Physics 207, Lecture 23, Nov. 22

- Agenda: Catch up
- Chapter 18, Superposition and Standing Waves
  - ❖ Superposition
  - ❖ Interference
  - ❖ Standing Waves
  - ❖ Nodes, Anti-nodes

Assignments:

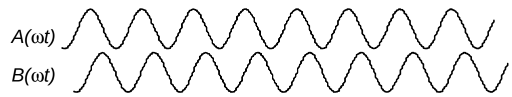
- Problem Set 9 due Tuesday, Dec. 5, 11:59 PM

Ch. 18: 3, 18, 30, 40, 58

- Mid-term 3, Tuesday, Nov. 28, Chapters 14-17, 90 minutes, 7:15-8:45 PM in rooms 105 and 113 Psychology
- Monday is a review for Tuesday's mid-term

Superposition & Interference (How do waves add)

- Consider two harmonic waves *A* and *B* meeting.
  - ❖ Same frequency and amplitudes, but phases differ (ϕ).
- The displacement versus time for each is shown below:



What does $C(t) = A(t) + B(t)$ look like ?

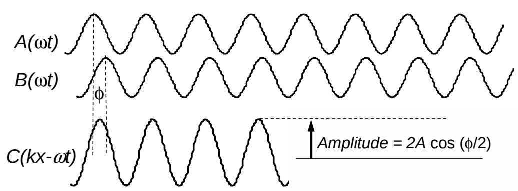
Wave Superposition

Superposition & Interference

- Consider A + B,

  $A(x,t)=A\cos(kx-\omega t)$ $B(x,t)=A\cos(kx-\omega t+\phi)$

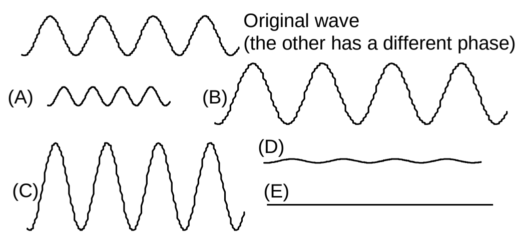
  - ❖ We can show: $C = 2A\cos(\phi/2)\cos(kx-\omega t+\phi/2)$
  - ❖ Using half-angle identities……see text 18.1



Phase shift = $\phi / 2$

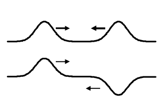
Lecture 23, *Exercise 1* Superposition

- Two continuous harmonic waves with the same frequency and amplitude but, at a certain time, have a phase difference of 170° are superimposed. Which of the following best represents the resultant wave at this moment?



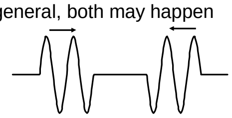
Superposition & Interference

- We have just seen that when waves combine (superimpose) the result can either be bigger or smaller than the original waves.
- Waves can add "constructively" or "destructively" depending on the relative sign of each wave.



- In general, both may happen

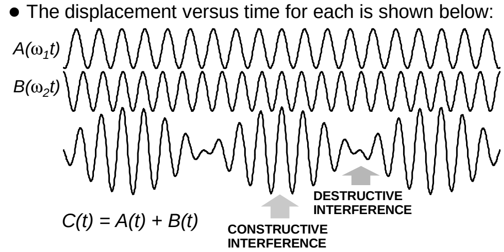
Pulse Superposition



Superposition & Interference

- Consider two harmonic waves *A* and *B* meet at $t=0$.
  - ❖ They have same amplitudes and phase, but

    $\omega_2 = 1.15 \times \omega_1$. Beat Superposition
- The displacement versus time for each is shown below:



$C(t) = A(t) + B(t)$

Aside: Why superposition works

- The equation governing waves (Chapter 16, "the Wave Equation") is linear. For linear equations, if we have two (or more) separate solutions, f_1 and f_2 , then $B f_1 + C f_2$ is also a solution .
- For linear equations, if we have two (or more) separate solutions, f_1 and f_2 , then $B f_1 + C f_2$ is also a solution
- This is called the "Superposition Principle"
- You have already seen this in the case of simple harmonic motion:

$$\frac{d^2x}{dt^2} = -\omega^2 x \qquad \text{linear in } x\text{ !}$$

$$\Rightarrow \quad x(t) = B\sin(\omega t) + C\cos(\omega t)$$

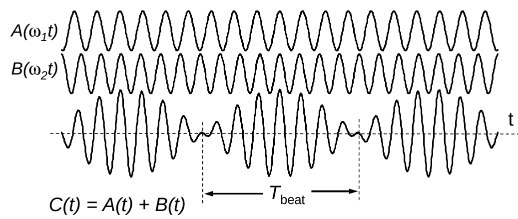
Superposition & Interference

- Consider A + B,

$y_A(x,t) = A\cos(k_1 x - \omega_1 t)$ $\qquad y_B(x,t) = A\cos(k_2 x - \omega_2 t)$

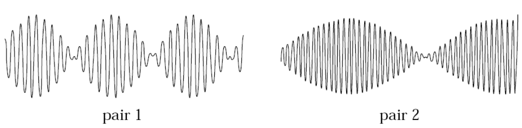
And let x=0, $y = y_A + y_B = 2A\cos[2\pi(f_1 - f_2)t/2]\cos[2\pi(f_1 + f_2)t/2]$

and $|f_1 - f_2| \equiv f_{beat} = 1 / T_{beat}$



Lecture 23, *Exercise 2*
Superposition

- The traces below show beats that occur when two different pairs of waves are added (the time axes are the same).
- For which of the two is the difference in frequency of the original waves greater?



