Physics 207, Lecture 21, Nov. 15

- Agenda: Chapter 16, Finish, Chapter 17, Sound
  - ❖ Traveling Waves
  - ❖ Reflection
  - ❖ Transmission
  - ❖ Power
- Chapter 17, Sound
  - ❖ Plane waves, spherical waves
  - ❖ Loudness

Assignments:

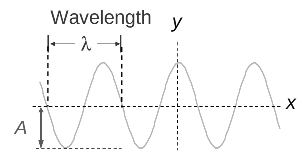
- Problem Set 8 due Nov. 21, Tuesday 11:59 PM

  Ch. 16: 3, 18, 30, 40, 58, **59** (Honors) Ch. 17: 3, 15, 34, 38, 40
- For Monday, Chapter 16, Doppler effect Start Chapter 17

Wave Properties

- Wavelength: The distance λ between identical points on the wave.
- Amplitude: The maximum displacement *A* of a point on the wave.
- A wave varies in time and space. *Animation 1*

$$y(x,t) = A\cos[(2\pi/\lambda)\,x - \omega t)]$$ *Animation*

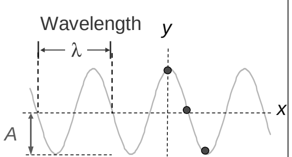


Wave Properties

$$y(x,t) = A\cos[(2\pi/\lambda)\,x - \omega t)]$$

Look at the spatial part (Let *t* =0).

$$y(x,0) = A\cos[(2\pi/\lambda)\,x)]$$ *Animation*



- $x = 0$ $y = A$
- $x = \lambda/4$ $y = A\cos(\pi/2) = 0$
- $x = \lambda/2$ $y = A\cos(\pi) = -A$

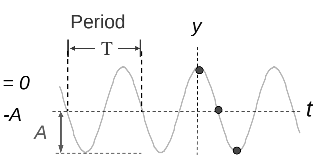
Look at the temporal (time-dependent) part

$$y(x,t) = A\cos[(2\pi/\lambda)\,x - \omega t)]$$

- Let x = 0 *Animation*

$$y(0,t) = A\cos(-\omega t) = A\cos[-(2\pi/T)\,t]$$

- $t = 0$ $y = A$
- $t = T/4$ $y = A\cos(-\pi/2) = 0$
- $t = T/2$ $y = A\cos(-\pi) = -A$



Wave Properties...

- Period: The time *T* for a point on the wave to undergo one complete oscillation.
- Speed: The wave moves one wavelength λ in one period *T* so its speed is *v* = λ / *T*.

$$v = \frac{\lambda}{T}$$

Animation

Lecture 21, *Exercise 1*
Wave Motion

- The speed of sound in air is a bit over 300 m/s, and the speed of light in air is about 300,000,000 m/s.
- Suppose we make a sound wave and a light wave that both have a wavelength of 3 meters.

  What is the ratio of the frequency of the light wave to that of the sound wave ? (Recall $v = \lambda / T = \lambda f$)

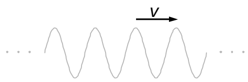
(A) About 1,000,000

(B) About 0.000,001

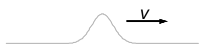
(C) About 1000

Wave Forms

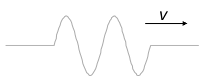
- So far we have examined "continuous waves" that go on forever in each direction !



- We can also have "pulses" caused by a brief disturbance of the medium:



- And "pulse trains" which are somewhere in between.



Lecture 20, *Exercise 2* Wave Motion

- A harmonic wave moving in the <u>positive x direction</u> can be described by the equation
- $v = \lambda / T = \lambda f = (\lambda/2\pi)(2\pi f) = \omega / k$ and, by definition, $\omega > 0$ and the "wavevector" or wave number" $k \equiv 2\pi/\lambda$
- $y(x,t) = A\cos((2\pi/\lambda)\, x - \omega t) = A\cos(k x - \omega t)$

  with $v = \omega / k$, if $\omega / k > 0$ then $v > 0$ or if $\omega / k < 0$ then $v < 0$

- Which of the following equations describes a harmonic wave moving in the <u>negative x direction</u> ?

