Physics 207, Lecture 20, Nov. 13

- Agenda: Chapter 15, Finish, Chapter 16, Begin
  - ❖ Simple pendulum
  - ❖ Physical pendulum
  - ❖ Torsional pendulum
  - ❖ Energy
  - ❖ Damping
  - ❖ Resonance
  - ❖ Chapter 16, Traveling Waves

Assignments:

- Problem Set 7 due Nov. 14, Tuesday 11:59 PM
- Problem Set 8 due Nov. 21, Tuesday 11:59 PM

  Ch. 16: 3, 18, 30, 40, 58, **59** (Honors) Ch. 17: 3, 15, 34, 38, 40
- For Wednesday, Finish Chapter 16, Start Chapter 17

Energy of the Spring-Mass System

We know enough to discuss the mechanical energy of the oscillating mass on a spring.

Remember,

$x(t) = A\cos(\omega t + \phi)$

$v(t) = -\omega A \sin(\omega t + \phi)$

$a(t) = -\omega^2 A \cos(\omega t + \phi)$

Kinetic energy is always

$K = ½\, mv^2$

$K = ½\, m\, [-\omega A \sin(\omega t + \phi)]^2$

And the potential energy of a spring is,

$U = ½\, k\, x^2$

$U = ½\, k\, [A\cos(\omega t + \phi)]^2$

Energy of the Spring-Mass System

Add to get E = K + U = constant.

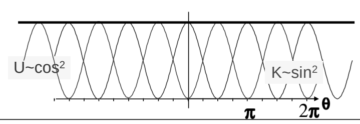
$½\, m\,(\omega A)^2 \sin^2(\omega t + \phi) + 1/2\, k\,(A\cos(\omega t + \phi))^2$

Recalling $\omega = \sqrt{\dfrac{k}{m}} \Rightarrow \omega^2 = \dfrac{k}{m}$

so, $E = ½\, k\, A^2 \sin^2(\omega t + \phi) + ½\, kA^2 \cos^2(\omega t + \phi)$

$= ½\, k\, A^2\, [\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)]$

$= ½\, k\, A^2$ with $\theta = \omega t + \phi$



Active Figure

SHM So Far

- The most general solution is $x = A\cos(\omega t + \phi)$

  where A = amplitude

  ω = (angular) frequency

  ϕ = phase constant
- For SHM without friction, $\omega = \sqrt{\dfrac{k}{m}}$
  - ❖ The frequency does *not* depend on the amplitude !
  - ❖ We will see that this is true of all simple harmonic motion!
- The oscillation occurs around the equilibrium point where the force is zero!
- Energy is a constant, it transfers between potential and kinetic.

The Simple Pendulum

- A pendulum is made by suspending a mass *m* at the end of a string of length *L*. Find the frequency of oscillation for small displacements.

$\Sigma F_y = ma_c = T - mg\cos(\theta) = m\, v^2/L$

$\Sigma F_x = ma_x = -mg\sin(\theta)$

If θ small then $x \cong L\,\theta$ and $\sin(\theta) \cong \theta$

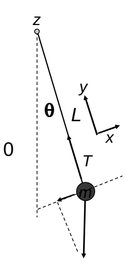
$dx/dt = L\; d\theta/dt$

$a_x = d^2x/dt^2 = L\, d^2\theta/dt^2$

so $a_x = -g\,\theta = L\, d^2\theta / dt^2 \rightarrow L\, d^2\theta / dt^2 - g\,\theta = 0$

and $\theta = \theta_0 \cos(\omega t + \phi)$ or $\theta = \theta_0 \sin(\omega t + \phi)$

with $\omega = (g/L)^{½}$



Lecture 20, *Exercise 1*
Simple Harmonic Motion

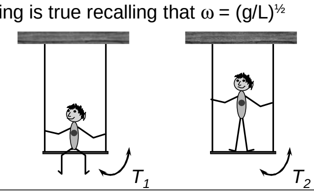
- You are *sitting* on a swing. A friend gives you a small push and you start swinging back & forth with period T_1.
- Suppose you were *standing* on the swing rather than sitting. When given a small push you start swinging back & forth with period T_2.

