Physics 207 – Lecture 27

**1st Law: Work & Heat**
- Two types of variables:
  - State variables: describe the system (e.g., T, P, V, U).
  - Transfer variables: describe the process (e.g., Q, W).
- Transfer variables involve change in state variables.
- Work done on gas (minus sign because system volume is referenced):
  \[ W = \int F \, dx \]
- Valid only for isobaric processes (P constant).
- If not, use average force or calculus:
  \[ W = \int F \, dx = \int P \, dV \]
- Work depends on the path taken in the PV-diagram:
- Same for Q (heat), depends on path.

**1st Law: Work (Area under the curve)**
- Work depends on the path taken in the PV-diagram:
- (a) \( W_a = W_{1b2} + W_{3b4} \) (here either P or V constant)
- \( W_a = P_1 (V_f - V_i) > 0 \) (work done on system)
- (b) \( W_b = W_{1b2} + W_{3b4} \) (here either P or V constant)
- \( W_b = 0 - P_i (V_f - V_i) > W_a > 0 \) (work done on system)
- (c) Need explicit form of P versus V but \( W_i > 0 \)

**Reversing the path (3 → 2 → 1)**
- Work depends on the path taken in the PV-diagram:
- (a) \( W_a = W_{1b2} + W_{2b3} \) (here either P or V constant)
- \( W_a = 0 - P_i (V_f - V_i) < 0 \) (work done on system)
- (b) \( W_b = W_{1b2} + W_{2b3} \) (here either P or V constant)
- \( W_b = P_i (V_f - V_i) < 0 \) (work done on system)
- (c) Need explicit form of P versus V but \( W_i < 0 \)

**1st Law: Work (going full cycle)**
- Work depends on the path taken in the PV-diagram:
- (a) \( W_a = W_{1b2} + W_{2b3} \) (here either P or V constant)
- \( W_a = P_i (V_f - V_i) > 0 \) (work done on system)
- (b) \( W_a = W_{1b4} + W_{4b5} \) (here either P or V constant)
- \( W_a = -P_i (V_f - V_i) < 0 \) (work done by system > 0)
- (a) & (b) \( W_j + W_{1b} = -P_i (V_f - V_i) - P_i (V_f - V_i) \) but net work done by system ... (what I get to use) ... is positive.
First Law of Thermodynamics with heat (Q) and/or work (W)

- First Law of Thermodynamics
  \[ \Delta U = Q + W \]
  - work done on the system
  - heat flow in (+) or out (-)
  - variation of internal energy

- \( \Delta U \) is independent of path in PV-diagram
- Depends only on state of the system \((P,V,T,\ldots)\)

- Isolated system is defined as one with
  - No interaction with surroundings
  - \( Q = W = 0 \Rightarrow \Delta U = 0 \)
  - \( U_f = U_i \): internal energy remains constant.

Other Applications

- Cyclic process:
  - Process that starts and ends at the same state \((P_i = P_f, T_i = T_f, V_i = V_f)\)
  - Must have \( \Delta U = 0 \Rightarrow Q = -W \).

- Adiabatic process:
  - No energy transferred through heat \( Q = 0 \).
  - So, \( \Delta U = W \).
  - Important for
    - Expansion of gas in combustion engines
    - Liquefaction of gases in cooling systems, etc.

  - Isobaric process: \((P \text{ is constant})\)
    - Work (on system) is:
      \[ W = - \int_{V_i}^{V_f} P \, dV = -nRT \ln \frac{V_f}{V_i} \]

  - \( PV \) is constant.
  - PV-diagram: isotherm

Other Applications (continued)

- Isovolumetric process:
  - Constant volume \( \Rightarrow W = 0 \).
  - So \( \Delta U = Q \Rightarrow all \text{ heat is transferred into internal energy} \)
    - e.g. heating a “can” (and no work done).

- Isothermal process:
  - \( T \) is constant
  - If ideal gas: \( PV = nRT \), we find \( P = nRT/V \).
  - Work (on system) becomes:
    \[ W = - \int_{V_i}^{V_f} P \, dV = -nRT \ln \frac{V_f}{V_i} \]
  - \( PV \) is constant.
  - PV-diagram: isotherm

Lecture 27: Exercise 2 Processes

- Identify the nature of paths A, B, C, and D
  - Isobaric, isothermal, isovolumetric, and adiabatic
We now try to do more than just raise the temperature of an object by adding heat. We want to add heat and get some work done!

