Physics 207, Lecture 8, Oct. 1, by Prof. Pupa Gilbert

Agenda:

- Chapter 7 (Circular Motion, Dynamics III)
  - Uniform and non-uniform circular motion
- Next time: Problem Solving and Review for MidTerm I

Assignment: (NOTE special time!)

- MP Problem Set 4 due Oct. 3,Wednesday, 4 PM
- MidTerm Thurs., Oct. 4, Chapters 1-6 & 7 (lite), 90 minutes, 7:15-8:45 PM

Rooms: B102 & B130 in **Van Vleck**.

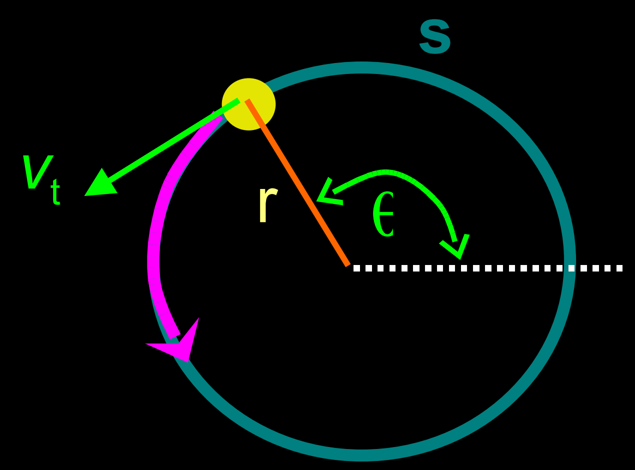
Honors class this Friday 8.50 AM, in **5310** Chamberlin Hall

Notice new location!!! this Friday only

Pr

Rotation requires a new lexicon

- Arc traversed **$s = \theta r$**
- Tangential velocity **v_t**
- Period and frequency
- Angular position
- Angular velocity



Period (T): The time required to do one full revolution, 360° or 2π radians

Frequency (f): 1/T, number of cycles per unit time

Angular velocity or speed **$\omega = 2\pi f = 2\pi/T$**, number of radians traced out per unit time

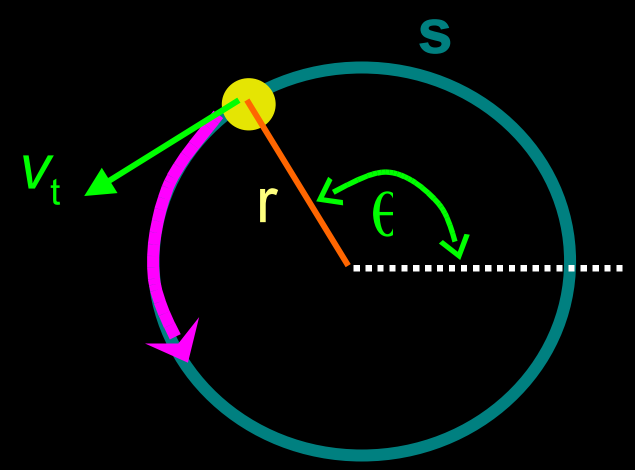
 demo

Relating rotation motion to linear velocity

- Assume a particle moves at constant tangential speed v_t around a circle of radius r.
- Distance = velocity x time
- Once around… $2\pi r = v_t\, T$

or, rearranging $(2\pi/T)\ r = v_t$

$$\boxed{\omega\, r = v_t}$$



Definition: UCM is uniform circular motion (**ω=constant**)

