Physics 207, Lecture 6, Sept. 24

Agenda:

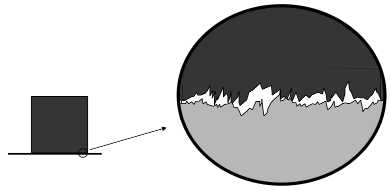
- Chapter 5
  - ❖ Friction (a external force that opposes motion)
- Chapter 6 (Dynamics II)
  - ❖ Motion in two (or three dimensions)
  - ❖ Frames of reference

Assignment: For Wednesday read Chapter 7

- MP Problem Set 3 due Wednesday
- MP Problem Set 4 available soon
- MidTerm Thursday, Oct. 4, Chapters 1-7, 90 minutes, 7:15-8:45 PM

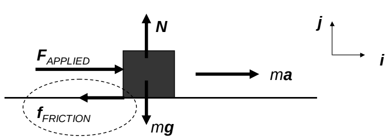
Friction...

- Friction is caused by the "microscopic" interactions between the two surfaces:



A special contact force, friction

- What does it do?
  - ❖ It opposes motion !
  - ❖ Parallel to a surface
  - ❖ Perpendicular to a surface **Normal** force
- How do we characterize this in terms we have learned?
  - ❖A resulting force in a direction opposite to the direction of motion (actual or implied)!



Sliding Friction (i.e., with motion)

- Direction: A force vector ⊥ to the normal force vector ***N***
- Magnitude: $|\boldsymbol{f}_K|$ is proportional to the magnitude of $|\boldsymbol{N}|$
  - ❖ $|\boldsymbol{f}_K| = \mu_K\,|\boldsymbol{N}|$ ($= \mu_K\,|m\boldsymbol{g}|$ in the previous example)
- The constant μ_K is called the "coefficient of kinetic friction"
- As the normal force varies so does the frictional force

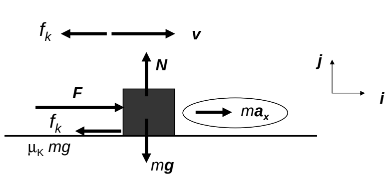
Case study ... big F

- Dynamics:

x-axis i : $ma_x = F - \mu_K N$

y-axis j : $ma_y = 0 = N - mg$ or $N = mg$

so $F - \mu_K mg = m\,a_x$



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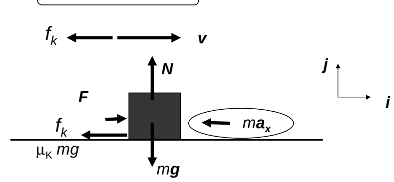
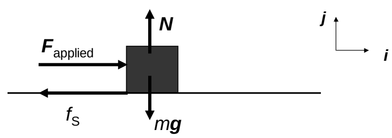


Static Friction...

- So far we have considered friction acting when something has a non-zero velocity
  - ❖ We also know that it acts in fixed or "static" systems:
- In these cases, the force provided by friction depends on the forces applied on the system (magnitude: $f_s \le \mu_s N$)
- Opposes motion that would occur if μ_s were zero



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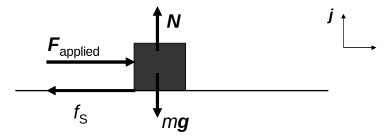
Static Friction...

- Just like in the sliding case except $a = 0$.

  i : $F_{applied} - f_S = 0$

  j : $N = mg$
- While the block is static: $f_S = F_{applied}$ ***(unlike kinetic friction)***



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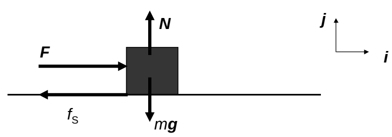
Static Friction...

- The maximum possible force that the friction between two objects can provide is $f_{MAX} = \mu_S N$, where μ_s is the "coefficient of static friction".
  - ❖ So $f_S \le \mu_S N$.
  - ❖ As one increases F, f_S gets bigger until $f_S = \mu_S N$ and the object "breaks loose" and starts to move.



