Physics 207, Lecture 11, Oct. 11

Agenda: Chapter 7, finish, Chapter 8, Potential Energy
- Work-Energy Theorem
- Work and Friction
- Power
  - ❖ $P = dW / dt = \mathbf{F} \cdot \mathbf{v}$ (a vector product!)
- Potential Energy
  - Conservative Forces and Potential Energy ($W = -\Delta U$)
  - Non-conservative Forces
  - Generalized Work Energy Theorem

Assignment: For Monday read Chapter 9
- WebAssign Problem Set 4 due Tuesday next week

See text: 7-4

Work Kinetic-Energy Theorem:

{***Net Work*** done on object}
=
{***change*** in ***kinetic energy*** of object}

$$\boxed{W_{net} = \Delta K}$$

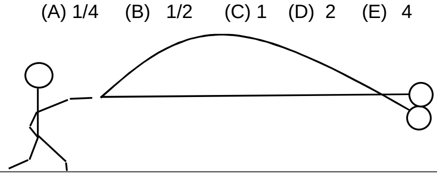
$$= K_2 - K_1 \qquad \text{(final – initial)}$$

$$= \frac{1}{2} m v_2^{\,2} - \frac{1}{2} m v_1^{\,2}$$

Lecture 11, *Exercise 1*
Kinetic Energy (= ½ mv^2)

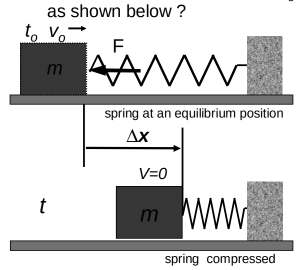
- To practice your pitching you use two baseballs. The first time you throw a slow curve and clock the speed at 50 mph (~25 m/s). The second time you go with high heat and the radar gun clocks the pitch at 100 mph. What is the ratio of the kinetic energy of the fast ball versus the curve ball ?

(A) 1/4 (B) 1/2 (C) 1 (D) 2 (E) 4



Example: Work Kinetic-Energy Theorem

- How much will the spring compress (i.e. $\Delta \mathbf{x}$) to bring the object to a stop (i.e., $v = 0$) if the object is moving initially at a constant velocity (v_o) on frictionless surface as shown below ?



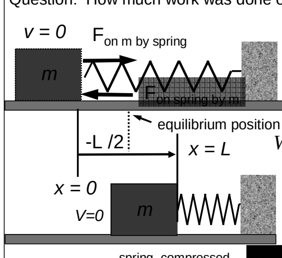
Notice that the spring force is opposite to the displacemant.

For the mass m, work is negative

For the spring, *work is positive*

Another Example

A mass m starts at rest and is attached to a spring which is expanded a distance ***L / 2*** to the left of its equilibrium position and then is allow to move a distance ***L*** to the right where the mass is once again at rest.
Question: How much work was done on the box? (Notice $\Delta K = 0$)



$$W_{\text{box}} = \int_{x_i}^{x_f} F(x)\,dx$$

$$W_{\text{box}} = \int_0^L -kx\,dx$$

$$W_{\text{box}} = -\tfrac{1}{2} kx^2 \Big|_0^L = -kL^2 ???$$

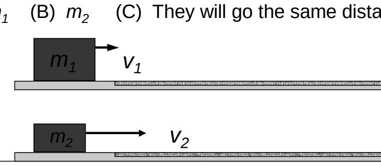
This doesn't make sense!

$$W_{\text{box}} = \int_{-L/2}^{L/2} -kx\,dx$$

Lecture 11, *Exercise 2*
Work & Friction

- Two blocks having mass m_1 and m_2 where $\mathbf{m_1 > m_2}$. They are sliding on a frictionless floor and have the <u>same kinetic energy</u> when they encounter a long rough stretch (i.e. $\mu > 0$) which slows them down to a stop.
- Which one will go farther before stopping?
- **Hint:** How much work does friction do on each block ?