Which of the following is true recalling that $\omega = (g/L)^{½}$

(A) $T_1 = T_2$

(B) $T_1 > T_2$

(C) $T_1 < T_2$



The Rod Pendulum

- A pendulum is made by suspending a thin rod of length L and mass M at one end. Find the frequency of oscillation for small displacements (i.e., $\theta \cong \sin\theta$).

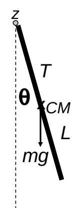
$\Sigma \tau_z = I \alpha = -| \mathbf{r} \times \mathbf{F} | = (L/2)\, mg \sin(\theta)$

(no torque from T)

$-[\, mL^2/12 + m (L/2)^2\,]\ \alpha \cong L/2\ mg\ \theta$

$-1/3\ L\ d^2\theta/dt^2 = ½\ g\ \theta$

The rest is for homework…



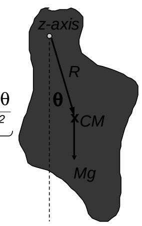
General Physical Pendulum

- Suppose we have some arbitrarily shaped solid of mass M hung on a fixed axis, that we know where the CM is located **and** what the moment of inertia I about the axis is.
- The torque about the rotation (z) axis for small θ is ($\sin\theta \cong \theta$)

$$\tau = -MgR \sin\theta \cong -MgR\theta \rightarrow \underbrace{-MgR\,\theta}_{\tau} = I \underbrace{\frac{d^2\theta}{dt^2}}_{\alpha}$$

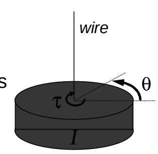
$\Rightarrow \dfrac{d^2\theta}{dt^2} = -\omega^2\theta$ where $\boxed{\omega = \sqrt{\dfrac{MgR}{I}}}$

$\Rightarrow \theta = \theta_0 \cos(\omega t + \phi)$



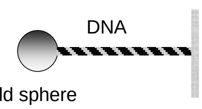
Torsion Pendulum

- Consider an object suspended by a wire attached at its CM. The wire defines the rotation axis, and the moment of inertia I about this axis is known.
- The wire acts like a "rotational spring".
  - ❖ When the object is rotated, the wire is twisted. This produces a torque that opposes the rotation.
  - ❖ In analogy with a spring, the torque produced is proportional to the displacement: $\tau = -\kappa\,\theta$ where κ is the torsional spring constant
  - ❖ $\omega = (\kappa / I)^{½}$



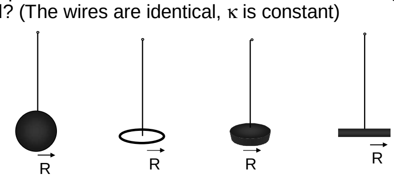
Torsional spring constant of DNA

- **<u>Session Y15: Biosensors and Hybrid Biodevices</u>**
- 11:15 AM–2:03 PM, Friday, March 25, 2005 LACC - 405
- **Abstract: Y15.00010 : Optical measurement of DNA torsional modulus under various stretching forces**
- Jaehyuck Choi, Kai Zhao, Y.-H. Lo Department of Electrical and Computer Engineering, [Department of Physics University of California at San Diego, La Jolla, California 92093-0407 We have measured the torsional spring modulus of a double stranded-DNA by applying an external torque around the axis of a vertically stretched DNA molecule. We observed that the torsional modulus of the DNA increases with stretching force. This result supports the hypothesis that an applied stretching force may raise the intrinsic torsional modulus of ds-DNA via elastic coupling between twisting and stretching. This further verifies that the torsional modulus value (C = 46.5 +/- 10 pN nm) of a ds-DNA investigated under Brownian torque (no external force and torque) could be the pure intrinsic value without contribution from other effects such as stretching, bending, or buckling of DNA chains.