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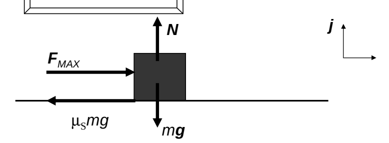
Static Friction...

- μ_S is discovered by increasing ***F*** until the block starts to slide:

  i : $F_{MAX} - \mu_S N = 0$

  j : $N = mg$

  $\mu_S = F_{MAX} / mg$



ActiveFigure
http://romano.physics.wisc.edu/winokur/fall2006/AF_0516.html

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Additional comments on Friction:

- The force of friction does not depend on the area of the surfaces in contact (a relatively good approximation if there is little surface deformation)
- Logic dictates that $\mu_S > \mu_K$ for any system

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Coefficients of Friction

Material on Material	μ_s = static friction	μ_k = kinetic friction
steel / steel	0.6	0.4
add grease to steel	0.1	0.05
metal / ice	0.022	0.02
brake lining / iron	0.4	0.3
tire / dry pavement	0.9	0.8
tire / wet pavement	0.8	0.7

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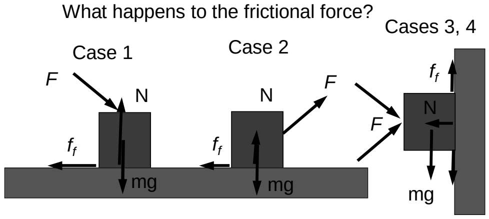
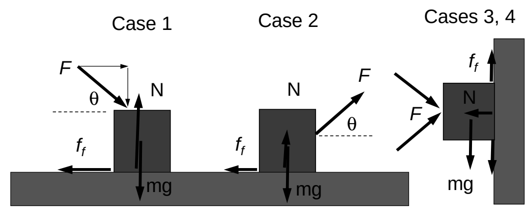
Forces at different angles

Case1: Downward angled force with friction
Case 2: Upwards angled force with friction
Cases 3,4: Up against the wall
Questions: Does it slide?
What happens to the normal force?
What happens to the frictional force?



Forces at different angles

1. Draw a Force Body Diagram
2. Choose directions for x, y and z axes
3. Write down Newton's 2nd Law for each axis
4. If no acceleration sum of the forces is zero, *ma* otherwise



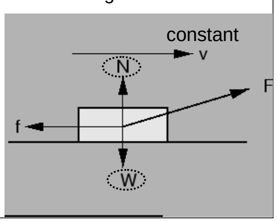
$N = W$

Lecture 6, *Exercise 1*
Dragging a block

A person pulls a block across a rough horizontal surface at constant velocity by applying a non-zero force F. The arrows in the diagram accurately indicate the direction of the forces but not their magnitude.

Which of the relationship between the magnitudes of **N** and **W** holds true ?

- ✗ A. N > W
- ✗ B. N = W
- ✓ C. N < W
- ✗ D. dependent on **f**
- ✗ E. dependent on F



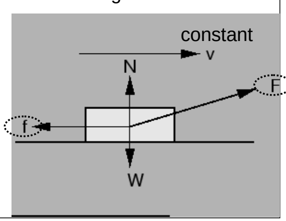
$N = W$

Lecture 6, *Exercise 2*
Dragging a block

A person pulls a block across a rough horizontal surface at constant velocity by applying a non-zero force F. The arrows in the diagram accurately indicate the direction of the forces but not their magnitude.

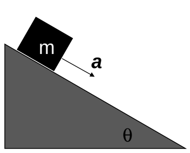
Which of the relationship between the magnitudes of **F** and **f** holds true ?

- ✗ A. f > F
- ✗ B. f = F
- ✓ C. f < F
- ✗ D. dependent on N
- ✗ E. dependent on W

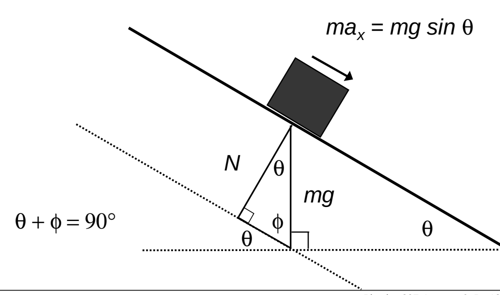


Frictionless inclined plane

- A block of mass *m* slides down a frictionless ramp that makes angle θ with respect to horizontal. What is its acceleration *a* ?