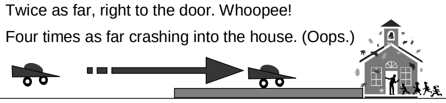
(A) m_1 (B) m_2 (C) They will go the same distance



Lecture 11, *Exercise 3*
Work & Friction

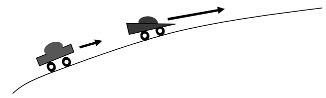
- You like to drive home fast, slam on your brakes at the start of the driveway, and screech to a stop "laying rubber" all the way. It's particularly fun when your mother is in the car with you. You practice this trick driving at 20 mph and with some groceries in your car with the same mass as your mother. You find that you only travel half way up the driveway. Thus when your mom joins you in the car, you try it driving twice as fast. How far will you go this time ?

A. The same distance. Not so exciting.
B. $\sqrt{2}$ times as far (only ~7/10 of the way up the driveway)
C. Twice as far, right to the door. Whoopee!
D. Four times as far crashing into the house. (Oops.)



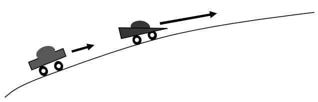
Work & Power:

- Two cars go up a hill, a Corvette and a ordinary Chevy Malibu. Both cars have the same mass.
- Assuming identical friction, both engines do the same amount of work to get up the hill.
- Are the cars essentially the same ?
- **NO.** The Corvette gets up the hill quicker
- It has a more **powerful** engine.



Work & Power:

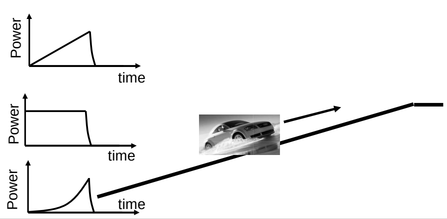
- Power is the rate at which work is done.
- Average Power is, $\overline{P} = \dfrac{W}{\Delta t}$
- Instantaneous Power is, $P = \dfrac{dW}{dt} = \vec{F} \bullet \vec{V}$



Lecture 11, *Exercise 4*
Work & Power

- Starting from rest, a car drives up a hill at constant acceleration and then suddenly stops at the top. The instantaneous power delivered by the engine during this drive looks like which of the following,

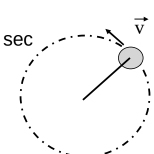
- (A)
- (B)
- (C)



Lecture 11, *Exercise 5*
Power for Circular Motion

- I swing a sling shot over my head. The tension in the rope keeps the shot moving in a circle. How much power must be provided by me, through the rope tension, to keep the shot in circular motion ?

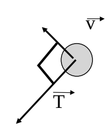
Note that: Rope Length = 1m
Shot Mass = 1 kg
Angular frequency = 2 rad / sec



(A) 16 J/s (B) 8 J/s (C) 4 J/s (D) 0

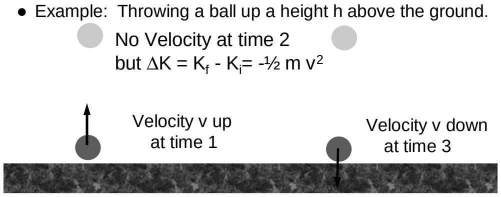
Lecture 11, *Exercise 5*
Power for Circular Motion

- Note that the string expends no power (because it does no work).
- By the work / kinetic energy theorem, work done equals change in kinetic energy.
- $K = 1/2\ mv^2$, thus since v doesn't change, neither does K.
- A force perpendicular to the direction of motion does not change speed, $|v|$, and so does no work.
- Answer is **(D)**



Chapter 8, Potential Energy

- What is "Potential Energy" ?
  It is a way of effecting energy transfer in a system so that it can be "recovered" (i.e. transferred out) at a later time or place.
- Example: Throwing a ball up a height h above the ground.



