Physics 207, Lecture 7, Sept. 26

Agenda:

- Chapter 6 (Dynamics II)
  - ❖ Motion in two (or three dimensions)
  - ❖ Frames of reference
- Start Chapter 7 (Dynamics III)
  - ❖ Circular Motion

Assignment: For Wednesday read Chapter 7

- MP Problem Set 3 due tonight
- MP Problem Set 4 available now
- MidTerm Thursday, Oct. 4, Chapters 1-7, 90 minutes, 7:15-8:45 PM

Rooms: B102 & B130 in **Van Vleck**.

Chapter 6:

Motion in 2 (and 3) dimensions, *Dynamics II*

- Recall instantaneous velocity and acceleration

$$\mathbf{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt}$$

$$\mathbf{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}$$

- These are vector expressions reflecting x, y and z motion

$\mathbf{r} = \mathbf{r}(t)$ $\mathbf{v} = d\mathbf{r} / dt$ $\mathbf{a} = d^2\mathbf{r} / dt^2$

Kinematics

- The position, velocity, and acceleration of a particle in 3-dimensions can be expressed as:

$\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$

$\mathbf{v} = v_x\,\mathbf{i} + v_y\,\mathbf{j} + v_z\,\mathbf{k}$ ($\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ unit vectors)

$\mathbf{a} = a_x\,\mathbf{i} + a_y\,\mathbf{j} + a_z\,\mathbf{k}$

$$x = x(t) \qquad y = y(t) \qquad z = z(t)$$

$$v_x = \frac{dx}{dt} \qquad v_y = \frac{dy}{dt} \qquad v_z = \frac{dz}{dt}$$

$$a_x = \frac{d^2x}{dt^2} \qquad a_y = \frac{d^2y}{dt^2} \qquad a_z = \frac{d^2z}{dt^2}$$

- All this complexity is hidden away in this compact notation

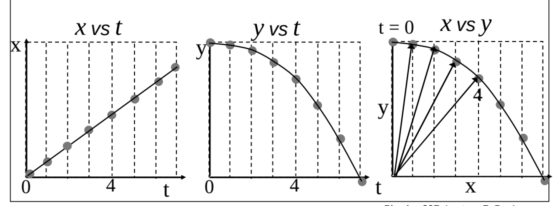
$\mathbf{r} = \mathbf{r}(t)$ $\mathbf{v} = d\mathbf{r} / dt$ $\mathbf{a} = d^2\mathbf{r} / dt^2$

Special Case

Throwing an object with *x* along the horizontal and *y* along the vertical.

x and *y* motion both coexist and *t* is common to both

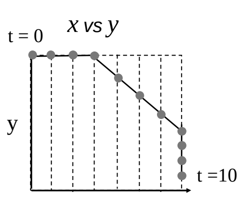
Let *g* act in the –*y* direction, $v_{0x} = v_0$ and $v_{0y} = 0$



A different trajectory

Can you identify the dynamics in this picture?

How many distinct regimes are there?



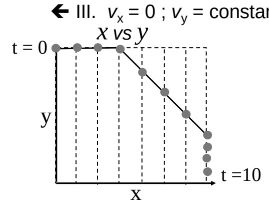
A different trajectory

Can you identify the dynamics in this picture?

How many distinct regimes are there?

0 < t < 3 3 < t < 7 7 < t < 10

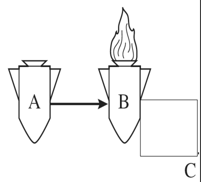
- 🡐 I. v_x = constant = v_0 ; $v_y = 0$
- 🡐 II. $v_x = v_y = v_0$
- 🡐 III. $v_x = 0$; v_y = constant < v_0



What can you say about the acceleration?

