Physics 207 – Lecture 2

Physics 207, Sept. 12, The inclined plane and unit conversion

Flight 173 ran out of fuel in flight….

So: "How does a jet run out of fuel at 26,000 feet?"

1. A maintenance worker found that the fuel gauge did not work on ground inspection. He incorrectly assured the pilot that the plane was certified to fly without a functioning fuel gauge if the crew checked the fuel tank levels.
2. Crew members measured the 2 fuel tank levels at 62 cm and 64 cm. This corresponded to 3758 L and 3924 L for a total of 7682 L according to the plane's manual.
3. The ground crew knew that the flight required 22,300 kg of fuel. The problem they faced was with 7,682 L of fuel on the plane, how many more liters were needed to total 22,300 kg of fuel?
4. One crew member informed the other that the "conversion factor" (being the fuel density) was 1.77. THE CRUCIAL FAULT BEING THAT NO ONE EVER INQUIRED ABOUT THE UNITS OF THE CONVERSION FACTOR. So it was calculated that the plane needed an additional 4,917 L of fuel for the flight. Alas that was too little.

Physics 207, Lecture 3, Sept. 12

Agenda

- Finish Chapters 2 & 3
  - ❖ One-Dimensional Motion with Constant Acceleration
  - ❖ Free-fall and Motion on an Incline
  - ❖ Coordinate systems

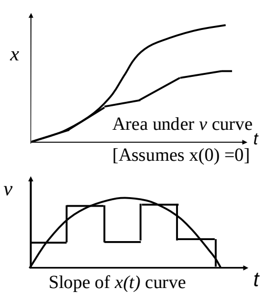
Assignment:

1. For Monday read Chapter 4
2. Homework Set 2 due Wednesday of next week (start ASAP)

Recall…in one-dimension

- If the position x is known as a function of time, then we can deduce the velocity v

$$x = x(t)$$
$$v = \frac{dx}{dt}$$
$$x - x(0) = \int_{t_0}^{t_1} dt \;\; v(t)$$



Representative examples of speed

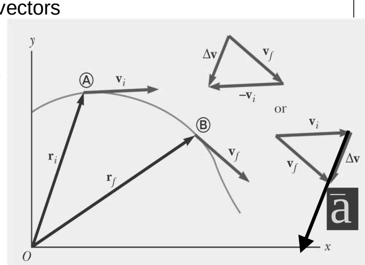
	Speed (m/s)
Speed of light	$3\text{x}10^8$
Electrons in a TV tube	10^7
Comets	10^6
Planet orbital speeds	10^5
Satellite orbital speeds	10^4
Mach 3	10^3
Car	10^1
Walking	1
Centipede	10^{-2}
Motor proteins	10^{-6}
Molecular diffusion in liquids	10^{-7}

Average Acceleration

- The average acceleration of a particle as it moves is defined as the change in the instantaneous velocity vector divided by the time interval during which that change occurs.
- Note: **bold** fonts are vectors

$$\overline{\mathbf{a}} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t_f - t_i} = \frac{\Delta\mathbf{v}}{\Delta t}$$

- The average acceleration is a vector quantity directed along $\Delta\mathbf{v}$



Instantaneous Acceleration

- The instantaneous acceleration is the limit of the average acceleration as $\Delta\mathbf{v}/\Delta t$ approaches zero

$$\mathbf{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}$$

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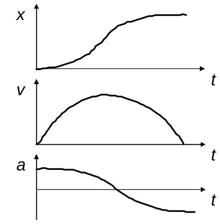
- Quick Comment: Instantaneous acceleration is a vector with components parallel (tangential) and/or perpendicular (radial) to the tangent of the path

(more in Chapter 6)

One step further…..in one dimension

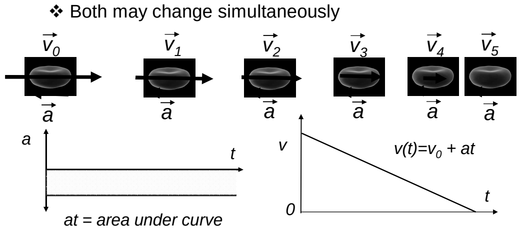
- If the position x is known as a function of time, then we can find both velocity v and acceleration a as a function of time!

