Physics 207, Lecture 13, Oct. 18

Agenda: Chapter 9, finish, Chapter 10 Start

- Chapter 9: Momentum and Collision
  - ❖ Impulse
  - ❖ Center of mass
- Chapter 10:
  - ❖ Rotational Kinematics
  - ❖ Rotational Energy
  - ❖ Moments of Inertia
  - ❖ Parallel axis theorem (Monday)
  - ❖ Torque, Work and Rotational Energy (Monday)

Assignment: For Monday read through Chapter 11

- WebAssign Problem Set 5 due Tuesday

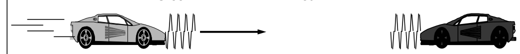
See text: 9.4

Example - Elastic Collision

- Suppose I have 2 identical bumper cars. One is motionless and the other is approaching it with velocity v_1. If they collide elastically, what is the final velocity of each car ?

Identical means $m_1 = m_2 = m$

Initially $v_{Green} = v_1$ and $v_{Red} = 0$



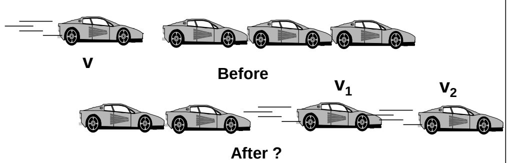
- COM → $mv_1 + 0 = mv_{1f} + mv_{2f}$ → $v_1 = v_{1f} + v_{2f}$
- COE → $½ mv_1^2 = ½ mv_{1f}^2 + ½ mv_{2f}^2$ → $v_1^2 = v_{1f}^2 + v_{2f}^2$
- $v_1^2 = (v_{1f} + v_{2f})^2 = v_{1f}^2 + 2v_{1f}v_{2f} + v_{2f}^2$ → $2 v_{1f} v_{2f} = 0$
- Soln 1: $v_{1f} = 0$ and $v_{2f} = v_1$ Soln 2: $v_{2f} = 0$ and $v_{1f} = v_1$

Lecture 13, *Exercise 1* Elastic Collisions

- I have a line of 3 bumper cars all touching. A fourth car smashes into the others from behind. Is it possible to satisfy both conservation of energy and momentum if two cars are moving after the collision?

All masses are identical, elastic collision.

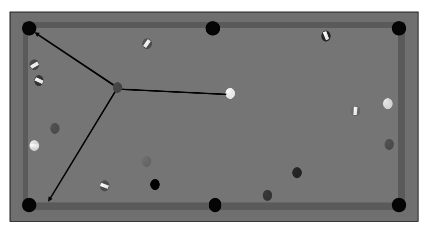
(A) Yes (B) No (C) Only in one special case



See text: Ex. 9.11

Example of 2-D Elastic collisions: Billiards

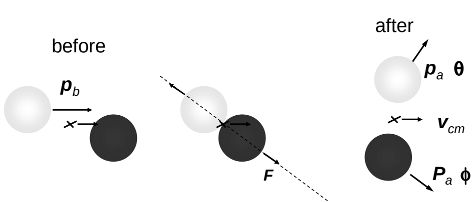
- If all we are given is the initial velocity of the cue ball, we don't have enough information to solve for the exact paths after the collision. But we can learn some useful things...



See text: Ex. 9.11

Billiards

- Consider the case where one ball is initially at rest.



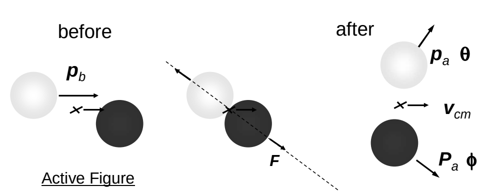
The final direction of the red ball will depend on where the balls hit.