Lecture 7, *Exercises 1 & 2*
Trajectories with acceleration

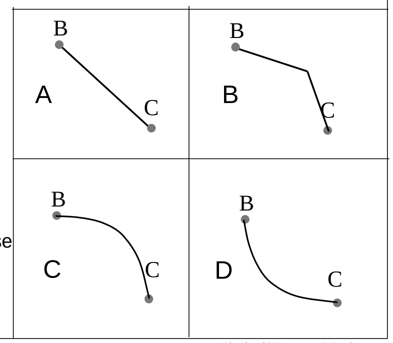
- A rocket is moving sideways in deep space, with its engine off, from A to B. It is not near any stars or planets or other outside forces.
- Its constant thrust engine (force is constant) is fired at point B and left on for 2 seconds in which time the rocket travels from point B to some point C
  - ❖ Sketch the shape of the path from B to C.
- At point C the engine is turned off.
  - ❖ Sketch the shape of the path after point C



Lecture 7, *Exercise 1*
Trajectories with acceleration

From B to C ?

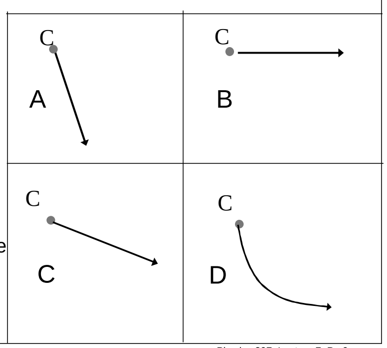
A. A
B. B
C. C
D. D
E. None of these



Lecture 7, *Exercise 2*
Trajectories with acceleration

After C ?

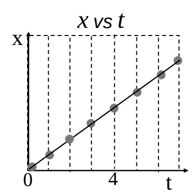
A. A
B. B
C. C
D. D
E. None of these

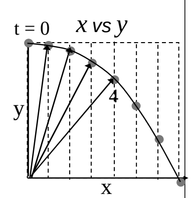


Trajectory with constant acceleration along the vertical

How does the trajectory appear to different observers ?

What if the observer is moving with the same x velocity?

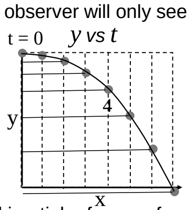


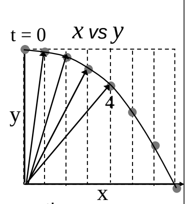


Trajectory with constant acceleration along the vertical

What if the observer is moving with the same x velocity?

This observer will only see the *y* motion



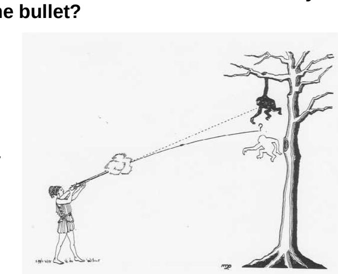


In all inertial reference frames everyone sees the same acceleration

Lecture 7, *Exercise 3*
Relative Trajectories: Monkey and Hunter

A hunter sees a monkey in a tree, aims his gun at the monkey and fires. At the same instant the monkey lets go. Does the bullet?

A. Go over the monkey
B. Hit the monkey
C. Go under the monkey

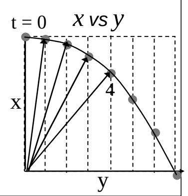


Trajectory with constant acceleration along the vertical

What do the velocity and acceleration vectors look like?

Velocity vector is always tangent to the trajectory curve!

Acceleration may or may not be!



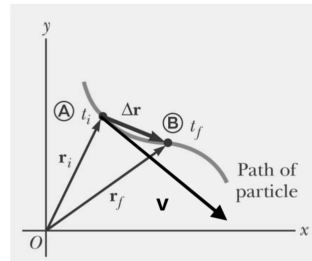
Instantaneous Velocity

- But how we think about requires knowledge of the path.
- The direction of the **instantaneous velocity** is **along** a line that is **tangent** to the path of the particle's direction of motion.

$$\mathbf{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt}$$

- The magnitude of the instantaneous velocity vector is the speed, *s*. (Knight uses *v*)

$s = (v_x^2 + v_y^2 + v_z^2)^{1/2}$

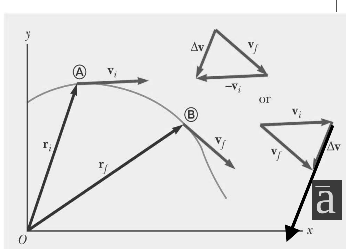


Average Acceleration: Review

- The average acceleration of particle motion reflects changes in the instantaneous velocity vector (divided by the time interval during which that change occurs).