$$x = x(t)$$
$$v = \frac{dx}{dt}$$
$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$



Acceleration

- Various changes in a particle's motion may produce an acceleration
  - ❖ The **magnitude** of the velocity vector may change
  - ❖ The **direction** of the velocity vector may change (Chapter 6, true even if the magnitude remains constant)
  - ❖ Both may change simultaneously



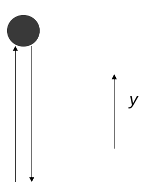
Acceleration has its limits

"High speed motion picture camera frame: John Stapp is caught in the teeth of a massive deceleration. One might have expected that a test pilot or an astronaut candidate would be riding the sled; instead there was Stapp, a mild mannered physician and diligent physicist with a notable sense of humor.
Source: US Air Force photo

Lecture 3, *Exercise 1*
Motion in One Dimension

When throwing a ball straight up, which of the following is true about its velocity v and its acceleration a at the highest point in its path?

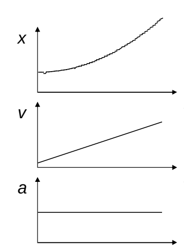
A. Both $v = 0$ and $a = 0$
B. $v \neq 0$, but $a = 0$
C. $v = 0$, but $a \neq 0$
D. None of the above



And given a <u>constant acceleration</u> we can integrate to get explicit *v* and *a*

$$x = x(t)$$
$$v = \frac{dx}{dt}$$
$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$v = v_0 + at$$
$$a = const$$



Rearranging terms gives two other relationships

- For constant acceleration:

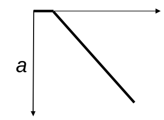
$$\boxed{\begin{aligned} x &= x_0 + v_0 t + \frac{1}{2}at^2 \\ v &= v_0 + at \\ a &= const \end{aligned}}$$

- From which we can show (caveat: **constant acceleration**):

$$\boxed{\begin{aligned} v^2 - v_0^2 &= 2a(x - x_0) \\ \bar{v}_{avg} &= \frac{1}{2}(v_0 + v) \end{aligned}}$$

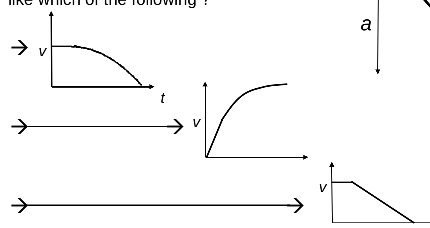
Lecture 3, Exercise 2
More complex Position *vs.* Time Graphs

In driving from Madison to Chicago, initially my speed is at a constant 65 mph. After some time, I see an accident ahead of me on I-90 and must stop quickly so I decelerate increasingly fast until I stop. The magnitude of my acceleration *vs* time is given by,



- Question: My velocity *vs* time graph looks like which of the following ?

A.
B.
C.



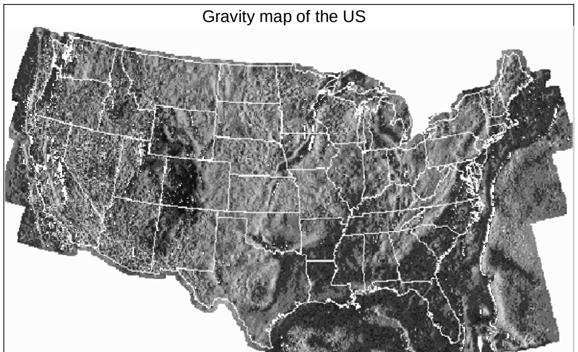
Free Fall

- When any object is let go it falls toward the ground !! The force that causes the objects to fall is called gravity.
- This acceleration caused by gravity is typically written as "little" ***g***
- Any object, be it a baseball or an elephant, experiences the same acceleration (***g***) when it is dropped, thrown, spit, or hurled, i.e. ***g*** is a constant.