See Figure 12-14

See text: Ex. 9.11

Billiards: All that really matters is Conservation of energy and momentum

- COE: $½ m v_b^2 = ½ m v_a^2 + ½ m V_a^2$
- x-dir COM: $m v_b = m v_a \cos\theta + m V_b \cos\phi$
- y-dir COM: $0 = m v_a \sin\theta + m V_b \sin\phi$



Active Figure

- The final directions are separated by 90°: $\theta - \phi = 90°$

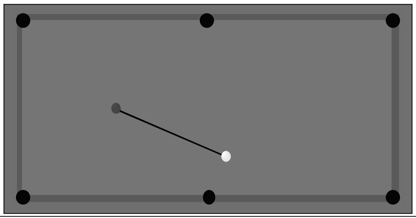
See Figure 12-14

See text: Ex. 9.11

Lecture 13 – *Exercise 2*
Pool Shark

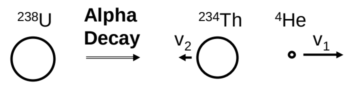
- Can I sink the red ball without scratching (sinking the cue ball) ?
  (Ignore spin and friction)

(A) Yes (B) No (C) More info needed

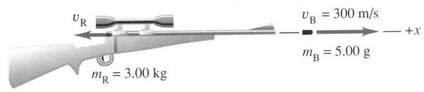


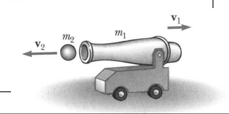
Applications of Momentum Conservation in Propulsion

Radioactive decay:



Guns, Cannons, etc.: (Recoil)





See text: 9-2

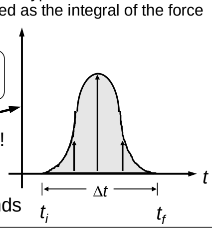
Force and Impulse
(A variable force applied for a given time)

- Gravity often provides a constant force to an object
- A spring provides a linear force (-kx) towards its equilibrium position
- A collision often involves a varying force F(t): 0 → maximum → 0
- The diagram shows the force vs time for a typical collision. The impulse, ***I***, of the force is a vector defined as the integral of the force during the collision.

$$\boldsymbol{I} = \int^t \boldsymbol{F}\, dt = \int^t (d\vec{p}/dt)dt = \int^p d\vec{p}$$

Impulse ***I*** = area under this curve !
(A change in momentum!)

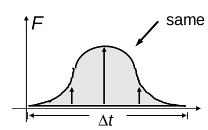
Impulse has units of Newton-seconds

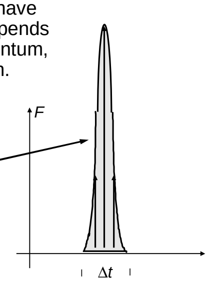


See text: 9-2

Force and Impulse

- Two different collisions can have the same impulse since ***I*** depends only on the change in momentum, not the nature of the collision.



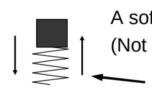


Δ*t* big, ***F*** small

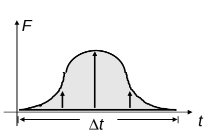
Δ*t* small, ***F*** big

See text: 9-2

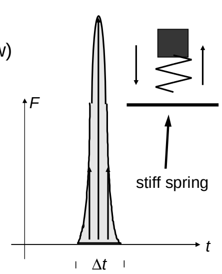
Force and Impulse



A soft "spring"
(Not Hooke's Law)



Δ*t* big, ***F*** small

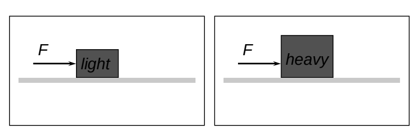


Δ*t* small, ***F*** big

Lecture 13, *Exercise 3*
Force & Impulse

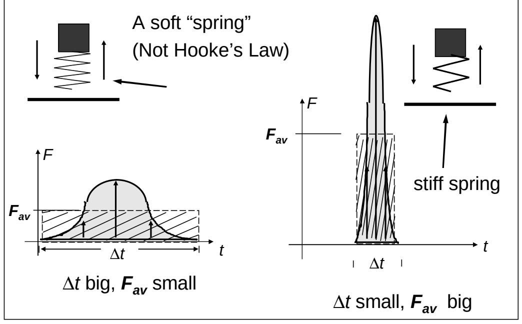
- Two boxes, one heavier than the other, are initially at rest on a horizontal frictionless surface. The same constant force *F* acts on each one for exactly 1 second.
  - ❖ Which box has the most momentum after the force acts ?