Angular displacement and velocity

- Arc traversed **$s = \theta r$**

  in time Δt then $\Delta s = \Delta\theta\ r$

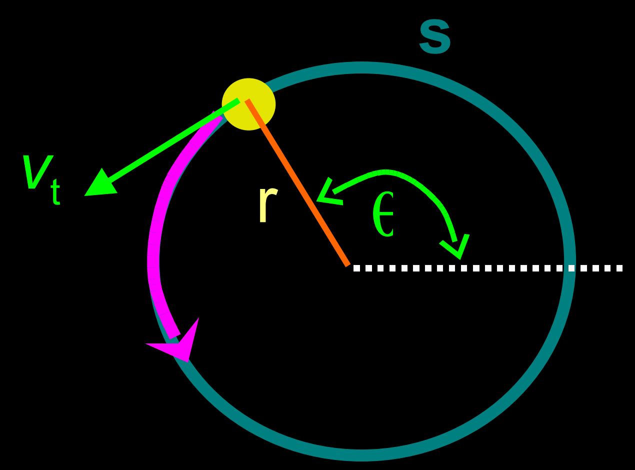
  so $\Delta s / \Delta t = (\Delta\theta / \Delta t)\ r$

  in the limit $\Delta t \rightarrow 0$

  one gets

  $$ds/dt = d\theta/dt \quad r$$

  $$v_t = \omega \quad r$$



$$\boldsymbol{\omega = d\theta/dt}$$

if ω is constant, integrating $\omega = d\theta/dt$,

we obtain: $\boldsymbol{\theta = \theta_o + \omega t}$

Counter-clockwise is positive, clockwise is negative

and we note that v_{radial} (or v_r) and v_z (are both zero if UCM)

bb3

Lecture 7, *Exercise 1*

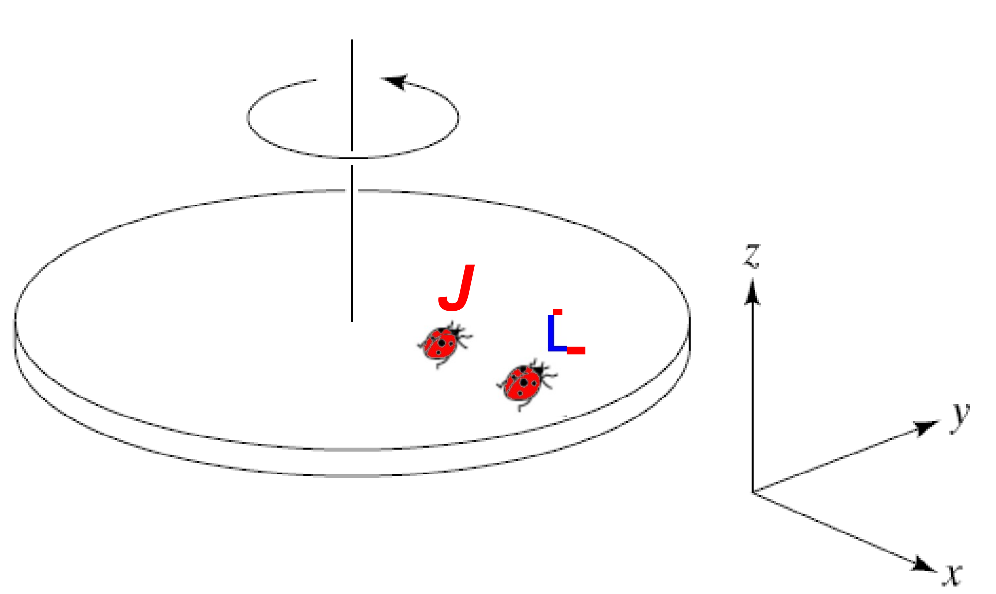
A Ladybug sits at the outer edge of a merry-go-round, and a June bug sits halfway between the outer one and the axis of rotation. The merry-go-round makes a complete revolution once each second.
What is the June bug's angular velocity?

