Physics 207, Lecture 16, Oct. 30

Agenda: Finish, Chapter 12, Begin midterm review

- Chapter 12
  - Statics
  - Young's Modulus
  - Shear Modulus
  - Bulk Modulus

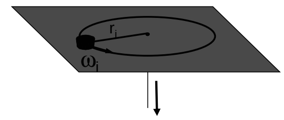
Assignments:

- WebAssign Problem Set 6 due Tuesday
- Problem Set 6, Ch 10-79, Ch 11-17,23,30,35,44abdef Ch 12-4,9,21,32,35

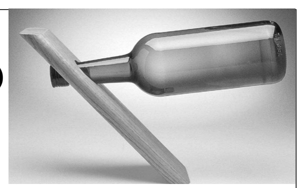
Lecture 16, *Exercise 0*

- A mass m=0.10 kg is attached to a cord passing through a small hole in a frictionless, horizontal surface as in the Figure. The mass is initially orbiting with speed ω_i = 5 rad/s in a circle of radius r_i = 0.20 m. The cord is then slowly pulled from below, and the radius decreases to r = 0.10 m. How much work is done moving the mass from r_i to r ?

(A) 0.15 J (B) 0 J (C) - 0.15 J



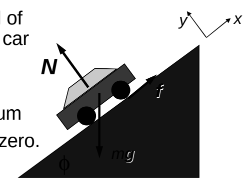
Statics (Chapter 12)



A repeat of Newton's Laws with systems having no net force and no net torque

Statics

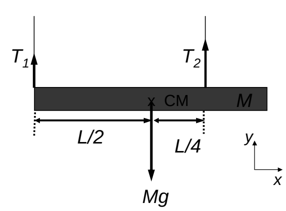
- As the name implies, "statics" is the study of systems that don't move.
  - Ladders, sign-posts, balanced beams, buildings, bridges, etc...
- Example: What are all of the forces acting on a car parked on a hill ?
- If the car is to remain motionless then the sum of the forces must be zero.



Statics: Using Torque

- Now consider a plank of mass *M* suspended by two strings as shown.
- We want to find the tension in each string:

$$\Sigma\vec{F} = 0 \qquad \Sigma\vec{\tau} = 0$$



Statics: Using Torque

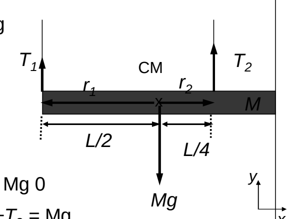
- We want to find the tension in each string:

$$\Sigma\vec{F} = 0 \qquad \Sigma\vec{\tau} = 0$$

y-dir: $\Sigma F_y = 0 = T_1+T_2 - Mg$

$T_1+T_2 = Mg$

and about the center of mass (x)



z-dir: $\Sigma\tau_z = 0 = -r_1T_1+r_2T_2 - Mg\ 0$

$T_1= r_2T_2\ /\ r_1 \rightarrow T_2\,(r_2\ /\ r_1) +T_2 = Mg$

Approach to Statics:

- In general, we can use the two equations

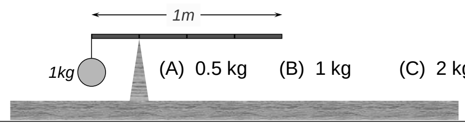
$$\boxed{\sum \boldsymbol{F} = 0} \qquad \boxed{\sum \tau = 0}$$

to solve any statics problems.

- When choosing axes about which to calculate torque, choose one that makes the problem easy....

Lecture 16, *Exercise 1*
Statics

- A 1 kg ball is hung at the end of a rod 1 m long. The system balances at a point on the rod 0.25 m from the end holding the mass.
  - ❖ What is the mass of the rod?
  - ❖ Process Hint 1: Use a free body diagram!
    Hint 2: Find centers of mass
    Hint 3: Choose a pivot point
    Hint 4: Draw in r vectors



(A) 0.5 kg (B) 1 kg (C) 2 kg

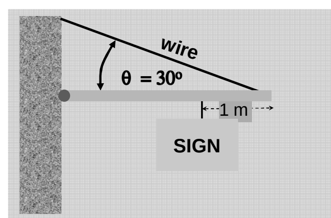
Example Problem: Hanging Lamp

- Your folks are making you help out on fixing up your house. They have always been worried that the walk around back is just too dark, so they want to hang a lamp. You go to the hardware store and try to put together a decorative light fixture. At the store you find
  (1) bunch of massless string (it costs nothing)
  (2) lamp of mass 2 kg
  (3) plank of mass 1 kg and length 2 m
  (4) hinge to hold the plank to the wall.

