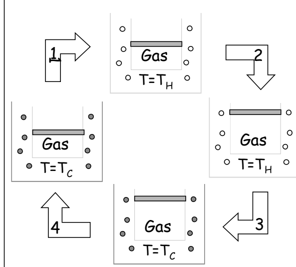
Physics 207, Lecture 29, Dec. 13

- Agenda: Finish Ch. 22, Start review, Evaluations
  - ❖ Heat engines and Second Law of thermodynamics
  - ❖ Carnot cycle
  - ❖ Reversible/irreversible processes and Entropy

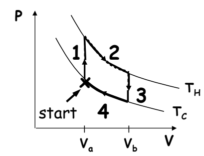
Assignments:
- Problem Set 11, Ch. 22: 6, 7, 17, 37, 46 (Due, Friday, Dec. 15, 11:59 PM)
- Friday, Review

Heat Engines

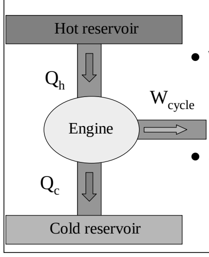
- Example: The Stirling cycle



We can represent this cycle on a P-V diagram:



Heat Engines and the 2nd Law of Thermodynamics



- A heat engine goes through a cycle (start and stop at the same point, same state variables)
  - ❖ 1st Law gives

    $\Delta U = Q + W = 0$
- What goes in must come out
  - ❖ 1st Law gives

    $Q_h = Q_c + W_{cycle}$ (Q's > 0)
- So (cycle mean net work on world)

  $Q_{net} = |Q_h| - |Q_c| = -W_{system} = W_{cycle}$

Efficiency of a Heat Engine

- How can we define a "figure of merit" for a heat engine?
- Define the efficiency ε as:

$$\varepsilon = \frac{W_{\text{cycle}}}{|Q_h|} = \frac{|Q_h| - |Q_c|}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|}$$

Lecture 29: *Exercise 1* Efficiency

- Consider two heat engines:
  - ❖ Engine I:
    - ➢ Requires Q_{in} = 100 J of heat added to system to get W=10 J of work (done on world in cycle)
  - ❖ Engine II:
    - ➢ To get W=10 J of work, Q_{out} = 100 J of heat is exhausted to the environment
- Compare ε_I, the efficiency of engine I, to ε_{II}, the efficiency of engine II.

$$\varepsilon = \frac{W_{\text{cycle}}}{|Q_h|} = \frac{|Q_h| - |Q_c|}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|}$$

(A) $\varepsilon_I < \varepsilon_{II}$

(B) $\varepsilon_I > \varepsilon_{II}$

(C) Not enough data to determine

Reversible/irreversible processes and the best engine, ever

- Reversible process:
  - ❖ Every state along some path is an equilibrium state
  - ❖ The system can be returned to its initial conditions along the same path
- Irreversible process;
  - ❖ Process which is not reversible !
- All real physical processes are irreversible
  - ❖ e.g. energy is lost through friction and the initial conditions cannot be reached along the same path
  - ❖ However, some processes are almost reversible
    - ➢ If they occur slowly enough (so that system is almost in equilibrium)

The Carnot cycle

Carnot Cycle

Named for Sadi Carnot (1796- 1832)

(1) Isothermal expansion
(2) Adiabatic expansion
(3) Isothermal compression
(4) Adiabatic compression

The Carnot Engine (the best you can do)

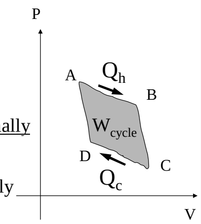
- No real engine operating between two energy reservoirs can be more efficient than a Carnot engine operating between the same two reservoirs.

A. A→B, the gas expands isothermally while in contact with a reservoir at T_h

B. B→C, the gas expands adiabatically ($Q=0$, $\Delta U=W_{B\to C}$, $T_h \to T_c$), PV^γ=constant

C. C→D, the gas is compressed isothermally while in contact with a reservoir at T_c

D. D→A, the gas compresses adiabatically ($Q=0$, $\Delta U=W_{D\to A}$, $T_c \to T_h$)



Carnot Cycle Efficiency

$\varepsilon_{Carnot} = 1 - Q_c/Q_h$

$Q_{A\to B} = Q_h = W_{AB} = nRT_h \ln(V_B/V_A)$

$Q_{C\to D} = Q_c = W_{CD} = nRT_c \ln(V_D/V_C)$

(here we reference work done by gas, $dU = 0 = Q - P\,dV$)