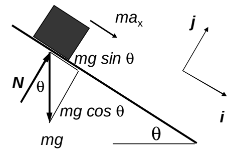


Angles of the inclined plane



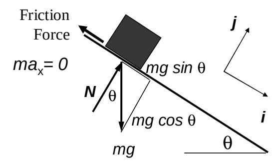
Frictionless inclined plane...

- Use a FBD and consider x and y components separately:
- F_x ***i***: $ma_x = mg \sin\theta$ ⇨ $a_x = g \sin\theta$
- F_y ***j***: $ma_y = 0 = N - mg \cos\theta$ ⇨ $N = mg \cos\theta$



Inclined plane...static friction

- Use a FBD and consider x and y components separately:\
- F_x ***i***: $ma_x = 0 = mg \sin\theta - f$ ⇨ $0 = g \sin\theta - f$
- F_y ***j***: $ma_y = 0 = N - mg \cos\theta$ ⇨ $N = mg \cos\theta$



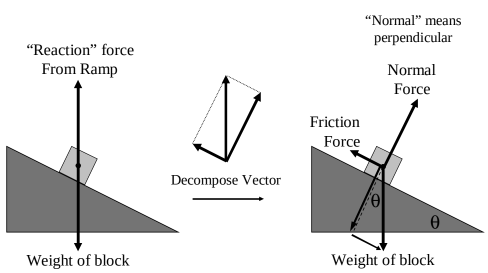
Special case:

At the breaking point

$f = \mu_s N = \mu_s mg \cos\theta$

$g \sin\theta = f = \mu_s mg \cos\theta$

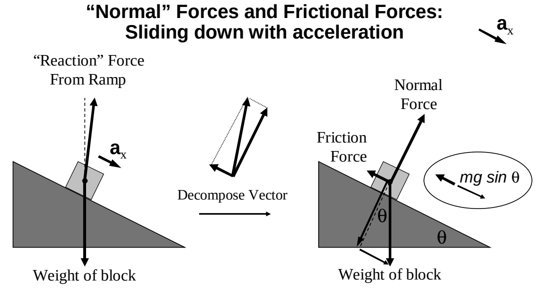
"Normal" Forces and Frictional Forces: Sitting still



Friction Force ≤ Normal Force × coeff. of static friction

$F_{friction} = mg \sin\theta \le N \mu_k$

"Normal" Forces and Frictional Forces: Sliding down with acceleration

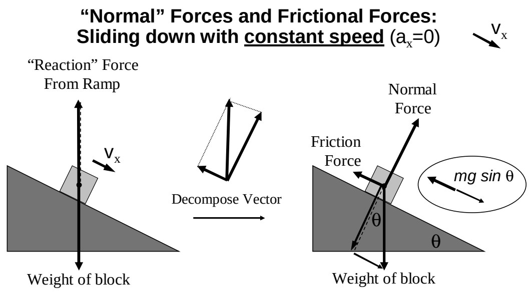


Sliding Friction Force < Normal Force × coeff. of static friction

$F_{friction} = N \mu_k < mg \sin\theta$

$\Sigma F_x = m a_x = mg \sin\theta - N \mu_k$

"Normal" Forces and Frictional Forces: Sliding down with <u>constant speed</u> (a_x=0)



Sliding Friction Force = weight vector component along the block

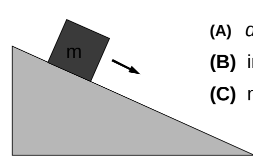
$F_{friction} = N \mu_k = mg \sin\theta$

$\Sigma F_x = 0 = mg \sin\theta - N \mu_k$

(with respect to previous slide we must increase μ_k and/or reduce θ)

Lecture 6, *Exercise 3* Test your intuition

- A block of mass m, is placed on a rough inclined plane ($\mu > 0$) and given a brief push. It motion thereafter is down the plane with a constant speed.
  - ❖ If a similar block (same μ) of mass $2m$ were placed on the same incline and given a brief push with v_0 down the block, it will