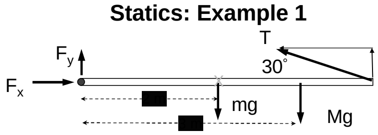
Your design is for the lamp to hang off one end of the plank and the other to be held to a wall by a hinge. The lamp end is supported by a massless string that makes an angle of 30º with the plank. (The hinge supplies a force to hold the end of the plank in place.) How strong must the string and the hinge be for this design to work ?

Statics: Example 1

A sign of mass M is hung 1 m from the end of a 4 m long beam (mass m) as shown in the diagram. The beam is hinged at the wall. What is the tension in the wire in terms of m, M, g and any other given quantity?



Statics: Example 1



Process: Make a FBD and note known / unknown forces.

Chose axis of rotation at support because F_x & F_y are not known

$\Sigma \mathbf{F} = 0 \rightarrow 0 = F_x - T \cos 30°$

$0 = F_y + T \sin 30° - mg - Mg$

z-dir: $\Sigma\tau_z = 0 = -mg\, 2r - Mg\, 3r + T \sin 30° 4r$ (r = 1m)

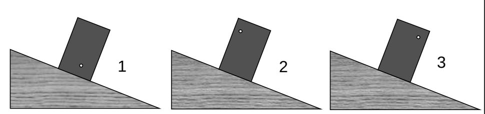
The torque equation get us where we need to go, T

$T = (2m + 3M)\, g / 2$

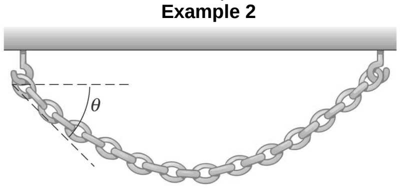
Lecture 16, *Exercise 2*
Statics

- Three different boxes are placed on a ramp in the configurations shown below. Friction prevents them from sliding. The center of mass of each box is indicated by a white dot in each case.
  - ❖ In which instances does the box tip over?

(A) all (B) 2 & 3 (C) 3 only

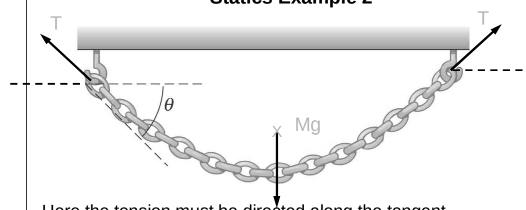


Lecture 16, Statics
Example 2



A freely suspended, flexible chain weighing Mg hangs between two hooks located at the same height. At each of the two mounting hooks, the tangent to the chain makes an angle $\theta = 42°$ with the horizontal. What is the magnitude of the force each hook exerts on the chain and what is the tension in the chain at its midpoint.

Statics Example 2



Here the tension must be directed along the tangent.

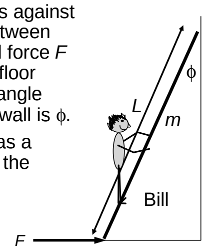
$\Sigma\mathbf{F} = 0 \rightarrow 0 = T_2 \cos 42° - T_1 \cos 42°$ let $T_1 = T_2 = T$

So $0 = 2\, T \sin 42° - Mg$

Statics requires that the net force in the x-dir be zero everywhere so T_x is the same everywhere or $T \cos 42°$

Lecture 16, *Exercise 3*
Statics: Ladder against smooth wall

- Bill (mass M) is climbing a ladder (length L, mass m) that leans against a smooth wall (no friction between wall and ladder). A frictional force F between the ladder and the floor keeps it from slipping. The angle between the ladder and the wall is ϕ.
- What is the magnitude of F as a function of Bill's distance up the ladder?