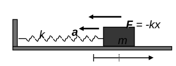
Lecture 20, *Exercise 2*
Period

- All of the following torsional pendulum bobs have the same mass and $\omega = (\kappa/I)^{½}$
- Which pendulum rotates the slowest, i.e. has the longest period? (The wires are identical, κ is constant)



Reviewing Simple Harmonic Oscillators

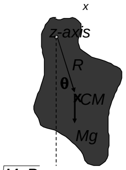
- Spring-mass system



$\dfrac{d^2x}{dt^2} = -\omega^2 x$ where $\omega = \sqrt{\dfrac{k}{m}}$

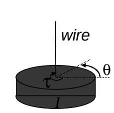
$\Rightarrow x(t) = A\cos(\omega t + \phi)$

- Pendulums $\dfrac{d^2\theta}{dt^2} = -\omega^2\theta$



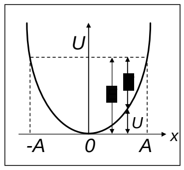
$\Rightarrow \theta = \theta_0 \cos(\omega t + \phi)$

- ❖ General physical pendulum $\omega = \sqrt{\dfrac{MgR}{I}}$
- ❖ Torsion pendulum $\omega = \sqrt{\dfrac{\kappa}{I}}$



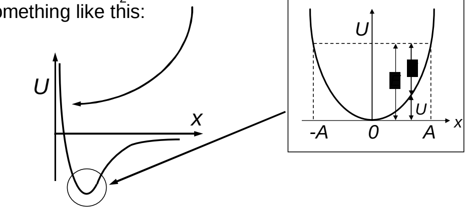
Energy in SHM

- For both the spring and the pendulum, we can derive the SHM solution and examine U and K
- The total energy ($K + U$) of a system undergoing SMH will always be constant !
- This is not surprising since there are only conservative forces present, hence mechanical energy ought be conserved.



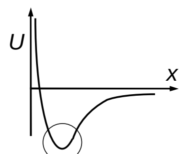
SHM and quadratic potentials

- SHM will occur whenever the potential is quadratic.
- For small oscillations this will be true:
- For example, the potential between H atoms in an H_2 molecule looks something like this:

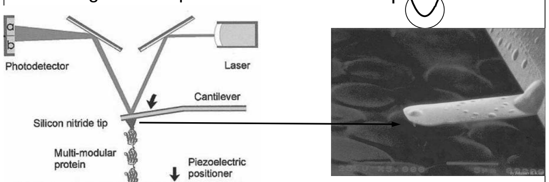


SHM and quadratic potentials

- Curvature reflects the spring constant or modulus (i.e., stress vs. strain or force vs. displacement)



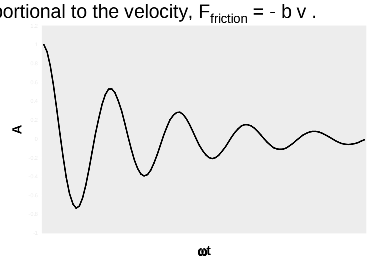
Measuring modular proteins with an AFM



See: http://hansmalab.physics.ucsb.edu

What about Friction?

- Friction causes the oscillations to get smaller over time
- This is known as DAMPING.
- As a model, we assume that the force due to friction is proportional to the velocity, $F_{friction} = - b v$.



What about Friction?

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2} \quad \Rightarrow \quad \frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

We can guess at a new solution.

$$x = \mathrm{A}\exp\left(-\tfrac{bt}{2m}\right)\cos(\omega t + \phi)$$ and now $\omega_0^2 \equiv k / m$

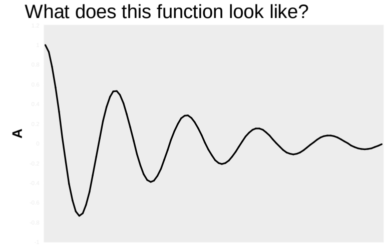
With,

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} = \sqrt{\omega_o^2 - \left(\frac{b}{2m}\right)^2}$$

What about Friction?

$$x(t) = \mathrm{A}\exp\left(-\tfrac{bt}{2m}\right)\cos(\omega t + \phi)$$ if $\omega_o > b/2m$

What does this function look like?

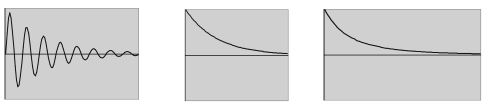


Damped Simple Harmonic Motion

$$\boxed{\omega = \sqrt{\omega_o^2 - (b/2m)^2}}$$

- There are three mathematically distinct regimes

$\omega_o > b/2m$ $\quad$ $\omega_o = b/2m$ $\quad$ $\omega_o < b/2m$

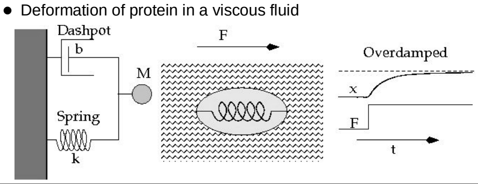


underdamped $\quad$ critically damped $\quad$ overdamped

Physical properties of a globular protein (mass 100 kDa)