Energy is conserved and so work must have been done

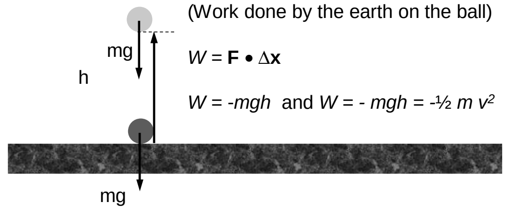
We can identify this work:

- Consider a ball moving up to height h above the ground. (from time 1 to time 2)

What work is done in this process ? (Work done by the earth on the ball)

$W = \mathbf{F} \bullet \Delta\mathbf{x}$

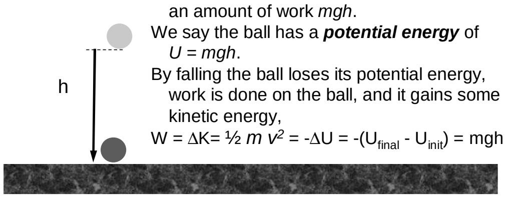
$W = -mgh$ and $W = -mgh = -½ m v^2$



Potential Energy

- Now consider the ball starting a height h above the ground and falling (from time 2 to time 3)

Before the ball falls it has the *potential* to do an amount of work *mgh*.
We say the ball has a ***potential energy*** of $U = mgh$.
By falling the ball loses its potential energy, work is done on the ball, and it gains some kinetic energy,
$W = \Delta K = ½ m v^2 = -\Delta U = -(U_{final} - U_{init}) = mgh$



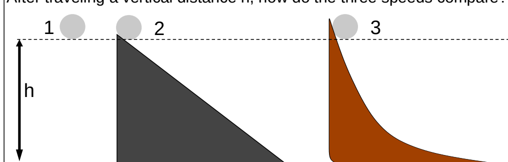
Notice that potential is measured as a difference with respect to an arbitrary reference (here it is the ground).

Potential Energy, Energy Transfer and Path

- A ball of mass m, initially at rest, is released and follows three difference paths. All surfaces are frictionless

1. The ball is dropped
2. The ball slides down a straight incline
3. The ball slides down a curved incline

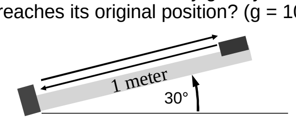
After traveling a vertical distance h, how do the three speeds compare?



(A) 1 > 2 > 3 (B) 3 > 2 > 1 (C) 3 = 2 = 1 (D) Can't tell

Lecture 11, *Exercise 7*
Work Done by Gravity

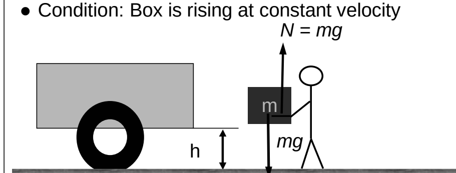
- An frictionless track is at an angle of 30° with respect to the horizontal. A cart (mass 1 kg) is released. It slides 1 meter downwards along the track bounces and then slides upwards to its original position.
- How much total work is done by gravity on the cart when it reaches its original position? (g = 10 m/s²)



(A) 5 J (B) 10 J (C) 20 J (D) 0 J

Work Done (by the person) Against <u>Gravity</u>

- Consider lifting a box onto the tail gate of a truck.
- Condition: Box is rising at constant velocity



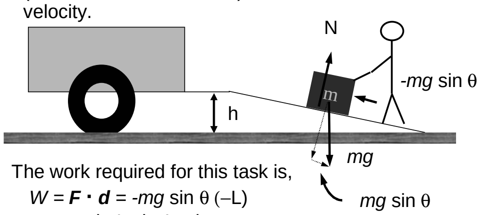
The work required for this task is,
$W = \mathbf{F} \cdot \mathbf{d} = N\,h$
$W = mgh$
Note: Holding the box level involves NO work

Work Done Against Gravity

- Now use a ramp, of length L, to help you with the task.