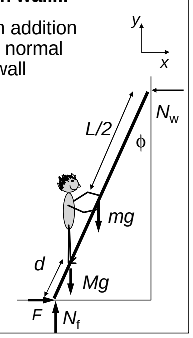


Ladder against smooth wall...

- Consider all of the forces acting. In addition to gravity and friction, there will be normal forces N_f and N_w by the floor and wall respectively on the ladder.
- First sketch the FBD
- Again use the fact that $F_{NET} = 0$ in both x and y directions:

x: $N_w = F$

y: $N_f = Mg + mg$



Ladder against smooth wall...

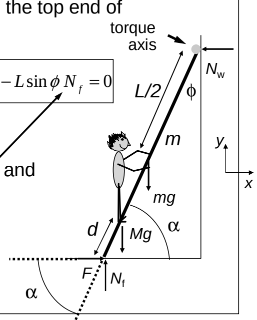
- Since we are not interested in N_w, calculate torques about an axis through the top end of the ladder, in the z direction.

$$\boxed{\frac{L}{2}\sin\phi\, mg + (L-d)\sin\phi\, Mg + L\sin\alpha\, F - L\sin\phi\, N_f = 0}$$

$\cos\phi$

- Substituting: $N_f = Mg + mg$ and solve for F :

$$\boxed{F = Mg \tan\phi \left[\frac{d}{L} + \frac{m}{2M}\right]}$$



Example: Ladder against smooth wall

We have just calculated that $\boxed{F = Mg \tan\phi \left[\frac{d}{L} + \frac{m}{2M}\right]}$

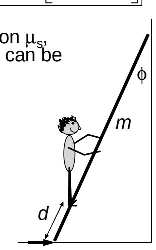
- For a given coefficient of static friction μ_s, the maximum force of friction F that can be provided is $\mu_s N_f = \mu_s\, g(M + m)$.

  ❖ The ladder will slip if F exceeds this value.

Cautionary note:

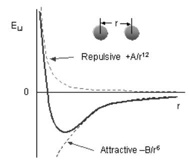
(1) Brace the bottom of ladders!

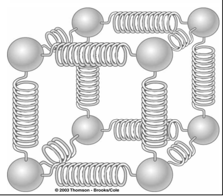
(2) Don't make ϕ too big!



States of Matter Solids

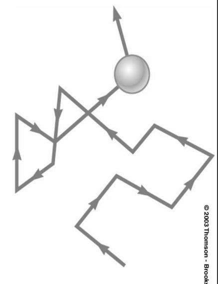
- Have definite volume
- Have definite shape
- Molecules are held in specific locations
  - ❖ by electrical forces
  - ❖ vibrate about equilibrium positions
  - ❖ can be modeled as springs connecting molecules (the potential energy curve <u>always</u> looks a parabola near the minimum!)
  - ❖ $U(r-r_{min}) = ½\, k\, \Delta r^2$





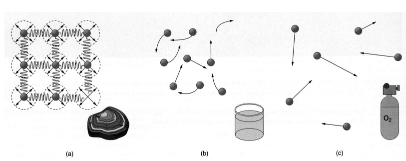
Liquid

- Has a definite volume
- No definite shape
- Exist at a higher temperature than solids
- The molecules “wander” through the liquid in a random fashion
  - ❖ The intermolecular forces are not strong enough to keep the molecules in a fixed position



Gas

- Has no definite volume
- Has no definite shape
- Molecules are in constant random motion
- The molecules exert only weak forces on each other
- Average distance between molecules is large compared to the size of the molecules



Question

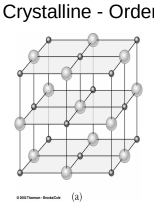
Are atoms in a solid always arranged in an ordered structure?