(A) heavier (B) lighter (C) same



See text 9-2

Average Force and Impulse



Back of the envelope calculation (Boxer)

(1) m_{arm} ~ 7 kg (2) **v_{arm}** ~7 m/s (3) **Impact time Δt** ~ 0.01 s

→ **Impulse** **$I = \Delta p$** ~ $m_{arm}\, v_{arm}$ ~ 49 kg m/s

→ **F ~ I / Δt ~ 4900 N** $I = \int^t F\, dt \approx F_{avg}\,\Delta t$

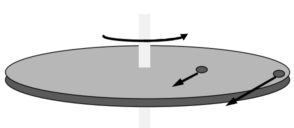
(1) m_{head} ~ 6 kg

→ **$a_{head} = F / m_{head}$ ~ 800 m/s^2 ~ 80 g !**

- **Enough to cause unconsciousness ~ 40% of fatal blow**

System of Particles:

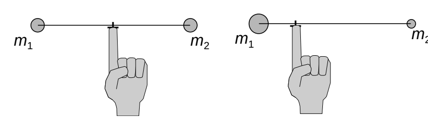
- Until now, we have considered the behavior of very simple systems (one or two masses).
- But real objects have distributed mass !
- For example, consider a simple rotating disk.



- An extended solid object (like a disk) can be thought of as a collection of parts. The motion of each little part depends on where it is in the object!

System of Particles: Center of Mass

- The center of mass is where the system is balanced !
  - ❖ Building a mobile is an exercise in finding centers of mass.



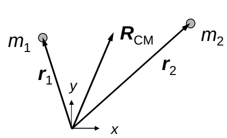
mobile

Active Figure

System of Particles: Center of Mass

- How do we describe the "position" of a system made up of many parts ?
- Define the **Center of Mass** (average position):
  - ❖ For a collection of N individual pointlike particles whose masses and positions we know:

$$\vec{R}_{CM} \equiv \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{M}$$



(In this case, $N = 2$)

See text: 9-6

Example Calculation:

- Consider the following mass distribution:

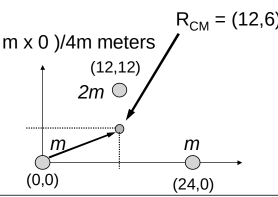
$$\vec{R}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{M} = X_{CM}\,\hat{\mathbf{i}} + Y_{CM}\,\hat{\mathbf{j}} + Z_{CM}\,\hat{\mathbf{k}}$$

$X_{CM} = (m \times 0 + 2m \times 12 + m \times 24)/4m$ meters

$Y_{CM} = (m \times 0 + 2m \times 12 + m \times 0)/4m$ meters

X_{CM} = 12 meters

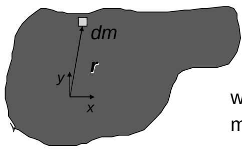
Y_{CM} = 6 meters



See text: 9-6

System of Particles: Center of Mass

- For a continuous solid, convert sums to an integral.



$$\vec{R}_{CM} = \frac{\int \vec{r}\, dm}{\int dm} = \frac{\int \vec{r}\, dm}{M}$$

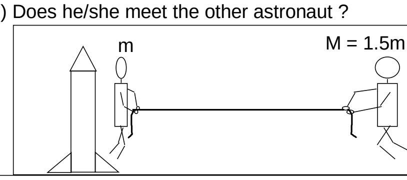
where *dm* is an infinitesimal mass element.

Center of Mass Example: Astronauts & Rope

- Two astronauts are initially at rest in outer space and 20 meters apart. The one on the right has 1.5 times the mass of the other (as shown). The 1.5 m astronaut wants to get back to the ship but his jet pack is broken. There happens to be a rope connected between the two. The heavier astronaut starts pulling in the rope.

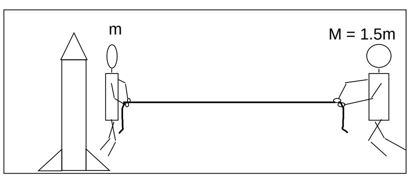
  (1) Does he/she get back to the ship ?