Gravity facts:

- *g* does not depend on the nature of the material!
  - Galileo (1564-1642) figured this out without fancy clocks & rulers!
- demo - feather & penny in vacuum
- Nominally, g = 9.81 m/s^2
  - At the equator g = 9.78 m/s^2
  - At the North pole g = 9.83 m/s^2

Gravity map of the US

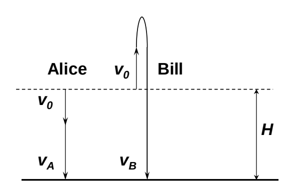


Red: Areas of stronger local g **Blue:** Areas of weaker local g
Due to **density variations** of the Earth's crust and mantle

Lecture 3, Exercise 3
1D Freefall

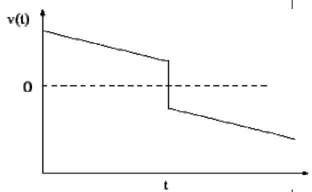
- Alice and Bill are standing at the top of a cliff of height *H*. Both throw a ball with initial speed v_0, Alice straight down and Bill straight up. The speed of the balls when they hit the ground are v_A and v_B respectively**.**

A. $v_A < v_B$
B. $v_A = v_B$
C. $v_A > v_B$



Lecture 3, Exercise 3A 1D Freefall

The graph at right shows the y velocity versus time graph for a ball. Gravity is acting downward in the -y direction and the x-axis is along the horizontal.



Which explanation best fits the motion of the ball as shown by the velocity-time graph below?

A. The ball is falling straight down, is caught, and is then thrown straight down with greater velocity.
B. The ball is rolling horizontally, stops, and then continues rolling.
C. The ball is rising straight up, hits the ceiling, bounces, and then falls straight down.
D. The ball is falling straight down, hits the floor, and then bounces straight up.
E. The ball is rising straight up, is caught and held for awhile, and then is thrown straight down.

Context Rich Problem: For discussion

- On a bright sunny day you are walking around the campus watching one of the many construction sites. To lift a bunch of bricks from a central area, they have brought in a helicopter. As the pilot is leaves he accidentally releases the bricks when they are 1000 m above the ground. A worker, directly below, stands for 10 seconds before walking away in 10 seconds. (Let $g = 10\ m/s^2$) There is no wind or other effects.

Does the worker live?

(Criteria for living…..the worker moves <u>before</u> the brick strike the ground)

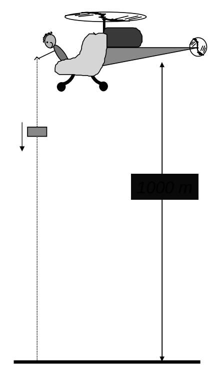
Problem Solution Method:

Five Steps:

1) **Focus the Problem**
   - **draw a picture – what are we asking for?**
2) **Describe the physics**
   - **what physics ideas are applicable**
   - **what are the relevant variables known and unknown**
3) **Plan the solution**
   - **what are the relevant physics equations**
4) **Execute the plan**
   - **solve in terms of variables**
   - **solve in terms of numbers**
5) **Evaluate the answer**
   - **are the dimensions and units correct?**
   - **do the numbers make sense?**

Problem:

1. We need to find the time it takes for the brick to hit the ground.
2. If $t > 10$ sec. then the worker is assured survival.



Tips:

- **Read !**
  - ❖Before you start work on a problem, read the problem statement thoroughly. Make sure you understand what information is given, what is asked for, and the meaning of all the terms used in stating the problem.
- **Watch your units (dimensional analysis) !**
  - ❖Always check the units of your answer, and carry the units along with your numbers during the calculation.
- **Ask questions !**

Problem #1 (At home)

- You are writing a short adventure story for your English class. In your story, two submarines on a secret mission need to arrive at a place in the middle of the Atlantic ocean at the same time. They start out at the same time from positions equally distant from the rendezvous point. They travel at different velocities but both go in a straight line. The first submarine travels at an average velocity of 20 km/hr for the first 500 km, 40 km/hr for the next 500 km, 30 km/hr for the next 500 km and 50 km/hr for the final 500 km. In the plot, the second submarine is required to travel at a constant velocity, which you wish to explicitly mention in the story. What is that velocity?
  - ❖a. Draw a diagram that shows the path of both submarines, include all of the segments of the trip for both boats.
  - ❖b. What exactly do you need to calculate to be able to write the story?
  - ❖c. Which kinematics equations will be useful?
  - ❖d. Solve the problem in terms of symbols.
  - ❖e. Does you answer have the correct dimensions (what are they)?
  - ❖f. Solve the problem with numbers.