(A) *decrease its speed*

(B) increase its speed

(C) move with constant speed

Lecture 6, *Exercise 3*
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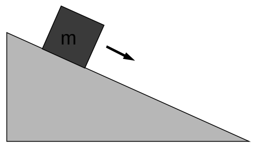
✘ A. *decrease its speed*
✘ B. increase its speed
✔ C. move with constant speed

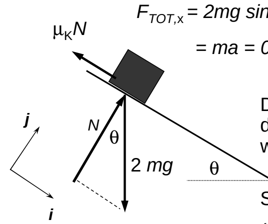


Lecture 6, *Exercise 3*
Solution

- Draw FBD and find the total force in the x-direction

$F_{TOT,x} = 2mg \sin\theta - \mu_K\ 2mg \cos\theta = 2\ ma$

$= ma = 0$ (case when just *m*)



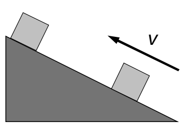
Doubling the mass will simply double both terms…net force will still be zero !

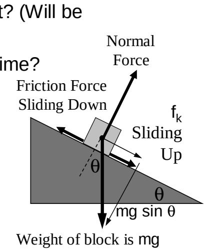
Speed will still be constant !

(C)

"Normal" Forces and Frictional Forces

1. At first the velocity is *v* up along the slide
2. Can you draw a velocity time plot? (Will be done in the review session)
3. What is the <u>acceleration</u> versus time?

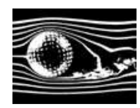




Friction Force = Normal Force × (coefficient of friction)

$F_{friction} = \mu_k F_{normal} = \mu_k\ mg \sin\theta$

Air Resistance and Drag



- So far we've "neglected air resistance" in physics
  - ❖ Can be difficult to deal with
- Affects projectile motion
  - ❖ Friction force opposes velocity through medium
  - ❖ Imposes horizontal force, additional vertical forces
  - ❖ Terminal velocity for falling objects
- Dominant energy drain on cars, bicyclists, planes
- This issue has been with a very long time.

Drag Force Quantified

- With a cross sectional area, *A* (in m²), coefficient of drag of 1.0 (most objects), ρ sea-level density of air, and velocity, *v* (m/s), the drag force is:

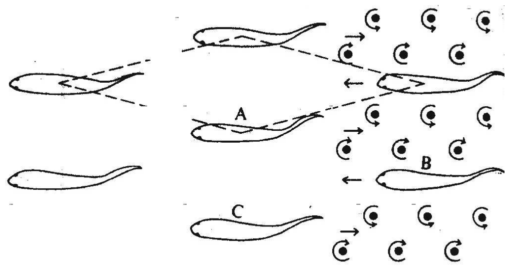
$D = ½\ C \rho A v^2 \cong c A v^2$ in Newtons

$c = ¼\ kg/m^3$

In falling, when D = mg, then at terminal velocity

- Example: Bicycling at 10 m/s (22 m.p.h.), with projected area of 0.5 m² exerts ~30 Newtons
  - ❖ Requires (*F v)* of *power* → 300 Watts to maintain speed
  - ❖ Minimizing drag is often important

Fish Schools



By swimming in synchrony in the correct formation, each fish can take advantage of moving water created by the fish in front to reduce drag.

Fish swimming in schools can swim 2 to 6 times as long as individual fish.

"Free" Fall

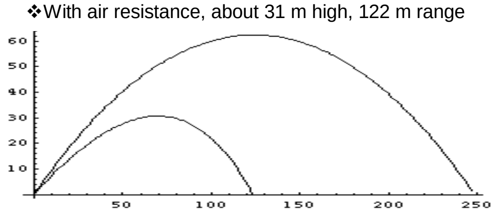
- Terminal velocity reached when $F_{drag} = F_{grav}$ (= mg)
- For 75 kg person with a frontal area of 0.5 m^2,

  $v_{term} \approx$ 50 m/s, or 110 mph

  which is reached in about 5 seconds, over 125 m of fall

Trajectories with Air Resistance

- Baseball launched at 45° with v = 50 m/s:
  - ❖Without air resistance, reaches about 63 m high, 254 m range
  - ❖With air resistance, about 31 m high, 122 m range



Vacuum trajectory vs. air trajectory for 45° launch angle.

