensions
  - Independence of x, y and/or z components
  - Circular motion
  - Curved paths and projectile motion
- Frames of reference
- Radial and tangential acceleration

Assignment: For Monday read Chapter 5 and look at Chapter 6
- WebAssign Problem Set 2 due Tuesday next week (start ASAP)

D Kinematics

3-D Kinematics
- The position, velocity, and acceleration of a particle in 3-dimensions can be expressed as:
  \[ r = x\hat{i} + y\hat{j} + z\hat{k} \]
  \[ v = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} \]
  \[ a = a_x\hat{i} + a_y\hat{j} + a_z\hat{k} \]

- The instantaneous acceleration is a vector with components
  - The speed is a scalar

- Which can be combined into the vector equations:
  \[ r = r(t) \quad v = dr/dt \quad a = d^2r/dt^2 \]

Instantaneous Velocity
- The instantaneous velocity is the limit of the average velocity as \( \Delta t \) approaches zero
- The direction of the instantaneous velocity is along a line that is tangent to the path of the particle’s direction of motion.

Instantaneous Acceleration
- The instantaneous acceleration is the limit of the average acceleration as \( \Delta t \) approaches zero

Average Acceleration
- The average acceleration of a particle as it moves is defined as the change in the instantaneous velocity vector divided by the time interval during which that change occurs.

Motion along a path (displacement, velocity, acceleration)
- 3-D Kinematics: vector equations:
  \[ \vec{r} = \vec{r}(t) \quad \vec{v} = d\vec{r}/dt \quad \vec{a} = d^2\vec{r}/dt^2 \]

- 3-D Kinematics:
  \[ \vec{v}_2 - \vec{v}_1 = \frac{\Delta \vec{v}}{\Delta t} \]

- Acceleration:
  \[ \vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} \quad \vec{a} = \frac{d\vec{v}}{dt} \]
General 3-D motion with non-zero acceleration:

\[ \vec{a} 
eq 0 \]

Two possible options:
- Change in the magnitude of \( \vec{v} \)
- Change in the direction of \( \vec{v} \)

Uniform Circular Motion

- What does it mean?
- How do we describe it?
- What can we learn about it?

Average acceleration in UCM:
- Even though the speed is constant, velocity is not constant since the direction is changing; must be some acceleration!
- Consider average acceleration in time \( \Delta t \): 

\[ \vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} \]

seems like \( \Delta \vec{v} \) (hence \( \Delta \omega \Delta t \)) points toward the origin!

Instantaneous acceleration in UCM:
- Again: Even though the speed is constant, velocity is not constant since the direction is changing.
- As \( \Delta t \) goes to zero in \( \frac{\Delta \vec{v}}{\Delta t} \): 

\[ \frac{d\vec{v}}{dt} = \vec{a} \]

Now \( \vec{a} \) points in the -R direction.

Acceleration in UCM:
- This is called Centripetal Acceleration.
- Calculating the magnitude:
- \( |\vec{v}| = |\vec{v}_2| = \vec{v} \)

\[ \frac{\Delta \vec{v}}{\Delta t} \]

Similar triangles:

\[ \frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta \vec{R}}{\Delta t} \]

But \( \Delta R = \omega \Delta t \) for small \( \Delta t \)

So: 

\[ \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v} \Delta \omega}{R} \]

\[ \vec{a} = \frac{\vec{v} \Delta \omega}{R} \]

Period and Frequency
- Recall that 1 revolution = 2\( \pi \) radians
  - Period (T) = seconds / revolution = distance / speed = 2\( \pi \)R / v
  - Frequency (f) = revolutions / second = 1/T (a)
  - Angular velocity (\( \omega \)) = radians / second (b)
- By combining (a) and (b)
  - \( \omega = 2\pi f \)
- Realize that:
  - Period (T) = seconds / revolution
  - So \( T = \frac{1}{f} = 2\pi \omega \)

\[ \omega = \frac{2\pi}{T} = 2\pi f \]
Recap: Centripetal Acceleration

- UCM results in acceleration:
  - Magnitude: \( a = \frac{v^2}{R} = \omega^2 R \)
  - Direction: toward center of circle

![Diagram of centripetal acceleration](physics207-lecture5-pg12)

Lecture 5, Exercise 1
Uniform Circular Motion

- A fighter pilot flying in a circular turn will pass out if the centripetal acceleration he experiences is more than about 9 times the acceleration of gravity \( g \). If his F-18 is moving at a speed of \( 300 \text{ m/s} \), what is the approximate radius of the tightest turn this pilot can make and survive to tell about it? (Let \( g = 10 \text{ m/s}^2 \))

UCM (recall)
- Magnitude: \( a = \frac{v^2}{R} \)
- Direction: toward center of circle

Exercise 1
- Solution
  - (a) \( 100 \text{ m} \)
  - (b) \( 1000 \text{ m} \)
  - (c) \( 10,000 \text{ m} \)
  - (d) \( 100,000 \text{ m} \)

Example: Newton & the Moon

- What is the acceleration of the Moon due to its motion around the earth?
  - \( T = 27.3 \text{ days} = 2.36 \times 10^6 \text{ s} \) (period ~ 1 month)
  - \( R = 3.84 \times 10^8 \text{ m} \) (distance to moon)
  - \( R_E = 6.35 \times 10^6 \text{ m} \) (radius of earth)

Moon...

