Physics 207, Lecture 12, Oct. 16

Agenda: Chapter 8, finish, Chapter 9

- Ch. 8: Generalized Work Energy Theorem
- Ch. 8: Energy Minimum
- Chapter 9: Momentum and Collision
  - ❖ Momentum conservation
  - ❖ Collisions
  - ❖ Systems of particles
  - ❖ Impulse
  - ❖ Center of mass (Wednesday)

Assignment: For Wednesday read through Chapter 10

- WebAssign Problem Set 4 due Tuesday, Problem Set 5 up today

See text: 8-4

Conservation of Energy

- If only conservative forces are present, the total energy (sum of potential and kinetic energies) of a system is conserved.

$E = K + U$

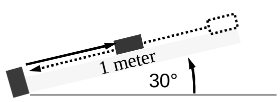
$E = K + U$ is ***constant !!!***

- Both K and U can change, but $E = K + U$ remains constant.

E is called "mechanical energy"

Lecture 12, *Exercise 1*
Work/Energy for Non-Conservative Forces

- The air track is once again at an angle of 30° with respect to horizontal. The cart (with mass 1 kg) is released 1 meter from the bottom and hits the bumper at a speed, v_1. This time the vacuum/ air generator breaks half-way through and the air stops. The cart only bounces up half as high as where it started.
- How much work did friction do on the cart ?(g=10 m/s²)



(A) 2.5 J (B) 5 J (C) 10 J (D) –2.5 J (E) –5 J (F) –10 J

Lecture 12, *Exercise 1*
Work/Energy for Non-Conservative Forces

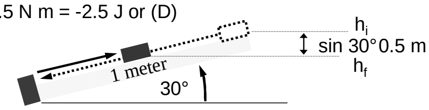
- How much work did friction do on the cart ? (g=10 m/s²)

W = F Δx is not easy to do…

- Work done is equal to the change in the energy of the system (U and/or K). $E_{final} - E_{initial}$ and is < 0. (E = U+K)

Use $W = U_{final} - U_{init} = mg (h_f - h_i) = - mg \sin 30° 0.5$ m

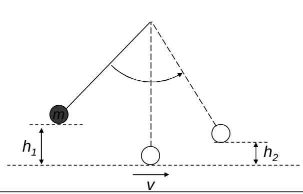
W = -2.5 N m = -2.5 J or (D)



(A) 2.5 J (B) 5 J (C) 10 J **(D) –2.5 J** (E) –5 J (F) –10 J

Another example of a conservative system: The simple pendulum.

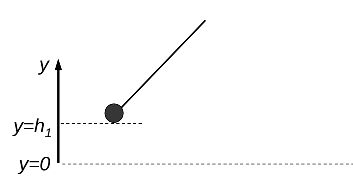
- Suppose we release a mass m from rest a distance h_1 above its lowest possible point.
  - ❖ What is the maximum speed of the mass and where does this happen ?
  - ❖ To what height h_2 does it rise on the other side ?



Example: The simple pendulum.

- ❖ What is the maximum speed of the mass and where does this happen ?

E = K + U = constant and so K is maximum when U is a minimum.



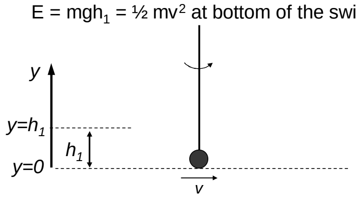
Example: The simple pendulum.

- What is the maximum speed of the mass and where does this happen ?

  E = K + U = constant and so K is maximum when U is a minimum

  $E = mgh_1$ at top

  $E = mgh_1 = \frac{1}{2} mv^2$ at bottom of the swing

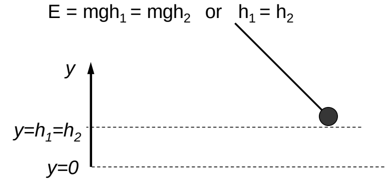


Example: The simple pendulum.