  (2) Does he/she meet the other astronaut ?



Example: Astronauts & Rope

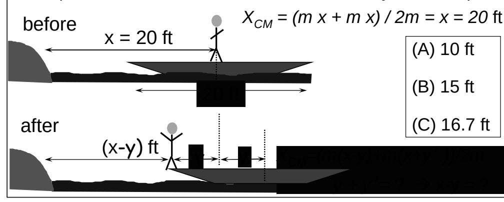
(1) There is no external force so if the larger astronaut pulls on the rope he will create an impulse that accelerates him/her to the left and the small astronaut to the right. The larger one's velocity will be less than the smaller one's so he/she doesn't let go of the rope they will either collide (elastically or inelastically) and thus never make it.



Lecture 13, *Exercise 4*
Center of Mass Motion

- A woman weighs exactly as much as her 20 foot long boat.
- Initially she stands in the center of the motionless boat, a distance of 20 feet from shore. Next she walks toward the shore until she gets to the end of the boat.
  - ❖ What is her new distance from the shore. (There is no horizontal force on the boat by the water).

$X_{CM} = (m\,x + m\,x) / 2m = x = 20$ ft



(A) 10 ft

(B) 15 ft

(C) 16.7 ft

See text: 9.6

Center of Mass Motion: Review

- We have the following rule for Center of Mass (CM) motion:

$$\vec{F}_{EXT} = M\vec{a}_{CM}$$

Active Figure

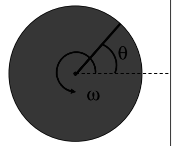
- This has several interesting implications:
- It tell us that the CM of an extended object behaves like a simple point mass under the influence of external forces:
  - ❖ We can use it to relate ***F*** and ***a*** like we are used to doing.
- It tells us that if $F_{EXT} = 0$, the total momentum of the system does not change.
  - ❖ As the woman moved forward in the boat, the boat went backward to keep the center of mass at the same place.

Chap. 10: Rotation

- Up until now rotation has been only in terms of circular motion ($a_c = v^2 / R$ and $| a_T | = d| v | / dt$)
  - ❖ We have not examined objects that roll.
  - ❖ We have assumed wheels and pulley are massless.
- Rotation is common in the world around us.
- Virtually all of the ideas developed for translational motion and are transferable to rotational motion.

Rotational Variables

- Rotation about a fixed axis:
  - ❖ Consider a disk rotating about an axis through its center:



- First, recall what we learned about Uniform Circular Motion:

$$\omega = \frac{d\theta}{dt} = \frac{2\pi}{T} \text{ (rad/s)}$$

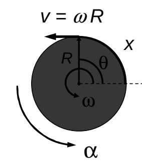
(Analogous to $v = \frac{dx}{dt}$)

Rotational Variables...

$\alpha = \text{constant}$ (angular accelation in rad/s^2)

$\omega = \omega_0 + \alpha t$ (angular velocity in rad/s)

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ (angular position in rad)



And taking the derivative of this we find

- Recall also that for a point a distance R away from the axis of rotation:
  - ❖ $x = \theta R$
  - ❖ $v = \omega R$
  - ❖ $a = \alpha R$

Summary
(with comparison to 1-D kinematics)

Angular	Linear
$\alpha = \text{constant}$	$a = \text{constant}$
$\omega = \omega_0 + \alpha t$	$v = v_0 + at$
$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	$x = x_0 + v_0 t + \frac{1}{2}at^2$

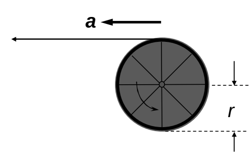
And for a point at a distance R from the rotation axis:

$x = R\,\theta$ $v = \omega R$ $a = \alpha R$

See text: 10.1

Example: Wheel And Rope

- A wheel with radius $r = 0.4$ m rotates freely about a fixed axle. There is a rope wound around the wheel. Starting from rest at $t = 0$, the rope is pulled such that it has a constant acceleration $a = 4 m/s^2$. How many revolutions has the wheel made after *10* seconds? (One revolution = 2π radians)