  Is less work needed to get the box into the truck?

  (It's "easier" to lift the box) and it moves at constant velocity.



The work required for this task is,

$W = \boldsymbol{F} \cdot \boldsymbol{d} = -mg \sin\theta\,(-L)$

and $L \sin\theta = h$

$W = mgh$ → Work is identical but force is not.

Important Definitions

- **Conservative Forces** - Forces for which the work done does not depend on the path taken, but only the initial and final position (no loss).
- **Potential Energy** - describes the amount of work that can potentially be done by one object on another under the influence of a conservative force

  - $W = -\Delta U$

  Only differences in potential energy matter.

Potential Energy

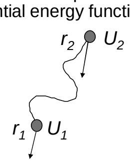
- For any conservative force F we can define a potential energy function U in the following way:

$$W = \int \boldsymbol{F} \cdot d\boldsymbol{r} = -\Delta U$$

  - The work done by a conservative force is equal and opposite to the change in the potential energy function.

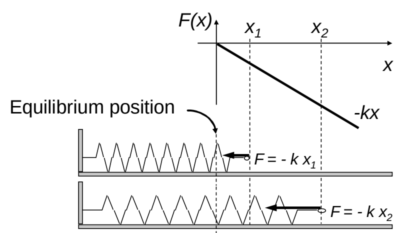
- This can be written as:

$$\Delta U = U_2 - U_1 = -W = -\int_{r_1}^{r_2} \boldsymbol{F} \cdot d\boldsymbol{r}$$



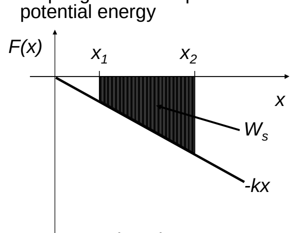
A Conservative Force Example: Hooke's Law Spring

- For a spring we know that $F_x = -kx$.



Spring...

- The work done *by the spring* W_s during a displacement from x_1 to x_2 is the area under the $F(x)$ vs x plot between x_1 and x_2.
- A spring under compression or tension has stored potential energy



Active Figure

See text: 8-4

Conservation of Energy

- If only conservative forces are present, the total energy (sum of potential and kinetic energies) of a system is conserved.

  $E = K + U$

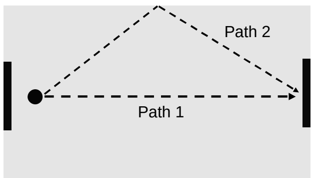
  $\boxed{E = K + U \text{ is } \textbf{\textit{constant !!!}}}$

- Both K and U can change, but $E = K + U$ remains constant.

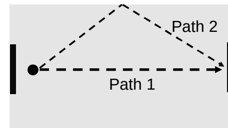
E is called "mechanical energy"

A Non-Conservative Force
Friction

- Looking down on an air-hockey table with no air flowing ($\mu > 0$).
- Now compare two paths in which the puck starts out with the same speed ($K_1 = K_2$) .



A Non-Conservative Force



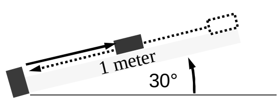
Since $path_2$ distance $>path_1$ distance the puck will be traveling slower at the end of path 2.

Work done by a non-conservative force irreversibly removes energy out of the “system”.

Here $W_{NC} = E_{final} - E_{initial} < 0$

Lecture 11, *Exercise 8*
Work/Energy for Non-Conservative Forces

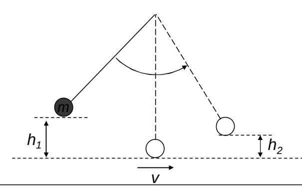
- The air track is once again at an angle of 30° with respect to horizontal. The cart (with mass 1 kg) is released 1 meter from the bottom and hits the bumper at a speed, v_1. This time the vacuum/ air generator breaks half-way through and the air stops. The cart only bounces up half as high as where it started.
- How much work did friction do on the cart ?($g=10$ m/s²)



(A) 2.5 J (B) 5 J (C) 10 J (D) –2.5 J (E) –5 J (F) –10 J

Another example of a conservative system: The simple pendulum.