$$\bar{\mathbf{a}} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t_f - t_i} = \frac{\Delta \mathbf{v}}{\Delta t}$$

- The average acceleration is a vector quantity directed along Δ**v**



Instantaneous Acceleration

- The instantaneous acceleration is the limit of the average acceleration as Δ**v**/Δ*t* approaches zero

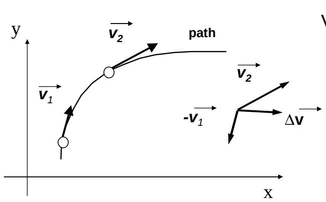
$$\mathbf{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}$$

- The instantaneous acceleration is a vector with components parallel (tangential) and/or perpendicular (radial) to the tangent of the path
- Changes in a particle's path may produce an acceleration
  - ❖ The **magnitude** of the velocity vector may change
  - ❖ The **direction** of the velocity vector may change (Even if the magnitude remains constant)
  - ❖ Both may change simultaneously (depends: path *vs* time)

Motion along a path (displacement, velocity, acceleration)

- 3-D Kinematics : **vector** equations:

$\vec{r} = \vec{r}(t)$ $\quad \vec{v} = d\vec{r} / dt$ $\quad \vec{a} = d^2\vec{r} / dt^2$



Velocity :

$\vec{v}_{av} = \Delta\vec{r} / \Delta t$

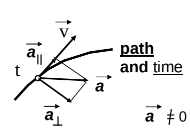
$\vec{v} = d\vec{r} / dt$

Acceleration :

$\vec{a}_{av} = \Delta\vec{v} / \Delta t$

$\vec{a} = d\vec{v} / dt$

General 3-D motion with non-zero acceleration:



$\vec{a} = \vec{a}_{\parallel} + \vec{a}_{\perp}$

$\vec{a} \neq 0$

Two possible options:

Change in the magnitude of $\vec{v}$ $\quad \vec{a}_{\parallel} \neq 0$

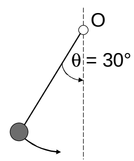
Change in the direction of $\vec{v}$ $\quad \vec{a}_{\perp} \neq 0$

Animation

http://romano.physics.wisc.edu/winokur/fall2006/AF_0418.html

- Uniform Circular Motion (Ch. 7) is one specific case:

Lecture 7, Exercise
The Pendulum revisited



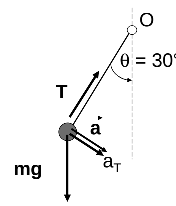
Which set of statements below best describes the motion of the pendulum bob at the instant of time drawn when the bob is at the **top** of its swing.

A) $v_r = 0$ $a_r = 0$
$v_T = 0$ $a_T \neq 0$

B) $v_r = 0$ $a_r \neq 0$
$v_T = 0$ $a_T = 0$

C) $v_r = 0$ $a_r \neq 0$
$v_T = 0$ $a_T \neq 0$

Lecture 7, Exercise
The Pendulum Solution



NOT uniform circular motion : But if curved motion then a_r not zero, if speed is increasing so a_T not zero

However, at the top of the swing the bob temporarily comes to rest, so ***v*** = 0 and the net tangential force is *mg* sin θ

C) $v_r = 0$ $a_r = 0$
$v_T = 0$ $a_T \neq 0$

Everywhere else the bob has a non-zero velocity and so then
(except at the bottom of the swing)

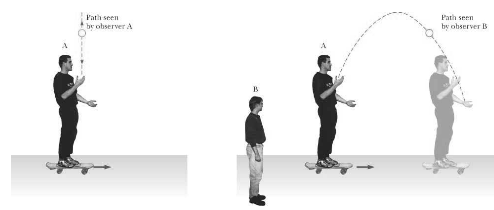
$v_r = 0$ $a_r \neq 0$
$v_T \neq 0$ $a_T \neq 0$

Returning to relative velocity, equations

- The positions as seen from the two reference frames are related through the velocity
  - **r**′ = **r** – $\mathbf{v}_o$ *t*
- The derivative of the position equation will give the velocity equation
  - **v**′ = **v** – $\mathbf{v}_o$
- These are called the **Galilean transformation equations**

Relative Velocity

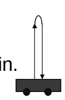
- Two observers moving relative to each other generally do not agree on the outcome of an experiment (path)
- For example, observers A and B below see different paths for the ball



Central concept for problem solving: "*x*" and "*y*" components of motion treated independently.