Problem #2 (At home)

- As you are driving to school one day, you pass a construction site for a new building and stop to watch for a few minutes. A crane is lifting a batch of bricks on a pallet to an upper floor of the building. Suddenly a brick falls off the rising pallet. You clock the time it takes for the brick to hit the ground at 2.5 seconds. The crane, fortunately, has height markings and you see the brick fell off the pallet at a height of 22 meters above the ground. A falling brick can be dangerous, and you wonder how fast the brick was going when it hit the ground. Since you are taking physics, you quickly calculate the answer.
  - ❖ a. Draw a picture illustrating the fall of the brick, the length it falls, and the direction of its acceleration.
  - ❖ b. What is the problem asking you to find?
  - ❖ c. What kinematics equations will be useful?
  - ❖ d. Solve the problem in terms of symbols.
  - ❖ e. Does you answer have the correct dimensions?
  - ❖ f. Solve the problem with numbers.

Coordinate Systems and vectors, Chapter 3

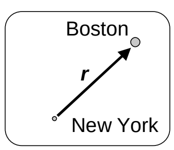
- In 1 dimension, only 1 kind of system,
  - ❖Linear Coordinates (x) +/-
- In 2 dimensions there are two commonly used systems,
  - ❖Cartesian Coordinates (x,y)
  - ❖Circular Coordinates (r,θ)
- In 3 dimensions there are three commonly used systems,
  - ❖Cartesian Coordinates (x,y,z)
  - ❖Cylindrical Coordinates (r,θ,z)
  - ❖Spherical Coordinates (r,θ,ϕ)

Vectors

- In 1 dimension, we can specify direction with a + or - sign.
- In 2 or 3 dimensions, we need more than a sign to specify the direction of something:
- To illustrate this, consider the ***position* vector r** in 2 dimensions.

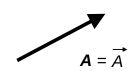
Example: Where is Boston?
- ❖ Choose **origin** at New York
- ❖ Choose coordinate system

Boston is 212 miles northeast of New York [in (r,θ)] OR
Boston is 150 miles north and 150 miles east of New York [in (x,y)]



Vectors...

- There are two common ways of indicating that something is a vector quantity:
  - ❖Boldface notation: ***A***



  - ❖“Arrow” notation: $\vec{A}$

Vectors have rigorous definitions

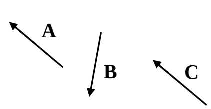
- A vector is composed of a magnitude and a direction
  - ❖ Examples: displacement, velocity, acceleration
  - ❖ Magnitude of **A** is designated |**A**|
  - ❖ Usually vectors include units (m, m/s, m/s^2)
- A vector has no particular position

(Note: the position vector reflects displacement from the origin)

- Two vectors are <u>equal</u> if their <u>directions</u>, <u>magnitudes</u> and <u>units</u> match.

A = **C**

A ≠ **B**, **B** ≠ **C**

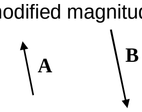


Comparing Vectors and Scalars

- A scalar is an ordinary number.
  - ❖ A magnitude **without** a direction
  - ❖ May have units (kg) or be just a number
  - ❖ Usually indicated by a regular letter, no bold face and no arrow on top.

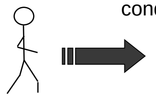
  Note: the lack of specific designation of a scalar can lead to confusion
- The **<u>product</u>** of a vector and a scalar is another vector in the same “direction” but with modified magnitude.

A = -0.75 **B**



Lecture 3, *Exercise 4 (Now for "homework")*
Vectors and Scalars

While I conduct my daily run, several quantities describe my condition



Which of the following is <u>cannot</u> be a vector ?