Example: Wheel And Rope

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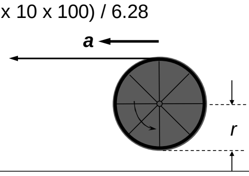
- Revolutions = **R** = $(\theta - \theta_0) / 2\pi$ and $a = \alpha\, r$

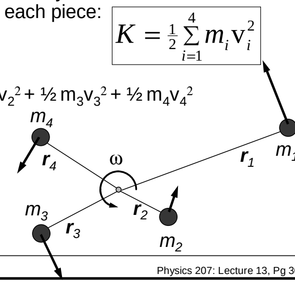
$\theta = \theta_0 + \omega_0\, t + ½\, \alpha\, t^2$ → **R** = $(\theta - \theta_0) / 2\pi = 0 + ½\,(a/r)\, t^2 / 2\pi$

R = (0.5 x 10 x 100) / 6.28



Rotation & Kinetic Energy

- Consider the simple rotating system shown below. (Assume the masses are attached to the rotation axis by massless rigid rods).
- The kinetic energy of this system will be the sum of the kinetic energy of each piece: $K = \frac{1}{2}\sum_{i=1}^{4} m_i v_i^2$
- $K = ½\, m_1 v_1^2 + ½\, m_2 v_2^2 + ½\, m_3 v_3^2 + ½\, m_4 v_4^2$



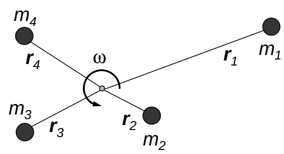
Rotation & Kinetic Energy

- Notice that $v_1 = \omega r_1$, $v_2 = \omega r_2$, $v_3 = \omega r_3$, $v_4 = \omega r_4$
- So we can rewrite the summation:

$$K = \tfrac{1}{2}\sum_{i=1}^{4} m_i v_i^2 = \tfrac{1}{2}\sum_{i=1}^{4} m_i \omega^2 r_i^2 = \tfrac{1}{2}\Big[\sum_{i=1}^{4} m_i r_i^2\Big]\omega^2$$

- We define a "new" quantity, the moment of inertia or I (we use "I" again….)

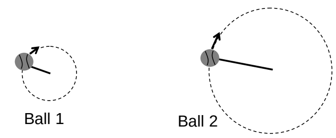
$$K = \tfrac{1}{2} I \omega^2$$



Lecture 14, *Exercise 1*
Rotational Kinetic Energy

- We have two balls of the same mass. Ball 1 is attached to a 0.1 m long rope. It spins around at 2 revolutions per second. Ball 2 is on a 0.2 m long rope. It spins around at 2 revolutions per second.
- What is the ratio of the kinetic energy of Ball 2 to that of Ball 1 ?

(A) 1/ (B) 1/2 (C) 1 (D) 2 (E) 4



Rotation & Kinetic Energy...

- The kinetic energy of a rotating system looks similar to that of a point particle:

Point Particle	Rotating System
$K = \frac{1}{2} m v^2$ *v* is "linear" velocity *m* is the mass.	$K = \frac{1}{2} I \omega^2$ ω is angular velocity *I* is the moment of inertia about the rotation axis. $I = \sum_i m_i r_i^2$

Moment of Inertia

- So $K = \frac{1}{2} I \omega^2$ where $I = \sum_i m_i r_i^2$
- Notice that the moment of inertia I depends on the distribution of mass in the system.
  - ❖ The further the mass is from the rotation axis, the bigger the moment of inertia.
- For a given object, the moment of inertia depends on where we choose the rotation axis (unlike the center of mass).
- In rotational dynamics, the moment of inertia I appears in the same way that mass *m* does in linear dynamics !

Physics 207, Lecture 13, Recap

Agenda: Chapter 9, finish, Chapter 10 Start

- Chapter 9: Momentum and Collision
  - ❖ Impulse
  - ❖ Center of mass
- Chapter 10:
  - ❖ Rotational Kinematics
  - ❖ Rotational Energy
  - ❖ Moments of Inertia

Assignment: For Monday read through Chapter 11

- WebAssign Problem Set 5 due Tuesday