Lecture 6 Recap

Assignment:

- Wednesday class: Read Chapter 6, Start Chapter 7
- MP Problem Set 3 due Wednesday
- MP Problem Set 4 available soon
- MidTerm Thursday, Oct. 4, Chapters 1-7, 90 minutes, 7:15-8:45 PM

Physics 207, Lecture 7, Sept. 26

Agenda:

- Chapter 6 (Dynamics II)
  - ❖ Motion in two (or three dimensions)
  - ❖ Frames of reference
- Start Chapter 7

Assignment: For Wednesday read Chapter 7

- MP Problem Set 3 due tonight
- MP Problem Set 4 available now
- MidTerm Thursday, Oct. 4, Chapters 1-7, 90 minutes, 7:15-8:45 PM

Rooms: B102 & B130 in **Van Vleck**.

Chapter 6:
Motion in 2 (and 3) dimensions, *Dynamics II*

- Recall instantaneous velocity and acceleration

$$\mathbf{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt}$$

$$\mathbf{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}$$

- These are vector expressions reflecting x, y and z motion

  $\mathbf{r} = \mathbf{r}(t)$ $\quad$ $\mathbf{v} = d\mathbf{r} / dt$ $\quad$ $\mathbf{a} = d^2\mathbf{r} / dt^2$

Kinematics

- The position, velocity, and acceleration of a particle in 3-dimensions can be expressed as:

$$\boldsymbol{r} = x\,\boldsymbol{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$$
$$\boldsymbol{v} = v_x\,\boldsymbol{i} + v_y\,\boldsymbol{j} + v_z\,\boldsymbol{k} \qquad (\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k} \text{ unit vectors})$$
$$\boldsymbol{a} = a_x\,\boldsymbol{i} + a_y\,\boldsymbol{j} + a_z\,\boldsymbol{k}$$

$$x = x(t) \qquad y = y(t) \qquad z = z(t)$$
$$v_x = \frac{dx}{dt} \qquad v_y = \frac{dy}{dt} \qquad v_z = \frac{dz}{dt}$$
$$a_x = \frac{d^2x}{dt^2} \qquad a_y = \frac{d^2y}{dt^2} \qquad a_z = \frac{d^2z}{dt^2}$$

- All this complexity is hidden away in

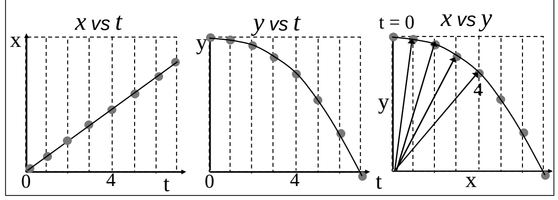
$\boldsymbol{r} = \boldsymbol{r}(t)$ $\quad \boldsymbol{v} = d\boldsymbol{r} / dt$ $\quad \boldsymbol{a} = d^2\boldsymbol{r} / dt^2$

Special Case

Throwing an object with *x* along the horizontal and *y* along the vertical.

x and *y* motion both coexist and *t* is common to both

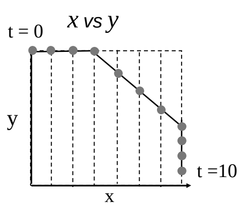
Let *g* act in the –*y* direction, $v_{0x} = v_0$ and $v_{0y} = 0$



Another trajectory

Can you identify the dynamics in this picture?

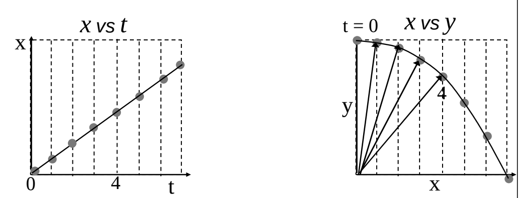
How many distinct regimes are there?