A) my <u>velocity</u> (3 m/s)

B) my <u>acceleration</u> downhill (30 m/s^2)

C) my <u>destination</u> (the pub - 100,000 m east)

D) my <u>mass</u> (150 kg)

Lecture 3, *Exercise 4*
Vectors and Scalars

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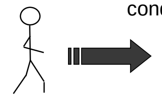

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C. my <u>destination</u> (the lab - 100,000 m east)
D. my <u>mass</u> (150 kg)

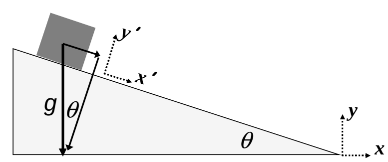
End of Class

- See you Monday!

Assignment:

- For Monday, Read Chapter 4
- Mastering Physics Problem Set 1, due tonight!
- Mastering Physics Problem Set 2, due next week but don't wait!

Resolving vectors, little g & the inclined plane

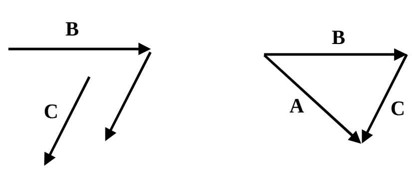


- **g** (bold face, vector) can be resolved into its *x,y* **or** *x′,y′* components
  - **g** = - g **j**
  - **g** = - g *cos θ* **j'** + g *sin θ* **i'**
  - The bigger the tilt the faster the acceleration…..along the incline

Vector addition

- The sum of two vectors is another vector.

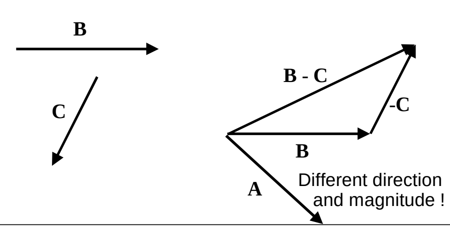
A = **B** + **C**



Vector subtraction

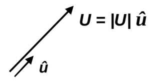
- Vector subtraction can be defined in terms of addition.

B - **C** = **B** + (-1)**C**



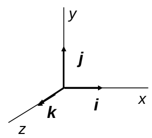
Unit Vectors

- A **Unit Vector** is a vector having length 1 and no units
- It is used to specify a direction.
- Unit vector ***u*** points in the direction of ***U***
  - ❖Often denoted with a "hat": ***u = û***



- Useful examples are the cartesian unit vectors [***i, j, k***]
  - ❖ Point in the direction of the *x*, *y* and *z* axes.

  $R = r_x \mathbf{i} + r_y \mathbf{j} + r_z \mathbf{k}$



Physics 207: Lecture 3, Pg 37

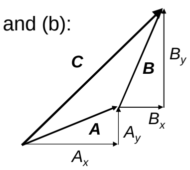
Vector addition using components:

- Consider ***C*** = ***A*** + ***B***.

(a) $C = (A_x \mathbf{i} + A_y \mathbf{j}) + (B_x \mathbf{i} + B_y \mathbf{j}) = (A_x + B_x)\mathbf{i} + (A_y + B_y)$

(b) $C = (C_x \mathbf{i} + C_y \mathbf{j})$

- Comparing components of (a) and (b):
  - ❖ $C_x = A_x + B_x$
  - ❖ $C_y = A_y + B_y$



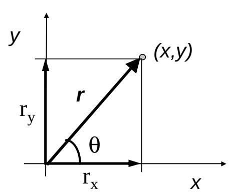
Physics 207: Lecture 3, Pg 38

Converting Coordinate Systems

- In ***polar*** coordinates the vector ***R*** = (r,θ)
- In Cartesian the vector ***R*** = (r_x, r_y) = (x,y)
- We can convert between the two as follows:

$$r_x = x = r\cos\theta$$
$$r_y = y = r\cos\theta$$
$$R = x\,\hat{\imath} + y\,\hat{\jmath}$$

$$r = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1}(y / x)$$



- In 3D cylindrical coordinates (*r*,θ,*z*), *r* is the same as the magnitude of the vector in the *x-y* plane [*sqrt*(x^2 +y^2)]

Physics 207: Lecture 3, Pg 39