  (A) $y(x,t) = A \sin(k x - \omega t)$

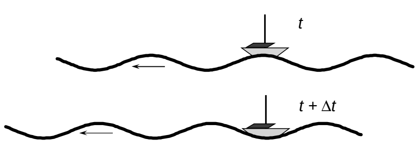
  (B) $y(x,t) = A \cos(k x + \omega t)$

  (C) $y(x,t) = A \cos(-k x + \omega t)$

Lecture 20, *Exercise 3* Wave Motion

- A boat is moored in a fixed location, and waves make it move up and down. If the spacing between wave crests is *20* meters and the speed of the waves is *5* m/s, how long Δt does it take the boat to go from the top of a crest to the bottom of a trough ? (Recall $v = \lambda / T = \lambda f$)

(A) 2 sec (B) 4 sec (C) 8 sec



Waves on a string

- What determines the speed of a wave ?
- Consider a pulse propagating along a string:

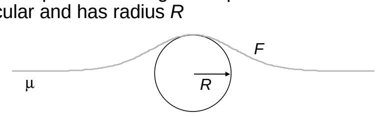
- "Snap" a rope to see such a pulse
- How can you make it go faster ?

<u>Animation</u>

Waves on a string...

Suppose:

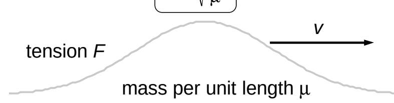
- The tension in the string is F
- The mass per unit length of the string is μ (kg/m)
- The shape of the string at the pulse's maximum is circular and has radius R



Waves on a string...

- So we find: $v = \sqrt{\dfrac{F}{\mu}}$

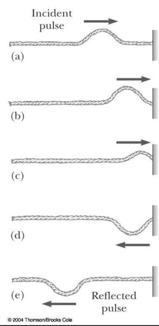
<u>Animation</u>



- Increasing the tension increases the speed.
- Increasing the string mass density decreases the speed.
- The speed depends only on the nature of the medium and <u>not</u> on amplitude, frequency, etc.

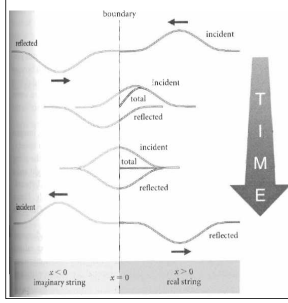
Reflection of a Wave, Fixed End

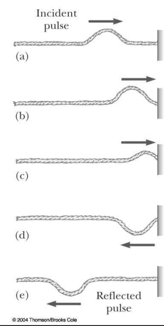
- When the pulse reaches the support, the pulse moves back along the string in the opposite direction
- This is the **reflection** of the pulse
- The pulse is inverted



Reflection of a Wave, Fixed End

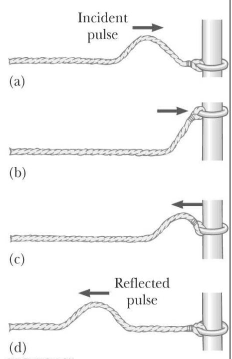
Animation





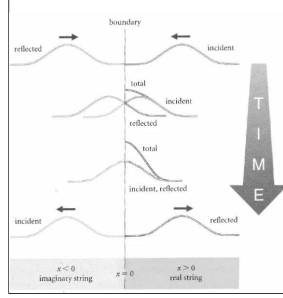
Reflection of a Wave, Free End

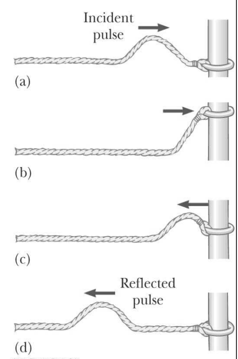
- With a free end, the string is free to move vertically
- The pulse is reflected
- The pulse is not inverted



Reflection of a Wave, Free End

Animation

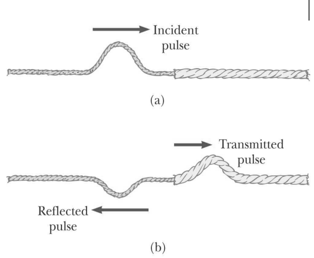




Transmission of a Wave, Case 1

- When the boundary is intermediate between the last two extremes (The right hand rope is massive or massless.) then part of the energy in the incident pulse is reflected and part is **transmitted**
- Some energy passes through the boundary
- Here $\mu_{rhs} > \mu_{lhs}$