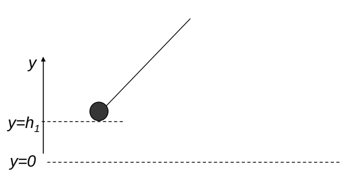
- Suppose we release a mass m from rest a distance h_1 above its lowest possible point.
  - What is the maximum speed of the mass and where does this happen ?
  - To what height h_2 does it rise on the other side ?



Example: The simple pendulum.

- What is the maximum speed of the mass and where does this happen ?

  E = K + U = constant and so K is maximum when U is a minimum.



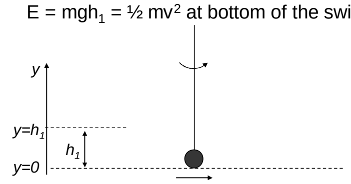
Example: The simple pendulum.

- What is the maximum speed of the mass and where does this happen ?

  E = K + U = constant and so K is maximum when U is a minimum

  $E = mgh_1$ at top

  $E = mgh_1 = \frac{1}{2} mv^2$ at bottom of the swing

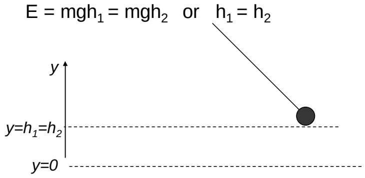


Example: The simple pendulum.

To what height *h2* does it rise on the other side?

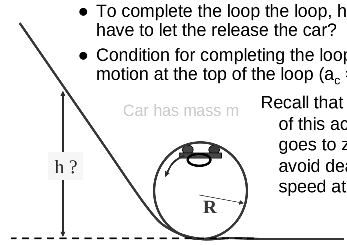
E = K + U = constant and so when U is maximum again (when K = 0) it will be at its highest point.

$E = mgh_1 = mgh_2$ or $h_1 = h_2$



Lecture 11, *Exercise 9*
The Loop the Loop … again

- To complete the loop the loop, how high do we have to let the release the car?
- Condition for completing the loop the loop: Circular motion at the top of the loop ($a_c = v^2 / R$)



Recall that "g" is the source of this acceleration and N goes to zero. that to avoid death, the minimum speed at the top is $v = \sqrt{gR}$

(A) 2R (B) 3R (C) 5/2 R (D) $2^{3/2}$ R

See text: 8.5

Non-conservative Forces :

- If the work done does not depend on the path taken, the force involved is said to be conservative.
- If the work done does depend on the path taken, the force involved is said to be non-conservative.
- An example of a non-conservative force is friction:
- Pushing a box across the floor, the amount of work that is done by friction depends on the path taken.
  - Work done is proportional to the length of the path !

Generalized Work Energy Theorem:

- Suppose $F_{NET} = F_C + F_{NC}$ (sum of conservative and non-conservative forces).
- The total work done is: $W_{TOT} = W_C + W_{NC}$
- The Work Kinetic-Energy theorem says that: $W_{TOT} = \Delta K$.
  - $W_{TOT} = W_C + W_{NC} = \Delta K$
  - $W_{NC} = \Delta K - W_C$
- But $W_C = -\Delta U$

So $\boxed{W_{NC} = \Delta K + \Delta U = \Delta E}$

Physics 207, Lecture 11, Recap

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- Power
  - $P = dW / dt = \mathbf{F} \cdot \mathbf{v}$ (a vector product!)
- Potential Energy
  - Conservative Forces and Potential Energy ($W = -\Delta U$)
  - Non-conservative Forces
  - Generalized Work Energy Theorem

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