To what height *h2* does it rise on the other side?

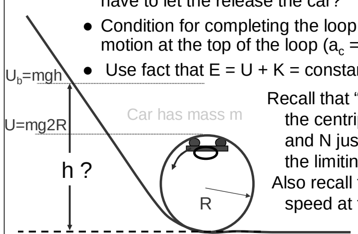
E = K + U = constant and so when U is maximum again (when K = 0) it will be at its highest point.

$E = mgh_1 = mgh_2$ or $h_1 = h_2$



Lecture 12, *Exercise 2*
The Loop-the-Loop … again

- To complete the loop the loop, how high do we have to let the release the car?
- Condition for completing the loop the loop: Circular motion at the top of the loop ($a_c = v^2 / R$)
- Use fact that E = U + K = constant !



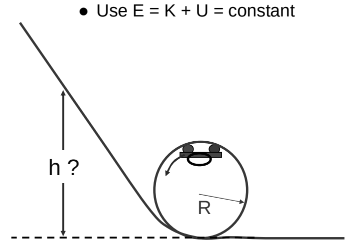
Recall that "g" is the source of the centripetal acceleration and N just goes to zero is the limiting case.
Also recall the minimum speed at the top is $v = \sqrt{gR}$

(A) 2R (B) 3R (C) 5/2 R (D) $2^{3/2}$ R

Lecture 12, *Exercise 2*
The Loop-the-Loop … again

- Use E = K + U = constant

$v = \sqrt{gR}$



(A) 2R (B) 3R **(C) 5/2 R** (D) $2^{3/2}$ R

See text: 8.5

Non-conservative Forces :

- If the work done does not depend on the path taken, the force involved is said to be conservative.
- If the work done does depend on the path taken, the force involved is said to be non-conservative.
- An example of a non-conservative force is friction:
- Pushing a box across the floor, the amount of work that is done by friction depends on the path taken.
  - Work done is proportional to the length of the path !
  - Friction is associated with negative work and an irreversible loss in the mechanical energy

Generalized Work Energy Theorem:

- Suppose $F_{NET} = F_C + F_{NC}$ (sum of conservative and non-conservative forces).
- The total work done is: $W_{TOTAL} = W_C + W_{NC}$
- The Work Kinetic-Energy theorem says that: $W_{TOTAL} = \Delta K$.
  - $W_{TOTAL} = W_C + W_{NC} = \Delta K$
  - $W_{NC} = \Delta K - W_C$
- But $W_C = -\Delta U$

So $W_{NC} = \Delta K + \Delta U = \Delta E$

See Text: section 8.6

Conservative Forces and Potential Energy

- We have defined potential energy for conservative forces
  $\Delta U = - W$
- But we also now that (in one-dimensional motion)
  $W = F_x \Delta x$
- Combining these two,
  $\Delta U = - F_x \Delta x$
- Letting small quantities go to infinitesimals (ΔU, $\Delta x \rightarrow 0$),
  $dU = - F_x dx$
- Or, in this limit,
  $F_x = -dU / dx$

There is a fundamental relationship between the potential energy and the force.

See Text: section 8.6

Example of the U - F relationship

- For a Hooke's Law spring,
  - ❖ $U(x) = (1/2)kx^2$
- Notice that the derivative gives Hooke's Law
  - ❖ $F_x = - d\ ((1/2)kx^2) / dx$
  - ❖ $F_x = - 2\ (1/2)\ kx$
  - ❖ $F_x = -kx$

Equilibrium

- Example
  - ❖ Spring: $F_x = 0$ => $dU / dx = 0$ for x=0

  The spring is in equilibrium position
- In general: $dU / dx = 0 \rightarrow$ for ANY function establishes equilibrium

stable equilibrium unstable equilibrium

Chapter 9: Linear Momentum

- **Definition:** For a single particle, the momentum ***p*** is defined as:

$$\boldsymbol{p} \equiv m\boldsymbol{v}$$

(***p*** is a vector since ***v*** is a vector)

So $p_x = mv_x$ etc.