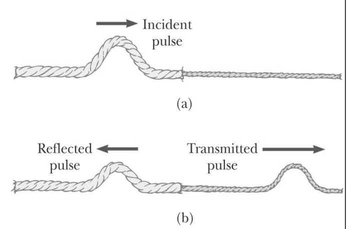
Animation



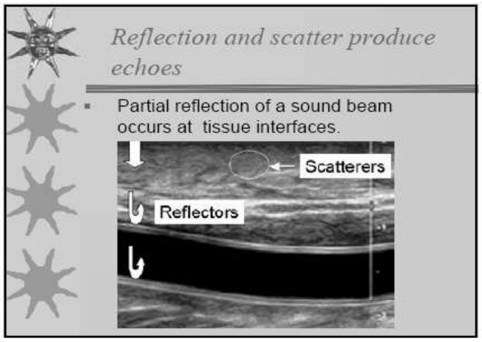
Transmission of a Wave, Case 2

- Now assume a heavier string is attached to a light string
- Part of the pulse is reflected and part is transmitted
- The reflected part is not inverted

Animation

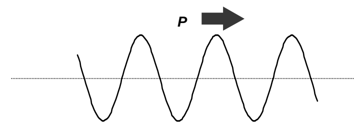


From Prof. Zagzebski's seminar on Ultrasound



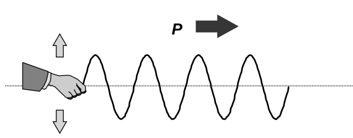
Wave Power

- A wave propagates because each part of the medium transfers its motion to an adjacent region.
  - ❖ Energy is transferred since work is done !
- How much energy is moving down the string per unit time. (i.e. how much *power* ?)



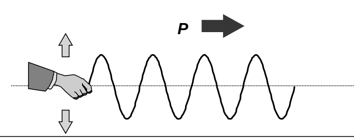
Wave Power...

- Think about grabbing the left side of the string and pulling it up and down in the *y* direction.
- You are clearly doing work since $\mathbf{F} \cdot d\mathbf{r} > 0$ as your hand moves up and down.
- This energy must be moving away from your hand (to the right) since the kinetic energy (motion) of the string stays the same.



Wave Power...

- Power is the energy transferred per unit time dE/dt
- So what is the energy density? (Energy / Length)
- For SHM $E = \frac{1}{2} k A^2$ with $\omega^2 = k / m$
- In one wavelength $E = \frac{1}{2} \Delta m \omega^2 A^2 = \frac{1}{2} \lambda\mu\, \omega^2 A^2$
- In one period $\mathcal{P}_{avg} = \Delta E / \Delta T = \frac{1}{2} \lambda\mu\, \omega^2 A^2 / T$ and $\lambda / T = v$
- So $\mathcal{P}_{avg} = \frac{1}{2} \mu\, \omega^2 A^2\, v$ and $v = (F/\mu)^{1/2}$



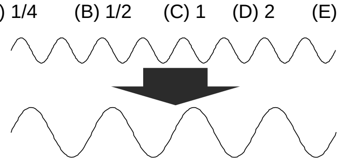
Lecture 21, *Exercise 4*
Wave Power

- A wave propagates on a string. If just the amplitude and the wavelength are doubled, by what factor will the average power carried by the wave change ?

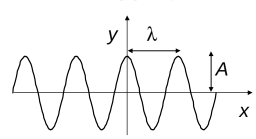
$P_{final}/P_{init} = ?$

Recall $\mathcal{P}_{avg} = \frac{1}{2} \mu\, \omega^2 A^2 v$ and $\lambda / T = v = \omega / k = \lambda\, \omega / 2\pi$