A. half the Ladybug's.

B. the same as the Ladybug's.

C. twice the Ladybug's.

D. impossible to determine.



Lecture 7, *Exercise 1*

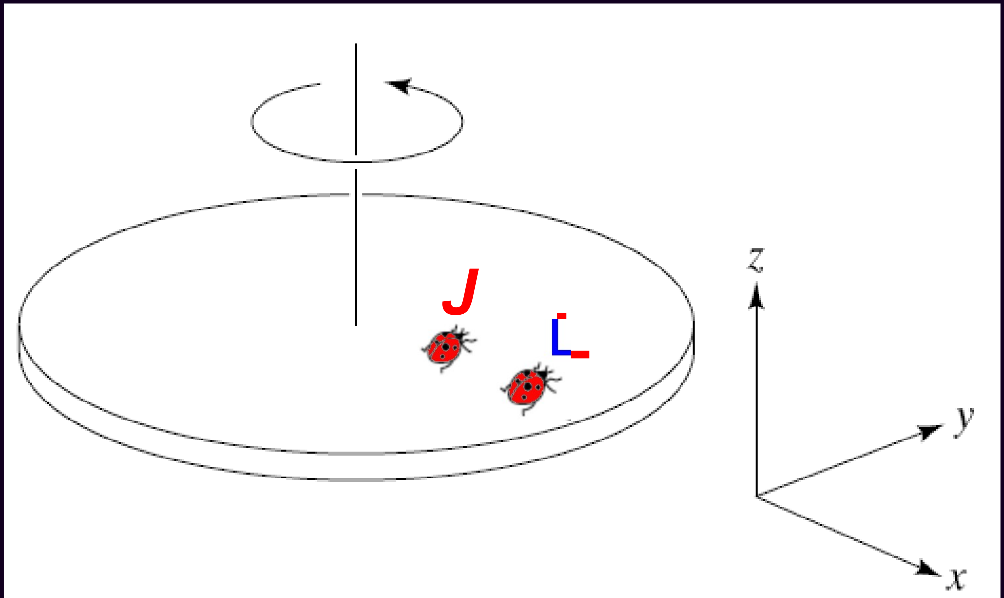
A **L**adybug sits at the outer edge of a merry-go-round, and a **J**une bug sits halfway between the outer one and the axis of rotation. The merry-go-round makes a complete revolution once each second.
What is the **J**une bug's angular velocity?

A. half the Ladybug's.

B. the same as the Ladybug's.

C. twice the Ladybug's.

D. impossible to determine.



demo

And if ω is increasing…

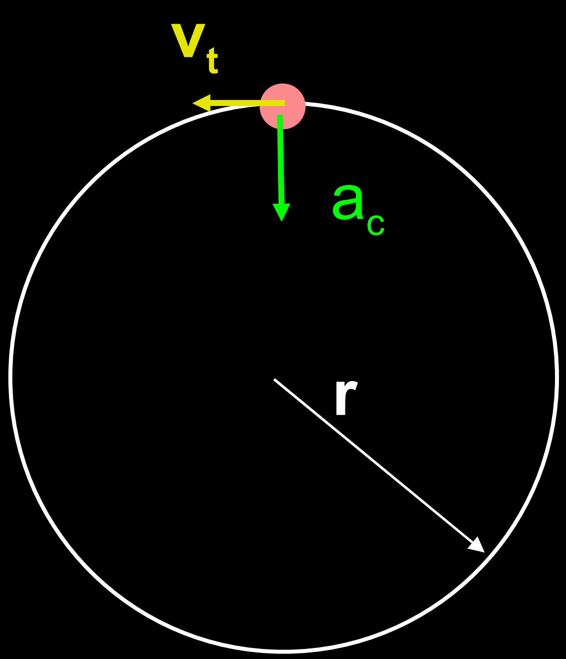
- Then angular velocity is no longer constant so $d\omega/dt \neq 0$
- Define tangential acceleration as **$a_t = dv_t/dt$**
- Can we relate a_t to $d\omega/dt$? bb4

$$\boxed{\begin{aligned} \theta &= \theta_o + \omega_o t + \frac{1}{2}\frac{a_t}{r}t^2 \\ \omega &= \omega_o + \frac{a_t}{r} t \end{aligned}}$$

- Many analogies to linear motion but it isn't one-to-one
- Most importantly: even if the angular velocity is constant, there is **always** a **radial acceleration**.

