Physics 207, Lecture 4, Sept. 17

Agenda
- Chapter 3, Chapter 4 (forces)
- Vector addition, subtraction and components
- Inclined plane
- Force
- Mass
- Newton’s 1st and 2nd Laws
- Free Body Diagrams

Assignment: Read Chapter 5
- MP Problem Set 2 due Wednesday (should have started)
- MP Problem Set 3, Chapters 4 and 5 (available soon)

Chapter 3, Chapter 4 (forces)
- Vector addition, subtraction and components
- Inclined plane
- Force
- Mass
- Newton’s 1st and 2nd Laws

Vector addition
- The sum of two vectors is another vector.

\[ A = B + C \]

Vector subtraction
- Vector subtraction can be defined in terms of addition.

\[ B - C = B + (-1)C \]

Unit Vectors
- A Unit Vector is a vector having length 1 and no units
- It is used to specify a direction.
- Unit vector \( \mathbf{u} \) points in the direction of \( \mathbf{U} \)
  - Often denoted with a “hat”: \( \hat{\mathbf{u}} \)
- Useful examples are the cartesian unit vectors \( \{ \mathbf{i}, \mathbf{j}, \mathbf{k} \} \)
  - Point in the direction of the \( x, y \) and \( z \) axes.
  - \( \mathbf{R} = r_1 \mathbf{i} + r_2 \mathbf{j} + r_3 \mathbf{k} \)

Vector addition using components:
- Consider \( \mathbf{C} = \mathbf{A} + \mathbf{B} \).
  - (a) \( \mathbf{C} = (A_x \mathbf{i} + A_y \mathbf{j}) + (B_x \mathbf{i} + B_y \mathbf{j}) = (A_x + B_x) \mathbf{i} + (A_y + B_y) \mathbf{j} \)
  - (b) \( \mathbf{C} = (C_x \mathbf{i} + C_y \mathbf{j}) \)
- Comparing components of (a) and (b):
  - \( C_x = A_x + B_x \)
  - \( C_y = A_y + B_y \)

Lecture 4, Exercise 1
Vector Addition
- Vector \( \mathbf{A} = (0, 2, 1) \)
- Vector \( \mathbf{B} = (3, 0, 2) \)
- Vector \( \mathbf{C} = (1, -4, 2) \)

What is the resultant vector, \( \mathbf{D} \), from adding \( \mathbf{A} + \mathbf{B} + \mathbf{C} \)?
- A) \( (3, -4, 2) \)
- B) \( (4, -2, 5) \)
- C) \( (5, -2, 4) \)
Lecture 4, Exercise 1
Vector Addition
- Vector A = (0, 2, 1)
- Vector B = (3, 0, 2)
- Vector C = (1, -4, 2)

What is the resultant vector, D, from adding A + B + C?

A. (3, -4, 2)
B. (4, -2, 5)
C. (5, -2, 4)
D. None of the above

Converting Coordinate Systems
- In polar coordinates the vector \( \mathbf{R} = (r, \theta) \)
- In Cartesian the vector \( \mathbf{R} = (x, y) \)
- We can convert between the two as follows:

\[
\begin{align*}
    r &= \sqrt{x^2 + y^2} \\
    \theta &= \tan^{-1} \left( \frac{y}{x} \right)
\end{align*}
\]

- In 3D cylindrical coordinates \((r, \theta, z)\), \(r\) is the same as the magnitude of the vector in the \(x-y\) plane \([\sqrt{x^2 + y^2}]\)

Exercise: Frictionless inclined plane
- A block of mass \(m\) slides down a frictionless ramp that makes angle \(\theta\) with respect to horizontal. What is its acceleration \(a\)?

Resolving vectors, little \(\mathbf{g}\) & the inclined plane
- \(\mathbf{g}\) (bold face, vector) can be resolved into its \(x\), \(y\) components
- \(\mathbf{g} = -g \mathbf{j}\)
- \(\mathbf{g} = -g \cos \theta \mathbf{j} + g \sin \theta \mathbf{i}\)
- The bigger the tilt the faster the acceleration..... along the incline

Lecture 4, Example
Vector addition
An experimental aircraft can fly at full throttle in still air at 200 m/s. The pilot has the nose of the plane pointed west (at full throttle) but, unknown to the pilot, the plane is actually flying through a strong wind blowing from the northwest at 140 m/s. Just then the engine fails and the plane starts to fall at 5 m/s².

What is the magnitude and directions of the resulting velocity (relative to the ground) the instant the engine fails?

Calculate: \(\mathbf{A} + \mathbf{B}\)

\[
A_x + B_x = -200 + 140 \times 0.71 \quad \text{and} \quad A_y + B_y = 0 - 140 \times 0.71
\]

And now, Chapter 4: Newton’s Laws and Forces
Sir Isaac Newton (1642 - 1727)
Dynamics

- Principia Mathematica published in 1687. This revolutionary work proposed three "laws" of motion:

**Law 1:** An object subject to no net external forces is at rest or moves with a constant velocity if viewed from an inertial reference frame.