A. Pair 1

B. Pair 2

C. The frequency difference was the samefor both pairs of waves.

D. Need more information.

Interference of Waves

- A path corresponds to a phase [recall the $\cos(2\pi x/\lambda)$]

  Path (or x) → phase = 2π (path/ λ) (modulo 2π)
- If two waves start out "in-phase" (at the same time) and then travel different distances before they are superimposed then the path difference, ΔL, corresponds to a phase difference with constructive or destructive interference.

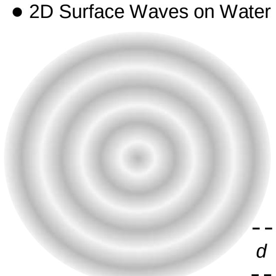
$$\Delta L = \frac{\phi}{2\pi}\lambda = 2n\frac{\lambda}{2} \quad \text{(constructive)}$$

$$\Delta L = \frac{\phi}{2\pi}\lambda = (2n+1)\frac{\lambda}{2} \quad \text{(destructive)}$$

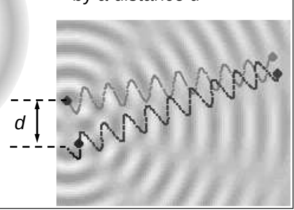
with $n = 0, 1, 2, \ldots$

Interference of Waves

- 2D Surface Waves on Water



In phase sources separated by a distance *d*



Interference of Sound

Sound waves interfere, just like transverse waves do. The resulting wave (displacement, pressure) is the sum of the two (or more) waves you started with.

$$\Delta L = |L_1 - L_2|$$

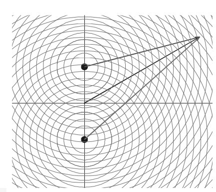
$$\phi = 2\pi\frac{\Delta L}{\lambda}$$

Constructive interference: $\phi = n(2\pi)$

$$\frac{\Delta L}{\lambda} = 0, 1, 2, 3, \ldots$$

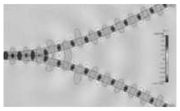
Destructive interference: $\phi = (2n+1)\pi$

$$\frac{\Delta L}{\lambda} = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \ldots$$

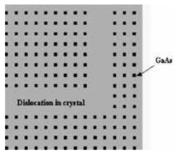


Interference of Waves, Splitting and Guiding

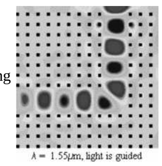
- Controlling wave sources is exploited in numerous applications



Optical Y Splitter



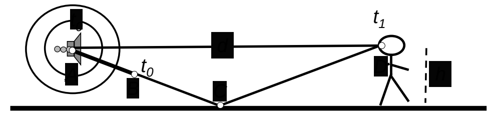
A Crystal with Line Defect Acting as a Waveguide:
Si (n=3.4); Period A = 0.58mm;
Filling Factor = 5/16;
Excitation λ = 1.55μm



Light turning a corner

Lecture 23, *Example* Interference

- A speaker sits on a pedestal 2 m tall and emits a sine wave at 343 Hz (the speed of sound in air is 343 m/s, so λ = 1m). Only the direct sound wave and that which reflects off the ground at a position half-way between the speaker and the person (also 2 m tall) makes it to the persons ear.
- How close to the speaker can the person stand (A to D) so they hear a maximum sound intensity assuming there is no phase change at the ground (this is a bad assumption)?



The distances AD and BCD have equal transit times so the sound waves will be in phase. The only need is for AB = 1 wavelength

Lecture 23, *Example* Interference

- The geometry dictates everything else.

AB = λ AD = BC+CD = BC + $(h^2 + (d/2)^2)^{½}$ = d

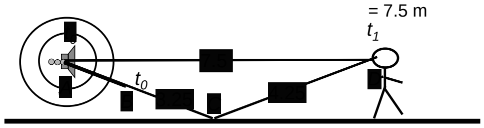
AC = AB+BC = λ +BC = $(h^2 + d/2^2)^{½}$

Eliminating BC gives λ+d = 2 $(h^2 + d^2/4)^{½}$

$\lambda + 2\lambda d + d^2 = 4 h^2 + d^2$

1 + 2d = 4 h^2 / λ → d = 2 h^2 / λ – ½

= 7.5 m



Because the ground is more dense than air there will be a phase change of π and so we really should set AB to λ/2 or 0.5 m.