Newton's Laws and Circular Motion
(Chapter 7)

Uniform circular motion involves only changes in the ***direction*** of the velocity vector, thus acceleration is perpendicular to the trajectory at any point, ***acceleration is only in the radial direction***. Quantitatively (see text)



Centripetal Acceleration

$$a_c = v_t^2/r$$

Circular motion involves continuous radial acceleration and this means a central force.

$$F_c = ma_c = mv_t^2/r = m\omega^2 r$$

No central force…no UCM!

Examples of central forces are:

- $\vec{T}$ on the string holding the ball
- Gravity (moon, solar system, satellites, space crafts….)
- Positive charge of the nucleus, for the e^-
- Centripetal friction acting on the ladybug
- Ball on a loop-the-loop track ($\vec{n}$ on the track)

Circular Motion

- UCM enables high accelerations (g’s) in a small space
- Comment: In automobile accidents involving rotation severe injury or death can occur even at modest speeds.

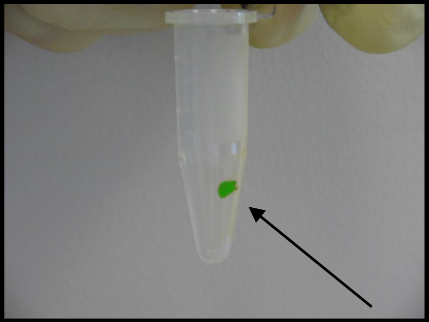
[In physics speed doesn’t kill….acceleration (i.e., force) does.]

demo: cut the string

Mass-based separation with a **centrifuge**

Before

After



$a_c = v^2 / r$ and $f = 10^4$ rpm is typical with

$r = 0.1$ m

and $v = \omega r = 2\pi f r$

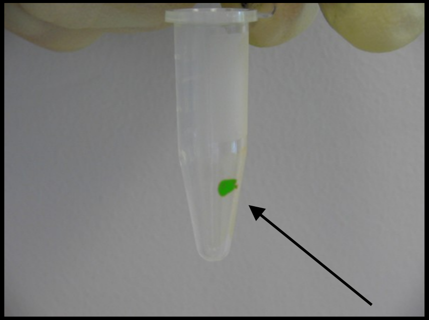
How many g's?

bb5

Mass-based separation with a centrifuge

Before

After



How many g's?

$a_c = v^2 / r$ and $f = 10^4$ rpm is typical with

$r = 0.1$ m

and $v = \omega r = 2\pi f r$

$v = (2\pi \times 10^4 / 60) \times 0.1$ m/s $= 100$ m/s

$a_c = 1 \times 10^4 / 0.1$ m/s^2 = **10,000 g**

bb5

g-forces with respect to humans

- 1 g Standing
- 1.2 g Normal elevator acceleration (up).
- 1.5-2g Walking down stairs.
- 2-3 g Hopping down stairs.
- 1.5 g Commercial airliner during takeoff run.
- 2 g Commercial airliner at rotation
- 3.5 g Maximum acceleration in amusement park rides (design guidelines).
- 4 g Indy cars in the second turn at Disney World (side and down force).
- 4+ g Carrier-based aircraft launch.
- 10 g Threshold for blackout during violent maneuvers in high performance aircraft.
- 11 g Alan Shepard in his historic sub orbital Mercury flight experience a maximum force of 11 g.
- 20 g Colonel Stapp's experiments on acceleration in rocket sleds indicated that in the 10-20 g range there was the possibility of injury because of organs moving inside the body. Beyond 20 g they concluded that there was the potential for death due to internal injuries. Their experiments were limited to 20 g.
- 30 g The design maximum for sleds used to test dummies with commercial restraint and air bag systems is 30 g.