Heat engines:
- Purpose: Convert heat into work using a cyclic process
- Example: Cycle a piston of gas between hot and cold reservoirs (Stirling cycle)
  1) hold volume fixed, raise temperature by adding heat
  2) hold temperature fixed, do work by expansion
  3) hold volume fixed, lower temperature by draining heat
  4) hold temperature fixed, compress back to original V

Example: The Stirling cycle

We can represent this cycle on a P-V diagram:

Heat is ADDED or REMOVED from the gas
Positive work is done BY or ON the gas for each step of the Stirling cycle:

\[ \Delta U = Q + W \text{ (references system)} \]

Ch. 21: Kinetic Theory of an Ideal Gas

Microscopic model for a gas
- Goal: relate T and P to motion of the molecules
- Assumptions for ideal gas:
  - Number of molecules N is large
  - They obey Newton's laws (but move randomly as a whole)
  - Short-range interactions during elastic collisions
  - Elastic collisions with walls (an impulse)
  - Pure substance: identical molecules
- This implies that temperature, for an ideal gas, is a direct measure of average kinetic energy of a molecule
Consider a fixed volume of ideal gas. When N or T is doubled the pressure increases by a factor of 2.

1. If T is doubled, what happens to the rate at which a single molecule in the gas has a wall bounce?
   (A) \(x_{1.4}\)  (B) \(x_2\)  (C) \(x_4\)

2. If N is doubled, what happens to the rate at which a single molecule in the gas has a wall bounce?
   (A) \(x_1\)  (B) \(x_{1.4}\)  (C) \(x_2\)

Theorem of equipartition of energy (A key result of classical physics)

Each degree of freedom contributes \(k_B T/2\) to the energy of a system (e.g., translation, rotation, or vibration)

Total translational kinetic energy of a system of N ideal gas molecules

\[
K_{\text{tot\ trans}} = N \left( \frac{1}{2} m v^2 \right) = \frac{3}{2} N k_B T = \frac{3}{2} nRT
\]

Internal energy of monoatomic gas: \(U = K_{\text{ideal\ gas}} = K_{\text{trans}}\)

Root-mean-square speed:

\[
\sqrt{\frac{m v_{\text{rms}}^2}{m}} = \sqrt{3k_B T/m}
\]

An atom in a classical solid can be characterized by three independent harmonic oscillators, one for the x, y and z-directions?

What is the classical molar heat capacity (\(\Delta V \approx 0\))?

\[
C_p = \frac{3}{2} R + R = \frac{3}{2} R + R
\]

An atom in a classical solid can be characterized by three independent harmonic oscillators, one for the x, y and z-directions (U per atom = 3 RT)?

What is the classical molar heat capacity (\(\Delta V \approx 0\))?

\[
C_p = \frac{3}{2} R + R
\]
**Adiabatic Processes**

- By definition a process in which no heat transfer \( (Q) \) occurs.
- For an ideal gas:
  \[
P V^\gamma = \text{const}
\]
  \[
\gamma \equiv \frac{C_p}{C_v}
\]

- Adiabatic process:
  - If ideal gas then \( PV \) is constant
  - \( PV = nRT \) but not isothermal
  - Work (on system) becomes:
  \[
  W = -\frac{\gamma}{\gamma - 1} P dV = \frac{\gamma}{\gamma - 1} V \left( V^{-\gamma} - V^{-1} \right)
  \]

**Recap, Lecture 27**

- Agenda: Ch. 20, 1st Law of Thermodynamics, Ch. 21
  - 1st Law of thermodynamics \( (\Delta U = Q + W) \)
  - Work done by an ideal gas in a piston
  - Introduction to thermodynamic cycles (Chapter 22)
  - Ideal gas at the molecular level, Internal Energy
  - Degree of Freedom and Equipartition theorem
  - Adiabatic processes (no heat transfer)

Assignments:

- Problem Set 10 (Ch. 20 & 21) due Tuesday, Dec. 12, 11:59 PM
  Ch. 20: 13, 22, 38, 43, 50, 68 Ch. 21: 2, 16, 29, 36, 70
- Finish Ch. 21, Monday, Read Chapter 22 (2nd Law of Thermodynamics)