- Newton's 2nd Law:

F = m***a***

$$= m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) \quad \Rightarrow \quad \vec{F} = \frac{d\vec{p}}{dt}$$

- Units of linear momentum are kg m/s.

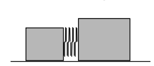
Momentum Conservation

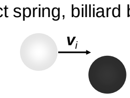
$$\boldsymbol{F}_{EXT} = \frac{d\boldsymbol{P}}{dt} \Rightarrow \frac{d\boldsymbol{P}}{dt} = 0 \Leftarrow \boldsymbol{F}_{EXT} = 0$$

- Momentum conservation (recasts Newton's 2nd Law when **F** = 0) is a fundamentally important principle.
- This is a component (vector) equation (P_x, P_y and P_z) .
  - ❖ Applicable in any situation in which there is no net external force applied.
- Many problems can be addressed through momentum conservation even if (mechanical) energy is not conserved (i.e., a non-conservative internal force exists).

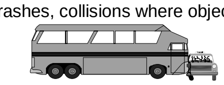
Elastic vs. Inelastic Collisions

- A collision is said to be *elastic* when energy as well as momentum is conserved before and after the collision. $K_{before} = K_{after}$
  - ❖ Carts colliding with a perfect spring, billiard balls, etc.



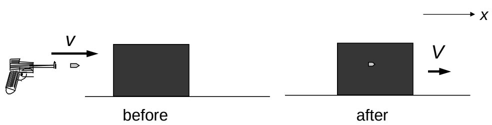


- A collision is said to be *inelastic* when energy is not conserved before and after the collision, but momentum is conserved. $K_{before} \neq K_{after}$
  - ❖ Car crashes, collisions where objects stick together, etc.



Inelastic collision in 1-D: Example 1

- A block of mass *M* is initially at rest on a frictionless horizontal surface. A bullet of mass *m* is fired at the block with a muzzle velocity (speed) *v*. The bullet lodges in the block, and the block ends up with a speed *V*. In terms of *m*, *M*, and *V* :
  - ❖ What is the momentum of the bullet with speed *v* ?
  - ❖ What is the initial energy of the system ?
  - ❖ What is the final energy of the system ?
  - ❖ Is energy conserved?



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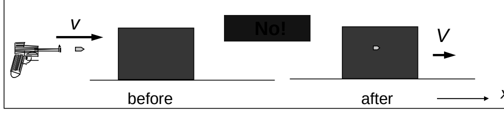
Inelastic collision in 1-D: Example 1

What is the momentum of the bullet with speed *v* ? $m\vec{v}$

- ❖ What is the initial energy of the system ? $\frac{1}{2}m\vec{v}\bullet\vec{v} = \frac{1}{2}mv^2$
- ❖ What is the final energy of the system ? $\frac{1}{2}(m+M)V^2$
- ❖ Is momentum conserved (yes)? $mv + M\,0 = (m+M)V$
- ❖ Is energy conserved? Examine E_{before}-E_{after}

$$\frac{1}{2}mv^2 - \frac{1}{2}[(m+M)V]V = \frac{1}{2}mv^2 - \frac{1}{2}(mv)\frac{m}{m+M}v = \frac{1}{2}mv^2\left(1-\frac{m}{m+M}\right)$$



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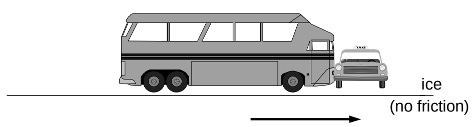
Lecture 12, Example 2
Inelastic Collision in 1-D with numbers

Do not try this at home!