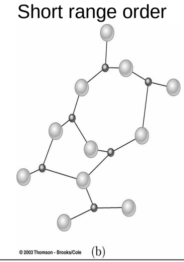
- ❖ Yes
- ❖ No ←

Amorphous –

Short range order

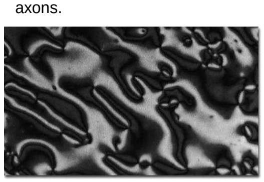
Crystalline - Ordered

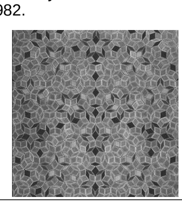




There are also complex states of mater:

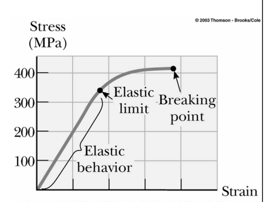
- **Liquid crystals** include properties of both liquids and solids (at the same time and place). Many of these are biological in nature. In fact, the very first liquid crystal to be heavily researched was myelin, a soft, fatty substance that sheathes certain nerve fibers and axons.
- **Quasicrystals** are a peculiar form of solid in which the atoms of the solid are arranged in a seemingly regular, yet non-repeating structure. They were first observed by Dan Shechtman in 1982.





Solid are not infinitely rigid, solids will always deform if a force is applied

- All objects are deformable, *i.e.* It is possible to change the shape or size (or both) of an object through the application of external forces
  - ❖ Sometimes when the forces are removed, the object tends to its original shape, called *elastic behavior*
  - ❖ Large enough forces will break the bonds between molecules and also the object



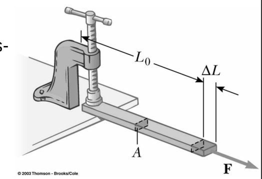
Elastic Properties

- *Stress* is related to the force causing the deformation
- *Strain* is a measure of the degree of deformation
- The *elastic modulus* is the constant of proportionality between stress and strain
  - ❖For sufficiently small stresses, the stress is directly proportional to the strain
  - ❖The constant of proportionality depends on the material being deformed and the nature of the deformation
  - ❖The elastic modulus can be thought of as the stiffness of the material

$$\text{Elastic modulus} = \frac{\text{stress}}{\text{strain}}$$

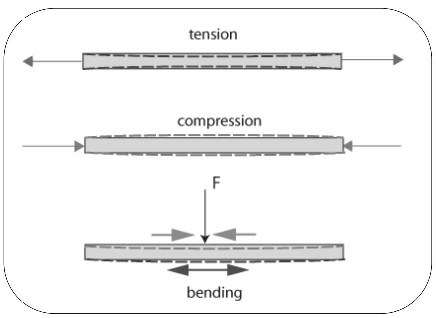
Young's Modulus: Elasticity in Length

- Tensile stress is the ratio of the external force to the cross-sectional area
  - ❖For both *tension and compression*
- The elastic modulus is called *Young's modulus*
- SI units of stress are Pascals, Pa
  - ❖1 Pa = 1 N/m^2
- The tensile strain is the ratio of the change in length to the original length
  - ❖Strain is dimensionless



$$Y \equiv \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F / A}{\Delta L / L_0}$$

Beams



Elastic vs. Plastic Behavior

- If the strain disappears when the stress is removed, the material is said to behave *elastically*.
- The largest stress for which this occurs is called the *elastic limit*
- When the strain does not return to zero after the stress is removed, the material is said to behave *plastically*.

(From C to D)

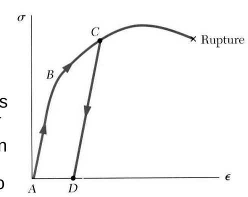
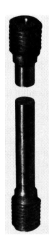


Fig. 2.18

Stress-Strain Diagram: Brittle Materials



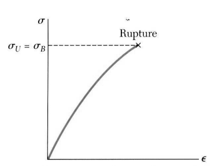
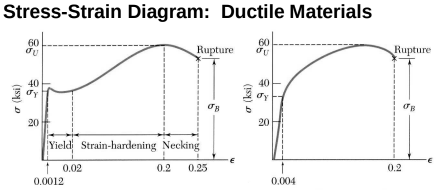
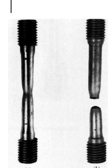


Fig. 2.11 Stress-strain diagram for a typical brittle material.