- Example: man on the cart tosses a ball straight up in the air.
- You can view the trajectory from two reference frames:

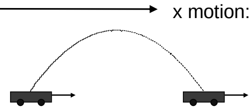
Reference frame on the moving train.



y(t) motion governed by
1) ***a*** = -g ***y***
2) $v_y = v_{0y} - g\,t$
3) $y = y_0 + v_{0y} - g\,t^2/2$

x motion: $x = v_x t$

Reference frame on the ground.



Net motion: **R** = *x(t)* **i** + *y(t)* **j** (vector)

Acceleration in Different Frames of Reference

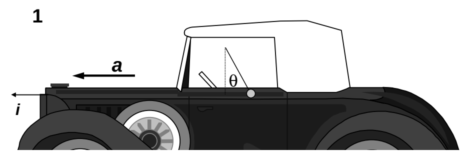
- The derivative of the velocity equation will give the acceleration equation
  - **v**′ = **v** – $\mathbf{v}_o$
  - **a**′ = **a**
- The acceleration of the particle measured by an observer in one frame of reference is the same as that measured by any other observer moving at a *constant velocity* relative to the first frame.

Accelerated Reference Frames:
The Accelerometer

- Your first job is with Ford. You are working on a project to design an accelerometer. The inner workings of this gadget consist of a weight of mass *m* that is hung inside a box that is attached to the ceiling of a car. You design the device with a very light string so that you can mathematically ignore it. The idea is that the angle the string makes with the vertical, θ, is determined by the car's acceleration. Your preliminary task is to think about calibration of the accelerometer when the car travels on a flat road.

What is the car's acceleration a when the hanging mass makes an angle θ with the vertical?

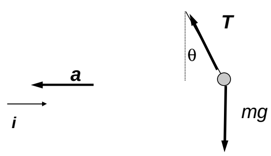
Accelerated Reference Frames:
The Accelerometer



We need to solve for the angle the plum bob makes with respect to vertical.

We will solve by using Newton's Second Law and checking x and y components.

Accelerated Reference Frames:
The Accelerometer



x-dir $F_x = -ma = -T \sin\theta$
y-dir $F_y = 0 = T\cos\theta - mg$

$T = mg / \cos\theta$
$a = T \sin\theta / m = g \tan\theta$

Lecture 7, *Exercise 4*
Accelerated Reference Frames

You are a passenger in a car and not wearing your seatbelt. Without increasing or decreasing speed, the car makes a sharp left turn, and you find yourself colliding with the right-hand door. Which is a correct description of the situation ?

(A) Before and after the collision there is a rightward force pushing you into the door.

(B) Starting at the time of the collision, the door exerts a leftward force on you.

(C) Both of the above.

(D) Neither of the above.

Lecture 7, *Exercise 4*
Accelerated Reference Frames

You are a passenger in a car and not wearing your seatbelt (poor judgement). Without increasing or decreasing speed, the car makes a sharp left turn, and you find yourself colliding with the right-hand door.

Which statement below is a correct description of the situation?

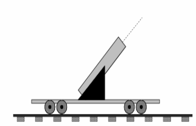
A. Before and after the collision there is a rightward force pushing you into the door.

B. Starting at the time of the collision, the door exerts a leftward force on you.

C. Both of the above

D. Neither of the above

Projection motion and a moving reference frame



A cannon, which can launch a 5. kg projectile with a velocity of 99. m/s, is mounted on a rail car which is moving at a fixed speed of 30. m/s along a horizontal track. Gravity acts along the vertical with $g = 10.$ m/s^2.

a. (10 pts) If the cannon is set to a 45.° angle of inclination (with respect to horizontal) and pointed in the direction of motion, what are the horizontal and vertical components of the projectile's velocity (with respect to a stationary observer on the ground) when it just exits the cannon?

(b) How far away from the launch point does the shell land?

Recap , Lecture 7

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