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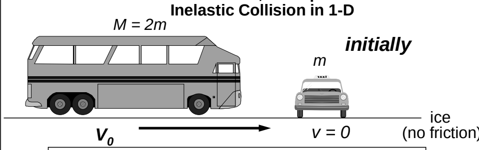
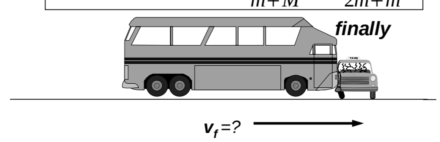
Lecture 12, Example 2
Inelastic Collision in 1-D



$$MV_0 = (m+M)V \text{ or } V = \frac{M}{m+M}V_0 = \frac{2m}{2m+m}V_0$$



V_f = (A) 0 (B) $V_o/2$ **(C) $2V_o/3$** (D) $3V_o/2$ (E) $2V_o$

Physics 207: Lecture 12, Pg 22

Lecture 12, *Exercise 3*
Momentum Conservation

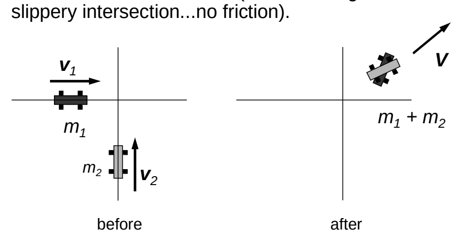
- Two balls of equal mass are thrown horizontally with the same initial velocity. They hit identical stationary boxes resting on a frictionless horizontal surface.
- The ball hitting box 1 bounces elastically back, while the ball hitting box 2 sticks.
  - ❖ Which box ends up moving fastest ?

(A) **Box 1** (B) **Box 2** (C) **same**

Physics 207: Lecture 12, Pg 23

Inelastic collision in 2-D

- Consider a collision in 2-D (cars crashing at a slippery intersection...no friction).



- If no external force momentum is conserved but energy is not. Momentum is a vector so p_x, p_y and p_z

Physics 207: Lecture 12, Pg 24

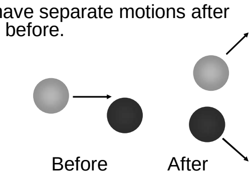
Comment on Energy Conservation

- We have seen that the total kinetic energy of a system undergoing an inelastic collision is not conserved.
  - ❖ Mechanical energy is lost:
    - ➢ Heat (friction)
    - ➢ Bending of metal and deformation
- Kinetic energy is not conserved since **negative work** is by a non-conservative force done during the collision !
- Momentum along a certain direction **is** conserved when there are no external forces acting in this direction.
  - ❖ In general, easier to satisfy than energy conservation.

See text: 9.4

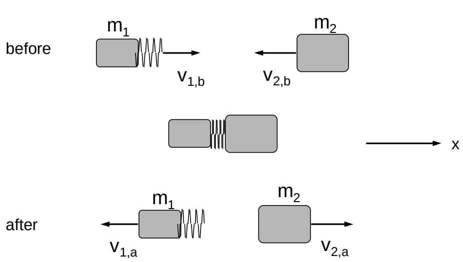
Elastic Collisions

- Elastic means that energy is conserved as well as momentum.
- This gives us more constraints.
  - ❖ We can solve more complicated problems !!
  - ❖ Billiards (2-D collision).
  - ❖ The colliding objects have separate motions after the collision as well as before.
- Start with a 1-D problem.



See text: 9.4

Elastic Collision in 1-D



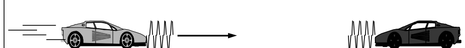
See text: 9.4

Example - Elastic Collision

- Suppose I have 2 identical bumper cars. One is motionless and the other is approaching it with velocity v_1. If they collide elastically, what is the final velocity of each car ?