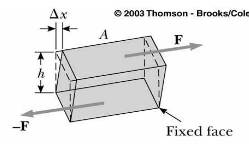
Stress-Strain Diagram: Ductile Materials





Shear Modulus: Elasticity of Shape

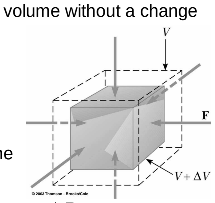
- Forces may be parallel to one of the objects faces
- The stress is called a *shear stress*
- The *shear strain* is the ratio of the horizontal displacement and the height of the object
- The shear modulus is S
- A material having a large shear modulus is difficult to bend



$$S \equiv \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x / h}$$

Bulk Modulus: Volume Elasticity

- Bulk modulus characterizes the response of an object to uniform squeezing
  - ❖ Suppose the forces are perpendicular to, and acts on, all the surfaces -- as when an object is immersed in a fluid
- The object undergoes a change in volume without a change in shape
- Volume stress, ΔP, is the ratio of the force to the surface area
  - ❖ This is also the *Pressure*
- The volume strain is equal to the ratio of the change in volume to the original volume



$$B \equiv \frac{\text{volume stress}}{\text{volume strain}} = -\frac{\Delta F/A}{\Delta V/V} = -\frac{\Delta P}{\Delta V/V}$$

Notes on Moduli

- Solids have Young’s, Bulk, and Shear moduli
- Liquids have only bulk moduli, they will not undergo a shearing or tensile stress
  - ❖The negative sign is included since an increase in pressure will produce a decrease in volume: B is always positive
  - ❖ But: “Composites with Inclusions of Negative Bulk Modulus: Extreme Damping and Negative Poisson’s Ratio” 2005 Article in J. Composite Materials

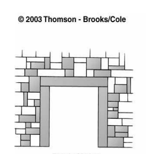
Ultimate Strength of Materials

- The *ultimate strength* of a material is the maximum stress the material can withstand before it breaks or factures
- Some materials are stronger in compression than in tension
- Linear to the *Elastic Limit*

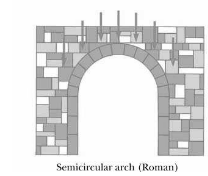
Arches

Which of the following two archways can you build bigger, assuming that the same type of stone is available in whatever length you desire?

❖Post-and-beam (Greek) arch
❖Semicircular (Roman) arch
❖You can build big in either type



Low ultimate tensile strength of sagging stone beams



Stability depends upon the compression of the wedge-shaped stones

Lecture 16, Statics Exercise 3

A plastic box is being pushed by a horizontal force at the top and it slides across a horizontal floor. The frictional force between the box and the floor causes the box to deform. To describe the relationship between stress and strain for the box, you would use

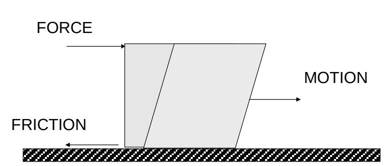
(A) Young’s modulus

(B) Shear modulus

(C) Bulk modulus

(D) None of the above

Lecture 16, Statics Exercise 3



(B) SHEAR MODULUS IS THE CHOICE!

Lecture 16, Statics
Exercises 4 and 5

1. A hollow cylindrical rod and a solid cylindrical rod are made of the same material. The two rods have the same length and outer radius. If the same compressional force is applied to each rod, which has the greater change in length?
(A) Solid rod
(B) Hollow rod
(C) Both have the same change in length

2. Two identical springs are connected end to end. What is the force constant of the resulting compound spring compared to that of a single spring?
(A) Less than
(B) Greater than
(C) Equal to

Lecture 16 Recap, Oct. 30

Agenda: Finish, Chapter 12, Begin midterm review

- Chapter 12
  - ❖ Statics
  - ❖ Young's Modulus
  - ❖ Shear Modulus
  - ❖ Bulk Modulus

Assignments:
- WebAssign Problem Set 6 due Tuesday
- Problem Set 6, Ch 10-79, Ch 11-17,23,30,35,44abdef Ch 12-4,9,21,32,35