Acceleration has its limits

“High speed motion picture camera frame: John Stapp is caught in the teeth of a massive deceleration. One might have expected that a test pilot or an astronaut candidate would be riding the sled; instead there was Stapp, a mild mannered physician and diligent physicist with a notable sense of humor.
Source: US Air Force photo

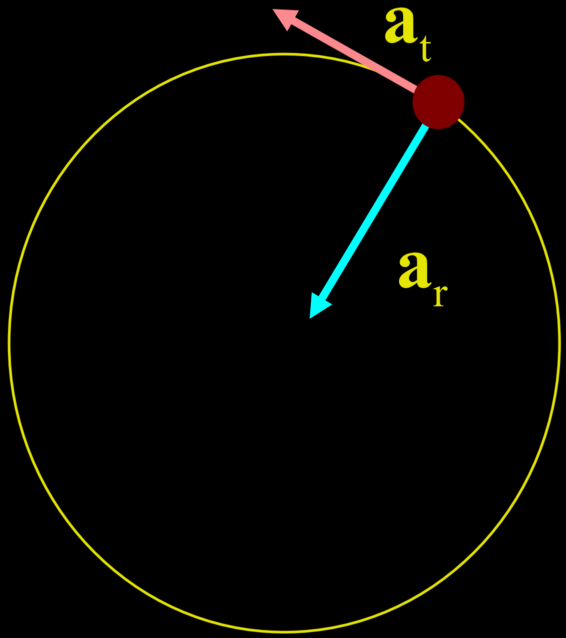
A bad day at the lab….

- In 1998, a Cornell campus laboratory was seriously damaged when the rotor of an ultracentrifuge failed while in use.
- Description of the Cornell Accident -- On December 16, 1998, milk samples were running in a Beckman. L2-65B ultracentrifuge using a large aluminum rotor. The rotor had been used for this procedure many times before. Approximately one hour into the operation, the rotor failed due to excessive mechanical stress caused by the g-forces of the high rotation speed. The subsequent explosion completely destroyed the centrifuge. The safety shielding in the unit did not contain all the metal fragments. The half inch thick sliding steel door on top of the unit buckled allowing fragments, including the steel rotor top, to escape. Fragments ruined a nearby refrigerator and an ultra-cold freezer in addition to making holes in the walls and ceiling. The unit itself was propelled sideways and damaged cabinets and shelving that contained over a hundred containers of chemicals. Sliding cabinet doors prevented the containers from falling to the floor and breaking. A shock wave from the accident shattered all four windows in the room. The shock wave also destroyed the control system for an incubator and shook an interior wall.

Non-uniform Circular Motion

For an object moving along a curved trajectory, with non-uniform speed

$\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t$ (radial and tangential)



$$a_r = \frac{v^2}{r}$$

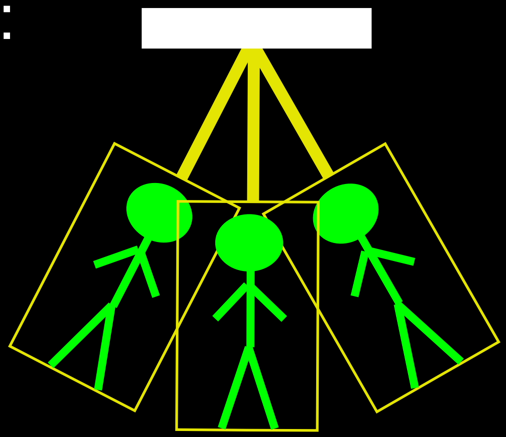
$$a_t = \frac{d|\vec{v}|}{dt}$$

What are F_r and F_t ?

ma_r and ma_t

Lecture 7, *Example*
Gravity, Normal Forces etc.