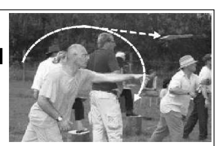
Trajectory with constant acceleration along the vertical

How does the trajectory appear to different observers ?

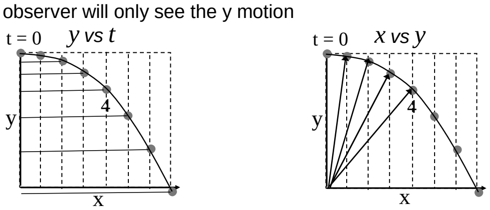
What if the observe is moving with the same x velocity?



Trajectory with constant acceleration along the vertical

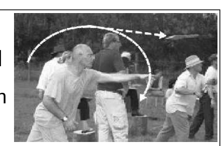


This observer will only see the y motion



In an inertial reference frames everyone sees the acceleration

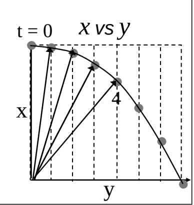
Trajectory with constant acceleration along the vertical



What do the velocity and acceleration vectors look like?

Velocity vector is always tangent to the curve!

Acceleration may or may not be!



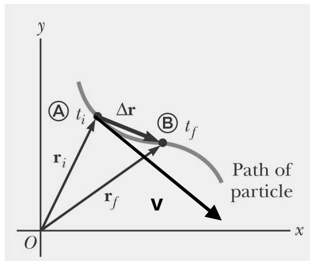
Instantaneous Velocity

- But how we think about requires knowledge of the path.
- The direction of the **instantaneous velocity** is **along** a line that is **tangent** to the path of the particle's direction of motion.

$$\mathbf{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt}$$

- The magnitude of the instantaneous velocity vector is the speed, *s*. (Knight uses *v*)

$s = (v_x^2 + v_y^2 + v_z^2)^{1/2}$

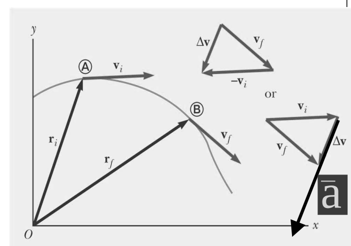


Average Acceleration: Review

- The average acceleration of particle motion reflects changes in the instantaneous velocity vector (divided by the time interval during which that change occurs).

$$\overline{\mathbf{a}} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t_f - t_i} = \frac{\Delta \mathbf{v}}{\Delta t}$$

- The average acceleration is a vector quantity directed along Δ**v**



Instantaneous Acceleration

- The instantaneous acceleration is the limit of the average acceleration as Δ**v**/Δ*t* approaches zero

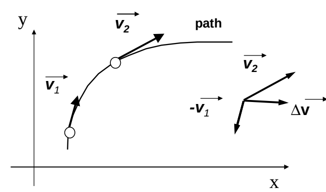
$$\mathbf{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}$$

- The instantaneous acceleration is a vector with components parallel (tangential) and/or perpendicular (radial) to the tangent of the path
- Changes in a particle's path may produce an acceleration
  - ❖ The **magnitude** of the velocity vector may change
  - ❖ The **direction** of the velocity vector may change (Even if the magnitude remains constant)
  - ❖ Both may change simultaneously (depends: path *vs* time)

Motion along a path (displacement, velocity, acceleration)

- 3-D Kinematics : **vector** equations:

$\vec{r} = \vec{r}(t)$ $\quad \vec{v} = d\vec{r} / dt \quad \vec{a} = d^2\vec{r} / dt^2$



Velocity :

$\vec{v}_{av} = \Delta\vec{r} \,/\, \Delta t$

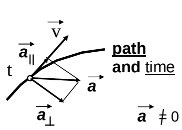
$\vec{v} = d\vec{r} \,/\, dt$

Acceleration :

$\vec{a}_{av} = \Delta\vec{v} \,/\, \Delta t$

$\vec{a} = d\vec{v} \,/\, dt$

General 3-D motion with non-zero acceleration:



$\vec{a} = \vec{a}_{||} + \vec{a}_{\perp}$

$\vec{a} \neq 0$

Two possible options:

Change in the magnitude of $\vec{v}$ $\quad \vec{a}_{||} \neq 0$

Change in the direction of $\vec{v}$ $\quad \vec{a}_{\perp} \neq 0$

Animation

- Uniform Circular Motion (Ch. 7) is one specific case:

Lecture 6 Recap

Assignment:

- Wednesday class: Read Chapter 7
- MP Problem Set 3 due Wednesday
- MP Problem Set 4 available soon
- MidTerm Thursday, Oct. 4, Chapters 1-7, 90 minutes, 7:15-8:45 PM

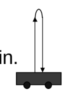
Relative Velocity, equations

- The positions as seen from the two reference frames are related through the velocity
  - ❖ $\mathbf{r}' = \mathbf{r} - \mathbf{v}_o\, t$
- The derivative of the position equation will give the velocity equation
  - ❖ $\mathbf{v}' = \mathbf{v} - \mathbf{v}_o$
- These are called the **Galilean transformation equations**

Central concept for problem solving: "*x*" and "*y*" components of motion treated independently.

- Again: man on the cart tosses a ball straight up in the air.
- You can view the trajectory from two reference frames:

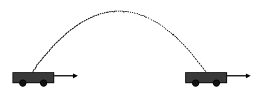
Reference frame on the moving train.



y(t) motion governed by
1) $\boldsymbol{a} = -g\,\boldsymbol{y}$
2) $v_y = v_{0y} - g\,t$
3) $y = y_0 + v_{0y} - g\,t^2/2$

x motion: $x = v_x t$

Reference frame on the ground.



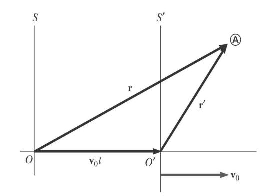
Net motion: $\mathbf{R} = x(t)\,\mathbf{i} + y(t)\,\mathbf{j}$ (vector)

Acceleration in Different Frames of Reference

- The derivative of the velocity equation will give the acceleration equation
  - ❖ $\mathbf{v}' = \mathbf{v} - \mathbf{v}_o$
  - ❖ $\mathbf{a}' = \mathbf{a}$
- The acceleration of the particle measured by an observer in one frame of reference is the same as that measured by any other observer moving at a *constant velocity* relative to the first frame.

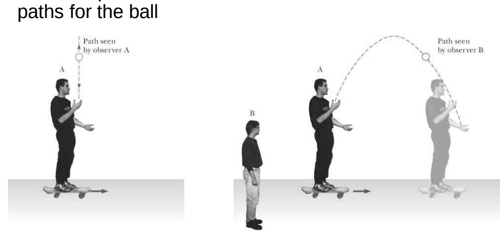
Relative motion and frames of reference

- Reference frame *S* is stationary
- Reference frame *S*′ is moving at $\mathbf{v}_o$
  This also means that *S* moves at $-\mathbf{v}_o$ relative to *S*′
- Define time $t = 0$ as that time when the origins coincide



Relative Velocity

- Two observers moving relative to each other generally do not agree on the outcome of an experiment (path)
- For example, observers A and B below see different paths for the ball



Relative Velocity, r, v, a and r′, v′ , a′

- The positions as seen from the two reference frames are related through the velocity
  (remember S is moving at a constant $-v_0$ relative to S′)
  - ❖ $\mathbf{r}' = \mathbf{r} - \mathbf{v}_o\, t$
- The derivative of the position equation will give the velocity equation
  - ❖ $\mathbf{v}' = \mathbf{v} - \mathbf{v}_o = d\,(\mathbf{r} - \mathbf{v}_o\, t)/dt$

Acceleration in Different Frames of Reference

- The derivative of the velocity equation will give the acceleration equation
  - $\mathbf{v}' = \mathbf{v} - \mathbf{v}_o$
  - $\mathbf{a}' = \mathbf{a}$
- The acceleration of the particle measured by an observer in one frame of reference is the same as that measured by any other observer moving at a *constant velocity* relative to the first frame.