But $P_AV_A=P_BV_B=nRT_h$ and $P_CV_C=P_DV_D=nRT_c$

so $P_B/P_A=V_A/V_B$ and $P_C/P_D=V_D/V_C$

as well as $P_BV_B^\gamma=P_CV_C^\gamma$ and $P_DV_D^\gamma=P_AV_A^\gamma$

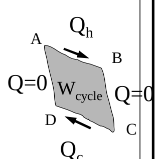
with $P_BV_B^\gamma/P_AV_A^\gamma=P_CV_C^\gamma/P_DV_D^\gamma$ thus

→ $(V_B/V_A)=(V_D/V_C)$

$Q_c/Q_h = T_c/T_h$

Finally

$\varepsilon_{Carnot} = 1 - T_c/T_h$



The Carnot Engine

- Carnot showed that the thermal efficiency of a Carnot engine is:

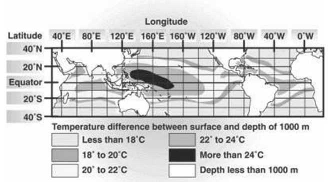
$$\varepsilon_{\text{Carnot cycle}} = 1 - \frac{T_{cold}}{T_{hot}}$$

- All real engines are less efficient than the Carnot engine because they operate irreversibly due to the path and friction as they complete a cycle in a brief time period.

Power from ocean thermal gradients… oceans contain large amounts of energy

Carnot Cycle Efficiency

$\varepsilon_{Carnot} = 1 - Q_c/Q_h = 1 - T_c/T_h$



See: http://www.nrel.gov/otec/what.html

Ocean Conversion Efficiency

$\varepsilon_{Carnot} = 1 - Q_c/Q_h = 1 - T_c/T_h$

$\varepsilon_{Carnot} = 1 - T_c/T_h = 1 - 275\text{ K}/300\text{ K}$

$= 0.083$ (even before internal losses and assuming a REAL cycle)

Still: "This potential is estimated to be about 10^{13} watts of base load power generation, according to some experts. The cold, deep seawater used in the OTEC process is also rich in nutrients, and it can be used to culture both marine organisms and plant life near the shore or on land."

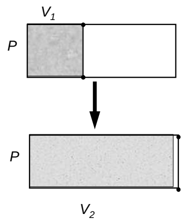
"Energy conversion efficiencies as high as 97% were achieved."

See: http://www.nrel.gov/otec/what.html

So $\varepsilon = 1 - Q_c/Q_h$ is always correct but

$\varepsilon_{Carnot} = 1 - T_c/T_h$ only reflects a Carnot cycle

Lecture 29: *Exercises 2 and 3*
Free Expansion and the 2^{nd} Law



You have an ideal gas in a box of volume V_1. Suddenly you remove the partition and the gas now occupies a larger volume V_2.

(1) How much work was done by the system?

(2) What is the final temperature (T_2)?

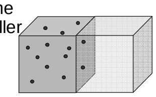
(3) Can the partition be reinstalled with all of the gas molecules back in V_1

1: (A) W > 0 (B) W =0 (C) W < 0

2: (A) $T_2 > T_1$ (B) $T_2 = T_1$ (C) $T_2 > T_1$

Entropy and the 2^{nd} Law

- Will the atoms go back without doing work?
  - ❖ Although possible, it is quite improbable
  - ❖ The are many more ways to distribute the atoms in the larger volume that the smaller one.
- Disorderly arrangements are much more probable than orderly ones



- Isolated systems tend toward greater disorder
  - ❖ Entropy (S) is a measure of that disorder
  - ❖ Entropy (ΔS) increases in all natural processes. (The 2^{nd} Law)
  - ❖ Entropy and temperature, as defined, guarantees the proper direction of heat flow.

Entropy and the 2^{nd} Law

$$dS \equiv \frac{dQ_r}{T}$$

- In a reversible process the total entropy remains constant, $\Delta S=0$!
- In a process involving heat transfer the change in entropy ΔS between the starting and final state is given by the heat transferred *Q* divided by the absolute temperature *T* of the system (if T is constant).

$$\Delta S = \frac{Q}{T}$$

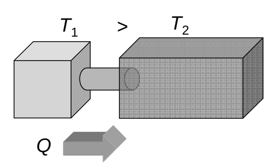
- The 2^{nd} Law of Thermodynamics
  "There is a quantity known as entropy that in a closed system always remains the same (reversible) or increases (irreversible)."
- Entropy, when constructed from a microscopic model, is a measure of disorder in a system.