(A) 1/4 (B) 1/2 (C) 1 (D) 2 (E) *4*



Recapping



- General harmonic waves

$$y(x,t) = A\cos(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda} \qquad \omega = 2\pi f = \frac{2\pi}{T}$$

$$v = \lambda f = \frac{\omega}{k}$$

- Waves on a string

$$v = \sqrt{\frac{F}{\mu}}$$ (F → tension, μ → mass / length)

$$\bar{P} = \frac{1}{2}\mu v \omega^2 A^2$$

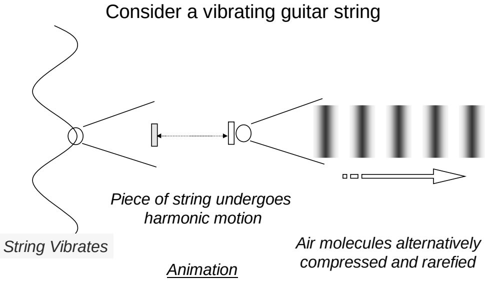
$$\frac{d\bar{E}}{dx} = \frac{1}{2}\mu\omega^2 A^2$$

Transverse and longitudinal waves

- Transverse waves: Displacement is perpendicular to the energy flow (velocity). Examples include water waves, waves in a rope, S-waves, ….
- Longitudinal waves: Amplitude and velocity have the same "direction".
  - ❖ Examples: Sound waves, P-waves
- Note: Longitudinal waves travel faster than transverse waves (i.e., a larger modulus or spring constant)!

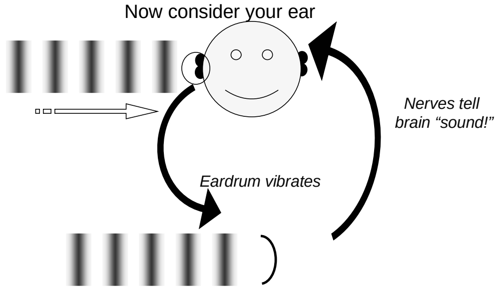
Chapter 17: Sound, A special kind of longitudinal wave

Consider a vibrating guitar string



Sound

Now consider your ear



Speed of Sound Waves, General

- The speed of sound waves in a medium depends on the compressibility and the density of the medium
- The compressibility can sometimes be expressed in terms of the elastic modulus of the material
- The speed of all mechanical waves follows a general form:

$$v = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}}$$

Waves on a string → $v = \sqrt{\frac{T}{\mu}}$

Speed of Sound in Liquid or Gas

- The bulk modulus of the material is *B*
- The density of the material is ρ
- The speed of sound in that medium is

$$v = \sqrt{\frac{B}{\rho}}$$

Speed of Sound in a Solid Rod

- The Young's modulus of the material is *Y*
- The density of the material is ρ
- The speed of sound in the rod is

$$v = \sqrt{\frac{Y}{\rho}}$$

Speed of Sound in Air

- The speed of sound also depends on the temperature of the medium
- This is particularly important with gases
- For air, the relationship between the speed and temperature is
  - ❖The 331 m/s is the speed at 0º C
  - ❖T_C is the air temperature in Centigrade

$$v = (331 \text{ m/s}) \sqrt{1 + \frac{T_c}{273^\circ \text{C}}}$$

Sound Level: How loud is loud?

- The range of intensities detectible by the human ear is very large
- It is convenient to use a logarithmic scale to determine the **intensity level**, β

$$\beta = 10 \ \log_{10} \left(\frac{I}{I_0} \right)$$

Sound Level

- I_0 is called the **reference intensity**
  - ❖ It is taken to be the threshold of hearing
  - ❖ I_0 = 1.00 x 10^{-12} W/ m^2
  - ❖ I is the intensity of the sound whose level is to be determined
- β is in decibels (dB)
- Threshold of pain: I = 1.00 W/m^2; β = 120 dB
- Threshold of hearing: I_0 = 1.00 x 10^{-12} W/ m^2 ; β = 0 dB

Sound Level, Example

- What is the sound level that corresponds to an intensity of 2.0 x 10^{-7} W/m^2 ?
- β = 10 log (2.0 x 10^{-7} W/m^2 / 1.0 x 10^{-12} W/m^2)
  = 10 log 2.0 x 10^5 = 53 dB
- Rule of thumb: An apparent "doubling" in the loudness is approximately equivalent to an increase of 10 dB.
- This factor is not linear with intensity

Loudness and Intensity

- Sound level in decibels relates to a *physical measurement* of the strength of a sound
- We can also describe a *psychological "measurement"* of the strength of a sound
- Our bodies "calibrate" a sound by comparing it to a reference sound
- This would be the threshold of hearing
- Actually, the threshold of hearing is this value for 1000 Hz

Lecture 21 Recap

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