Consider a person on a swing:

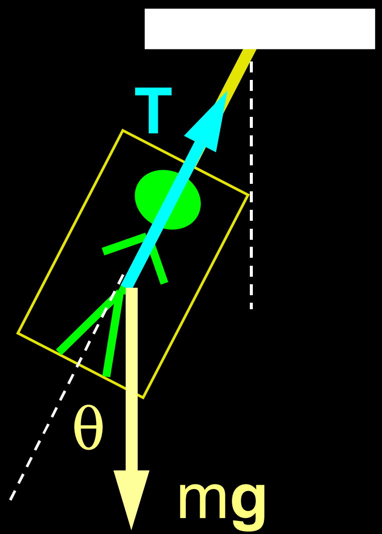


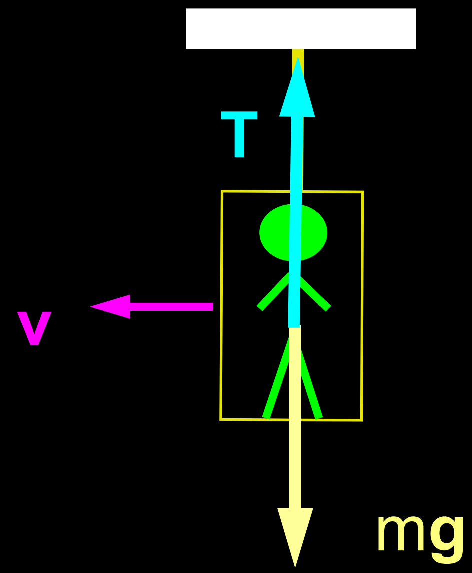
When is the tension on the rope largest?
And at that point is it :

(A) greater than
(B) the same as
(C) less than

the force due to gravity acting on the person?

Lecture 7, *Example*
Gravity, Normal Forces etc.