  Identical means $m_1 = m_2 = m$

  Initially $v_{Green} = v_1$ and $v_{Red} = 0$



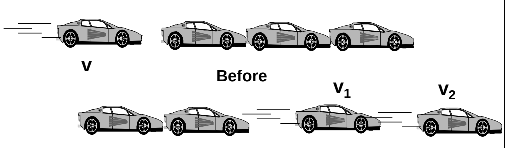
- COM → $mv_1 + 0 = mv_{1f} + mv_{2f}$ → $v_1 = v_{1f} + v_{2f}$
- COE → $½ mv_1^2 = ½ mv_{1f}^2 + ½ mv_{2f}^2$ → $v_1^2 = v_{1f}^2 + v_{2f}^2$
- $v_1^2 = (v_{1f} + v_{2f})^2 = v_{1f}^2 + 2v_{1f}v_{2f} + v_{2f}^2$ → $2\, v_{1f}\, v_{2f} = 0$
- Soln 1: $v_{1f} = 0$ and $v_{2f} = v_1$ Soln 2: $v_{2f} = 0$ and $v_{1f} = v_1$

Lecture 12, *Exercise 4*
Elastic Collisions

- I have a line of 3 bumper cars all touching. A fourth car smashes into the others from behind. Is it possible to satisfy both conservation of energy and momentum if two cars are moving after the collision?

  All masses are identical, elastic collision.

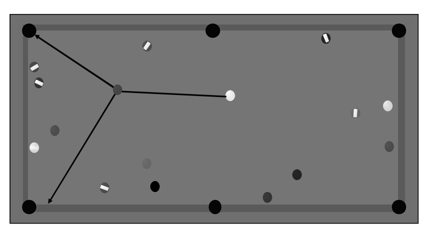
  (A) Yes (B) No (C) Only in one special case



See text: Ex. 9.11

Example of 2-D Elastic collisions: Billiards

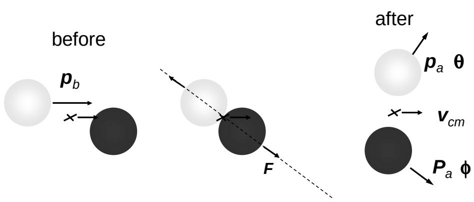
- If all we are given is the initial velocity of the cue ball, we don't have enough information to solve for the exact paths after the collision. But we can learn some useful things...



See text: Ex. 9.11

Billiards

- Consider the case where one ball is initially at rest.



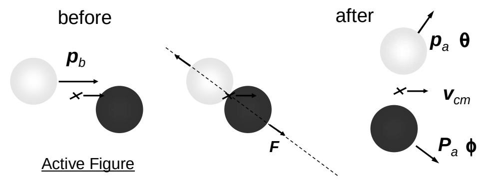
The final direction of the red ball will depend on where the balls hit.

See Figure 12-14

See text: Ex. 9.11

Billiards: All that really matters is conservation of energy and momentum

- COE: $\frac{1}{2} m v_b^2 = \frac{1}{2} m v_a^2 + \frac{1}{2} m V_a^2$
- x-dir COM: $m v_b = m v_a \cos\theta + m V_b \cos\phi$
- y-dir COM: $0 = m v_a \sin\theta + m V_b \sin\phi$



Active Figure

- The final directions are separated by 90°: $\theta - \phi = 90°$

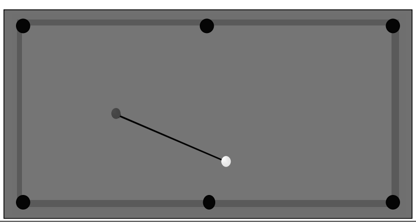
See Figure 12-14

See text: Ex. 9.11

Lecture 12 – *Exercise 4* Pool Shark

- Can I sink the red ball without scratching (sinking the cue ball) ?
  (Ignore spin and friction)

(A) Yes (B) No (C) More info needed



Physics 207, Recap

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- Chapter 9: Momentum and Collision
  - Momentum conservation
  - Collisions
  - Systems of particles
  - Impulse (Wednesday)
  - Center of mass (Wednesday)

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