Entropy, Temperature and Heat

- Example: Q joules transfer between two thermal reservoirs as shown below
- Compare the total change in entropy.

$$\Delta S = (-Q/T_1) + (+Q / T_2) > 0$$

because $T_1 > T_2$



Entropy and Thermodynamic processes

Examples of Entropy Changes:

Assume a reversible change in volume and temperature of an ideal gas by expansion against a piston held infinitesimally below the gas pressure ($dU = dQ – P\,dV$ with $PV = nRT$ and $dU/dT = C_v$):

$$\Delta S = \int_i^f dQ/T = \int_i^f (dU + PdV) / T \quad (=0)$$

$$\Delta S = \int_i^f \{C_v dT / T + nR(dV/V)\}$$

$$\Delta S = nC_v \ln (T_f /T_i) + nR \ln (V_f /V_i)$$

Ice melting:

$$\Delta S = \int_i^f dQ/T= Q/T_{melting} = m L_f /T_{melting}$$

Entropy and Thermodynamic processes

Examples of Entropy Changes:

Assume a reversible change in volume and temperature of an ideal gas by expansion against a piston held infinitesimally below the gas pressure ($dU = dQ – P\,dV$ with $PV = nRT$ and $dU/dT = C_v$):

So does $\Delta S = 0$?

$$\Delta S = nC_v \ln (T_f /T_i) + nR \ln (V_f/V_i)$$

$PV=nRT$ and PV^{γ} = constant → $TV^{\gamma-1}$ = constant

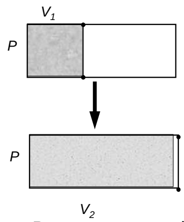
$$T_iV_i^{\gamma-1} = T_fV_f^{\gamma-1}$$

$T_f/T_i = (V_i/V_f)^{\gamma-1}$ and let $\gamma = 5/3$

$$\Delta S = 3/2\ nR \ln ((V_i/V_f)^{2/3}) + nR \ln (V_f/V_i)$$

$$\Delta S = nR \ln (V_i/V_f) - nR \ln (V_i/V_f) = 0 !$$

Lecture 29: ***Exercise***
Free Expansion and the 2nd Law



You have an ideal gas in a box of volume V_1. Suddenly you remove the partition and the gas now occupies a larger volume V_2.

Does the entropy of the system increase and by how much?

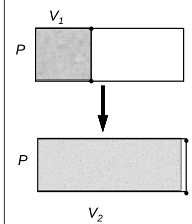
$\Delta S = nC_v \ln (T_f/T_i) + nR \ln (V_f/V_i)$

Because entropy is a state variable we can choose any path that get us between the initial and final state.

(1) Adiabatic reversible expansion (as above)

(2) Heat transfer from a thermal reservoir to get to T_i

Lecture 29: ***Exercise***
Free Expansion and the 2nd Law



You have an ideal gas in a box of volume V_1. Suddenly you remove the partition and the gas now occupies a larger volume V_2.

Does the entropy of the system increase and by how much?

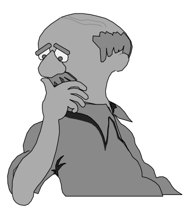
$\Delta S_1 = nC_v \ln (T_f/T_i) + nR \ln (V_f/V_i)$

(1) Heat transfer from a thermal reservoir to get to T_i

$\Delta S_2 = \int_i^f dQ/T = \int_i^f (dU + PdV)/T = \int_{T_f}^{T_i} nC_v\, dT/T = nC_v \ln(T_i/T_f)$

$\Delta S = \Delta S_1 + \Delta S_2 =$ ~~$nC_v \ln (T_f/T_i)$~~ $+ nR \ln (V_f/V_i)$ ~~$- nC_v \ln (T_f/T_i)$~~

The Laws of Thermodynamics



- First Law
  You can't get something for nothing.
- Second Law
  You can't break even.
- **Do not forget: Entropy, S, is a <u>state variable</u> just like P, V and T!**

Recap , Lecture 29

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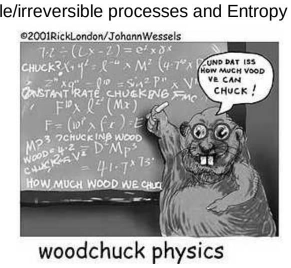
  - ❖ Heat engines and Second Law of thermodynamics
  - ❖ Reversible/irreversible processes and Entropy



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