- Calculate angular frequency:
  \[
  \omega = \frac{1 \text{ rot}}{27.3 \text{ day}} = \frac{1 \text{ day}}{86400 \text{ s}} \times \frac{2\pi \text{ rad}}{\text{rot}} = 2.66 \times 10^{-5} \text{ rad s}^{-1}
  \]

- So \( \omega = 2.66 \times 10^{-5} \text{ rad s}^{-1} \).
- Now calculate the acceleration.
  - \( a = \omega^2 R = 0.00272 \text{ m/s}^2 = 0.000272 \text{ g} \)
  - direction of \( a \) is toward center of earth (-\( R \)).
Radial and Tangential Quantities

What about non-uniform circular motion?
\[
\ddot{\mathbf{r}} = \mathbf{a}_r 
\]
\[
\dot{\mathbf{t}} = \mathbf{a}_t 
\]
\[
\mathbf{a}_r \text{ is along the direction of motion} 
\]
\[
\mathbf{a}_t \text{ is perpendicular to the direction of motion} 
\]

Lecture 5, Exercise 2
The Pendulum

Relative velocity and frames of reference

- Reference frame S is stationary
- Reference frame S' is moving at \( v_r \)
- This also means that S moves at \(-v_r\) relative to \( S' \)
- Define time \( t = 0 \) as that time when the origins coincide

Relative Velocity

- Two observers moving relative to each other generally do not agree on the outcome of an experiment
- For example, observers A and B below see different paths for the ball

Relative Velocity, equations

- The positions as seen from the two reference frames are related through the velocity
  \[
  \mathbf{r}' = \mathbf{r} - \mathbf{v}_r \, t 
  \]
- The derivative of the position equation will give the velocity equation
  \[
  \dot{\mathbf{v}}' = \mathbf{v}' - \mathbf{v}_r 
  \]
- These are called the Galilean transformation equations
Central concept for problem solving: “x” and “y” components of motion treated independently.
- Again: man on the cart tosses a ball straight up in the air.
- You can view the trajectory from two reference frames:

\[
\begin{align*}
1) & \quad a = -g \\
2) & \quad v_y = v_{0y} - gt \\
3) & \quad y = y_0 + v_{0y} - \frac{1}{2}gt^2
\end{align*}
\]

x motion: \(x = v_xt\)

Net motion: \(R = x(t) \hat{i} + y(t) \hat{j}\) (vector)

Again: man on the cart tosses a ball straight up in the air. You can view the trajectory from two reference frames:

Reference frame on the ground.

Reference frame on the moving train.

\[
\begin{align*}
\text{y(t) motion governed by} \\
1) & \quad a = -g \\
2) & \quad v_y = v_{0y} - gt \\
3) & \quad y = y_0 + v_{0y} - \frac{1}{2}gt^2
\end{align*}
\]

\[
\begin{align*}
x \text{ motion: } & \quad x = v_xt \\
\text{Net motion: } & \quad R = x(t) \hat{i} + y(t) \hat{j} \text{ (vector)}
\end{align*}
\]

Acceleration in Different Frames of Reference
- The derivative of the velocity equation will give the acceleration equation

\[
\begin{align*}
\dot{v} &= \dot{v}_0 - \dot{v}_0 \\
\dot{a} &= \ddot{a}_0
\end{align*}
\]

- The acceleration of the particle measured by an observer in one frame of reference is the same as that measured by any other observer moving at a constant velocity relative to the first frame.

Lecture 5, Exercise 3
Relative Motion
- You are swimming across a 50 m wide river in which the current moves at 1 m/s with respect to the shore. Your swimming speed is 2 m/s with respect to the water.
- How many seconds does it take you to get across?

\[
\begin{align*}
a) & \quad 50/2 = 25 \text{ s} \\
b) & \quad 50/1 = 50 \text{ s} \\
c) & \quad 50/\sqrt{3} = 29 \text{ s} \\
d) & \quad 50/\sqrt{2} = 35 \text{ s}
\end{align*}
\]

Lecture 5, Exercise 3
Solution
- The y component of your velocity with respect to the water is \(\sqrt{3}\) m/s
- The time to get across is \(\frac{50}{\sqrt{3}\text{ m/s}} = 29\text{ s}\)

Answer (c)

Recap
- First mid-term exam in just two weeks, Thursday Oct. 5
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