Main point

- Path differences will give phase differences.
- This will lead to a superposition with constructive or destructive interference.
- If two waves start out “in-phase” (at the same time) and then travel different distances before they are superimposed then the path difference, ΔL, corresponds to a phase difference with:

$$\Delta L = \frac{\phi}{2\pi}\lambda = 2n\frac{\lambda}{2} \text{ (constructive)}$$

$$\Delta L = \frac{\phi}{2\pi}\lambda = (2n+1)\frac{\lambda}{2} \text{ (destructive)}$$

Standing Waves: A special kind of superposition

- Consider A + B, same λ and ω but traveling to the left and right.

A(x,t)=A cos(*kx*–ωt) *B(x,t)=A* cos(*kx*+ωt+π)

Now C(x,t) = 2A cos(2π *x*/λ) cos(ωt) and there is no net energy flow. If $\phi = \pi/2$ then

C′(x,t) = 2A sin(2π *x*/λ) sin(ωt)

These are “standing waves”.

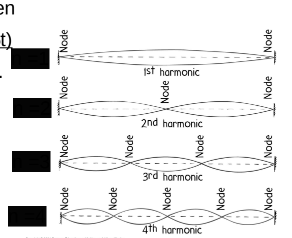
- This describes motion on a bound string (length *L*)

C(0,t) = C(L,t) = 0 if

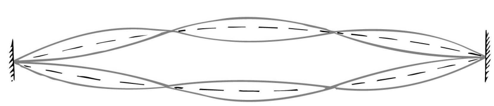
$L = n\,\lambda/2$ → $\lambda = 2\,L/n$

- Or more generally

C′(x,t) = 2A sin(πn *x/L*) sin(ωt)



Guitar Strings

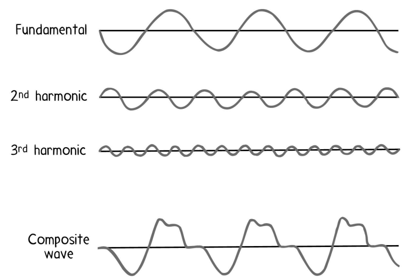


A combination wave composed of the 1st harmonic and the third harmonic.

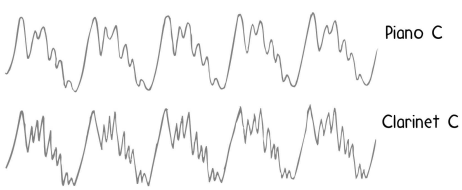
Music

- What makes instruments unique is the combination of harmonics produced by the different instruments.
- Flutes produce primarily the 1st harmonic
- They have a very pure tone
- Oboes produce a broad range of harmonics and sound very different

Combining Waves Revisited



Combining Waves



Fourier Synthesis

Musical Instruments

- Three ways to make sound
- Vibrate a string
- Vibrate an air column
- Vibrate a membrane

Vibrating Strings

- Violin, viola, cello, string bass
- Guitars
- Ukuleles
- Mandolins
- Banjos
- All vibrate a structure to "amplify" the sound

Vibrating Air Columns

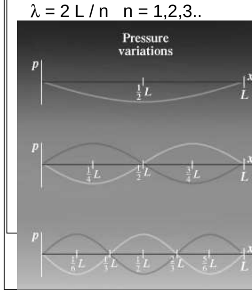
- Pipe Organs
- Brass Instruments
- Woodwinds
- Whistles

Vibrating Membranes

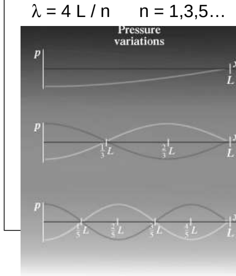
- Percussion Instruments
- Drums
- Bongos

Standing Waves in Pipes

Open at both ends:
Pressure(speed) Node at ends
Displacement AntiNode at ends
$\lambda = 2L/n \quad n = 1,2,3..$



Open at one end:
Pressure AntiNode at closed end
Displacement Node at closed end
$\lambda = 4L/n \quad n = 1,3,5\ldots$



Organ Pipe Example

A 0.9 m organ pipe (open at both ends) is measured to have it's first harmonic (i.e., its fundamental) at a frequency of 382 Hz. What is the speed of sound (refers to energy transfer) in the pipe?

L=0.9 m

$f = 382\ Hz$ and $f\,\lambda = v$ with $\lambda = 2\ L / n$ (n = 1)

$v = 382 \times 2(0.9)$ m → $v = 687$ m/s

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Lecture 23, *Exercise 3*
Standing Waves

- What happens to the fundamental frequency of a pipe, if the air (v =300 m/s) is replaced by helium (v = 900 m/s)?

  Recall: $f\,\lambda = v$

(A) Increases (B) Same (C) Decreases

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Recap, Lecture 23

- Agenda: Catch up
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  - ❖ Superposition
  - ❖ Interference
  - ❖ Standing Waves
  - ❖ Nodes, Anti-nodes

Assignments:

- Problem Set 9 due Tuesday, Dec. 5, 11:59 PM

Ch. 18: 9, 17, 21, 39, 53a (tentative)

- Mid-term 3, Tuesday, Nov. 28, Chapters 14-17, 90 minutes, 7:15-8:45 PM in rooms 105 and 113 Psychology
- Monday is a review session for Tuesday's mid-term
- Have a good Thanksgiving holiday and see you Monday!

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