**Law 2:** For any object, \( F_{\text{NET}} = \sum F = ma \)

*Important: Force is a vector and this is a vector sum*

**Law 3:** Forces occur in pairs: \( F_{A,B} = -F_{B,A} \)

(Deferred until later)

So...What is a force and how do we know it is there?

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**Force**

- We have a general notion of forces is from everyday life.

- In physics the definition must be precise.

  *A force is an action which causes a body to accelerate.*

  (Newton’s Second Law)

**Examples:**
- Contact Forces
- Field Forces (Non-Contact)
  - (physical contact (action at a distance) between objects)
  - Kicking a ball
  - Moon and Earth

- On a microscopic level, all forces are non-contact

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**Mass**

- We have an idea of what mass is from everyday life.

- In physics:

  *Mass (in Phys 207) is a quantity that specifies how much inertia an object has (i.e. a scalar that relates force to acceleration)*

  (Newton’s Second Law)

  - Mass is an inherent property of an object.
  - Mass and weight are different quantities; weight is usually the magnitude of a gravitational (non-contact) force.
  - "Pound" (lb) is a definition of weight (i.e., a force), not a mass!

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**Inertia and Mass**

- The tendency of an object to resist any attempt to change its velocity is called *inertia*

- **Mass** is that property of an object that specifies how much resistance an object exhibits to changes in its velocity (acceleration)

  If mass is constant then \( \vec{a} \propto \frac{F_{\text{NET}}}{m} \)

- If force constant \( \left| \vec{a} \right| \propto \frac{1}{m} \)

- Mass is an inherent property of an object
- Mass is independent of the object’s surroundings
- Mass is independent of the method used to measure it
- Mass is a scalar quantity
- The SI unit of mass is kg

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**Lecture 4, Exercise 2**

**Newton’s Laws and context**

- An object is moving to the right, and experiencing a net force that is directed to the right. The magnitude of the force is decreasing with time.

- The speed of the object is

  A. increasing
  B. decreasing
  C. constant in time
  D. Not enough information to decide

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**Lecture 4, Sept. 17, Recap**

**Assignments:**

- For Wednesday class: Read Chapter 5
- MP Problem Set 2 due Wednesday (should have started)
- MP Problem Set 3, Chapters 4 and 5 (available soon)
An object subject to no external forces moves with a constant velocity if viewed from an inertial reference frame (IRF).

If no net force acting on an object, there is no acceleration.

- The above statement can be used to define inertial reference frames.
- An IRF is a reference frame that is not accelerating (or rotating) with respect to the "fixed stars".
- If one IRF exists, infinitely many exist since they are related by any arbitrary constant velocity vector!
- The surface of the Earth may be viewed as an IRF

Important notes
- Contact forces are conditional, they are not necessarily constant
- The SI units of force are Newtons with 1 N = 1 kg m/s²

Now recall
- If net force is non-zero & constant then the change in the velocity is simply acceleration times time.
- If we double the time we double, keeping the force constant, then the change in velocity (assuming mass is constant)

Lecture 4, Exercise 3
Newton's Second Law
A constant force is exerted on a cart that is initially at rest on an air table. The force acts for a short period of time and gives the cart a certain final speed s.

4 \times \text{far}

In a second trial, we apply a force only half as large.

To reach the same final speed, how long must the same force be applied (recall acceleration is proportional to force if mass fixed)?

A. 4 x as long
B. 2 x as long
C. 1/2 as long
D. 1/4 as long

Lecture 4, Exercise 4
Newton's Second Law
A force of 2 Newtons acts on a cart that is initially at rest on an air track with no air and pushed for 1 second. Because there is friction (no air), the cart stops immediately after 1 finish pushing. It has traveled a distance, D.

Next, the force of 2 Newtons acts again but is applied for 2 seconds.

The new distance the cart moves relative to D is:

A. 8 x as far
B. 4 x as far
C. 2 x as far
D. 1/4 x as far
Lecture 4, Exercise 4
Solution

We know that under constant acceleration, 
\[ \Delta x = a \left( \Delta t \right)^2 / 2 \]  
(when \( v_0 = 0 \))

Here \( \Delta t_2 = 2 \Delta t_1 \), \( F_2 = F_1 \) \( \Rightarrow \) \( a_2 = a_1 \)

\[ \frac{\Delta x_2}{\Delta x_1} = \frac{\frac{1}{2} a \Delta t_2^2}{\frac{1}{2} a \Delta t_1^2} = \frac{(2 \Delta t_1)^2}{\Delta t_1^2} = 4 \]

(B) 4 x as long