- Mass: 166×10^{-24} kg
- Density: 1.38×10^{3} kg / m3
- Volume: 120 nm^3
- Radius: 3 nm
- Drag Coefficient: 60 pN-sec / m

- Deformation of protein in a viscous fluid

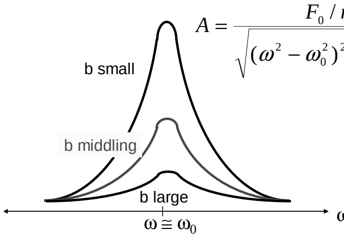


Driven SHM with Resistance

- Apply a sinusoidal force, $F_0 \cos(\omega t)$, and now consider what *A* and *b* do,

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F}{m}\cos\omega t$$

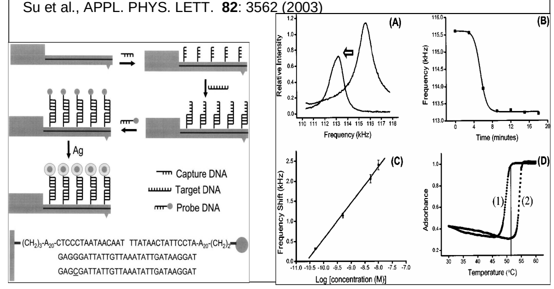
$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + (\frac{b\omega}{m})^2}}$$



Microcantilever resonance-based DNA detection with nanoparticle probes

Change the mass of the cantilever and change the resonant frequency and the mechanical response.

Su et al., APPL. PHYS. LETT. **82**: 3562 (2003)

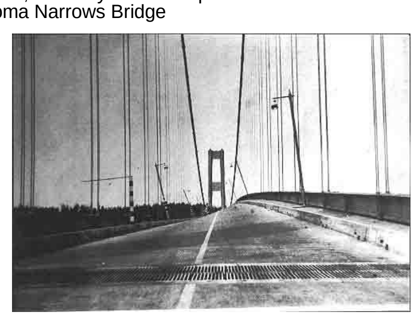


Stick - Slip Friction

- How can a constant motion produce resonant vibrations?
- Examples:
  - ❖ Violin
  - ❖ Singing / Whistling
  - ❖ Tacoma Narrows Bridge
  - ❖…

Dramatic example of resonance

- In 1940, a steady wind set up a torsional vibration in the Tacoma Narrows Bridge



A short clip

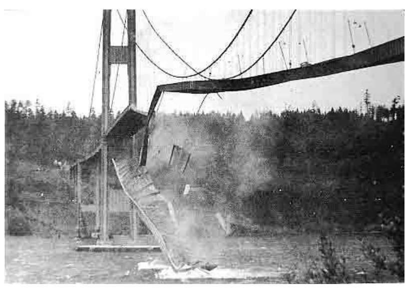
- In 1940, a steady wind sets up a torsional vibration in the Tacoma Narrows Bridge

Dramatic example of resonance

- Large scale torsion at the bridge's natural frequency

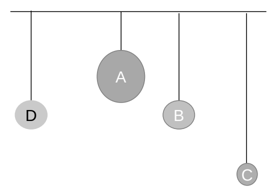
Dramatic example of resonance

- Eventually it collapsed



Lecture 20, *Exercise 3*
Resonant Motion

- Consider the following set of pendulums all attached to the same string



If I start bob D swinging which of the others will have the largest swing amplitude ?
(A) (B) (C)

Waves (Chapter 16)

- Oscillations:
  - ❖ Movement around one equilibrium point
- Waves:
  - ❖ Look only at one point: oscillations
  - ❖ But: changes in time and space (i.e., in 2 dimensions!)

What is a wave ?

- A definition of a wave:
  - ❖ A wave is a traveling disturbance that transports energy but not matter.
- Examples:
  - ❖ Sound waves (air moves back & forth)
  - ❖ Stadium waves (people move up & down)
  - ❖ Water waves (water moves up & down)
  - ❖ Light waves (an oscillating electromagnetic field)

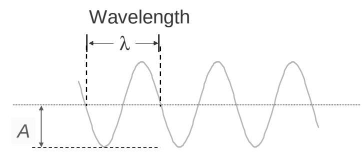
Animation

Types of Waves

- **Transverse:** The medium's displacement is perpendicular to the direction the wave is moving.
  - ❖ Water (more or less)
  - ❖ String waves
- **Longitudinal:** The medium's displacement is in the same direction as the wave is moving
  - ❖ Sound
  - ❖ Slinky

Wave Properties

- Wavelength: The distance λ between identical points on the wave.
- Amplitude: The maximum displacement A of a point on the wave.



Animation

Wave Properties...

- Period: The time T for a point on the wave to undergo one complete oscillation.
- Speed: The wave moves one wavelength λ in one period T so its speed is $v = \lambda / T$.

$$\boxed{v = \frac{\lambda}{T}}$$

Animation

Lecture 20, *Exercise 4*
Wave Motion

- The speed of sound in air is a bit over 300 m/s, and the speed of light in air is about 300,000,000 m/s.
- Suppose we make a sound wave and a light wave that both have a wavelength of 3 meters.

  What is the ratio of the frequency of the light wave to that of the sound wave ? (Recall $v = \lambda / T = \lambda f$)

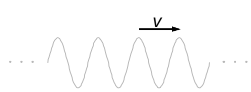
(A) About 1,000,000

(B) About 0.000,001

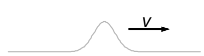
(C) About 1000

Wave Forms

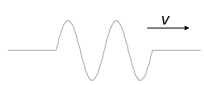
- So far we have examined "continuous waves" that go on forever in each direction !



- We can also have "pulses" caused by a brief disturbance of the medium:



- And "pulse trains" which are somewhere in between.



Lecture 20, *Exercise 5*
Wave Motion

- A harmonic wave moving in the <u>positive x direction</u> can be described by the equation

  (The wave varies in space and time.)
- $v = \lambda / T = \lambda f = (\lambda/2\pi)(2\pi f) = \omega / k$ and, by definition, $\omega > 0$
- $y(x,t) = A \cos((2\pi/\lambda)\, x - \omega t) = A \cos(k x - \omega t)$
- Which of the following equation describes a harmonic wave moving in the <u>negative x direction</u> ?

(A) $y(x,t) = A \sin(k x - \omega t)$

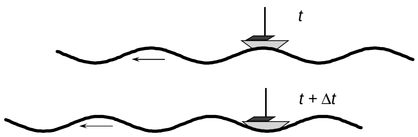
(B) $y(x,t) = A \cos(k x + \omega t)$

(C) $y(x,t) = A \cos(-k x + \omega t)$

Lecture 20, *Exercise 6*
Wave Motion

- A boat is moored in a fixed location, and waves make it move up and down. If the spacing between wave crests is *20* meters and the speed of the waves is *5* m/s, how long Δt does it take the boat to go from the top of a crest to the bottom of a trough ? (Recall $v = \lambda / T = \lambda f$)

(A) 2 sec (B) 4 sec (C) 8 sec



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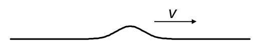
Waves on a string

- What determines the speed of a wave ?
- Consider a pulse propagating along a string:



- "Snap" a rope to see such a pulse
- How can you make it go faster ?

Animation

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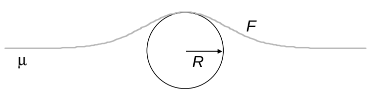
Waves on a string...

Suppose:

- The tension in the string is F
- The mass per unit length of the string is μ (kg/m)
- The shape of the string at the pulse's maximum is circular and has radius R



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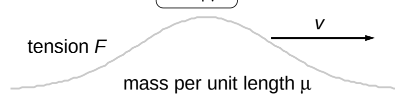
Waves on a string...

- So we find: $\boxed{v = \sqrt{\dfrac{F}{\mu}}}$

Animation



- Making the tension bigger increases the speed.
- Making the string heavier decreases the speed.
- The speed depends only on the nature of the medium, not on amplitude, frequency etc of the wave.

Physics 207: Lecture 20, Pg 40

Lecture 20, Recap

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  - ❖ Resonance
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