at top of swing $v_t = 0$

$F_c = m\,0^2 / r = 0 = T - mg \cos\theta$

$T = mg \cos\theta$

$T < mg$

at bottom of swing v_t *is max*

$F_c = m\,a_c = m\,v_t^2 / r = T - mg$

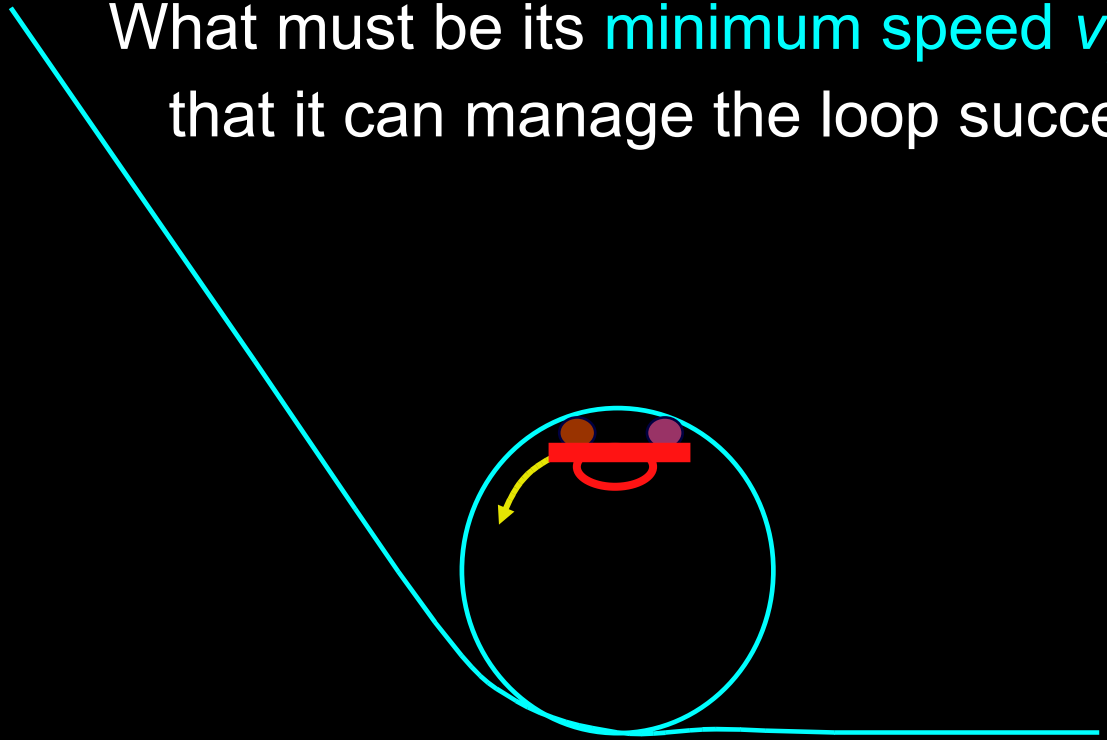
$T = mg + m\,v_t^2 / r$

$T > mg$

Loop-the-loop 1

A match box car is going to do a loop-the-loop of radius r.

What must be its minimum speed v_t at the top so that it can manage the loop successfully ?



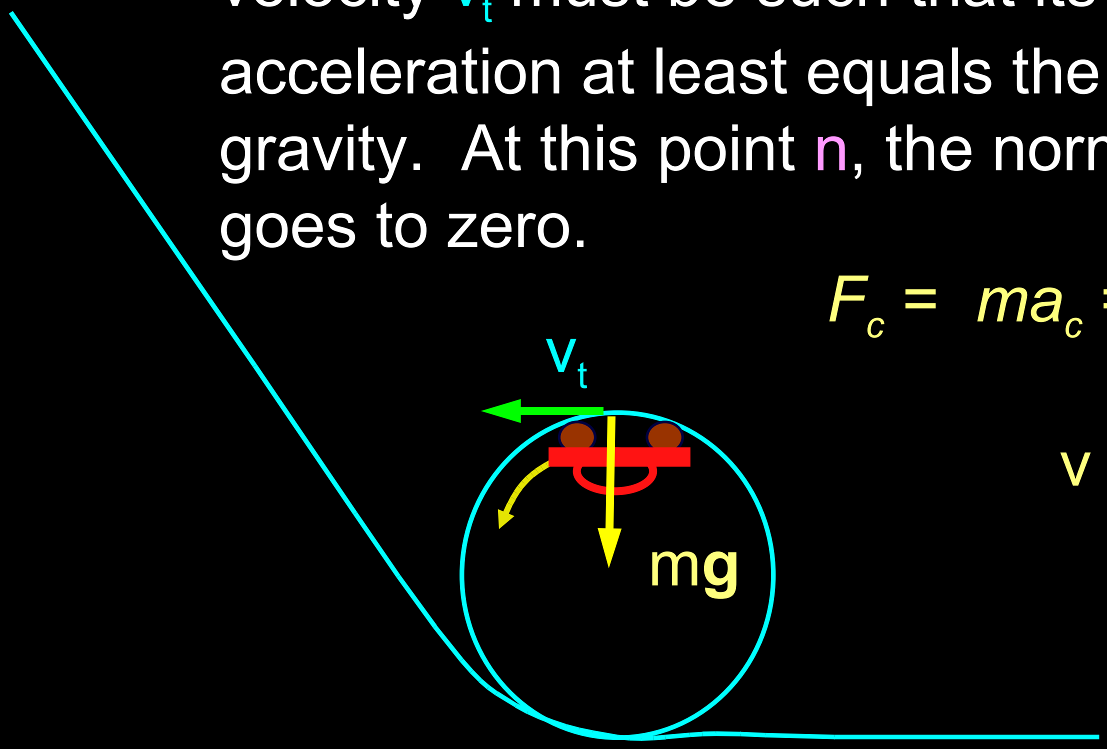
 demo

Loop-the-loop 1

To navigate the top of the circle its tangential velocity v_t must be such that its centripetal acceleration at least equals the force due to gravity. At this point n, the normal force, goes to zero.

$$F_c = ma_c = mg = mv^2/r$$

$$v = (gr)^{1/2}$$



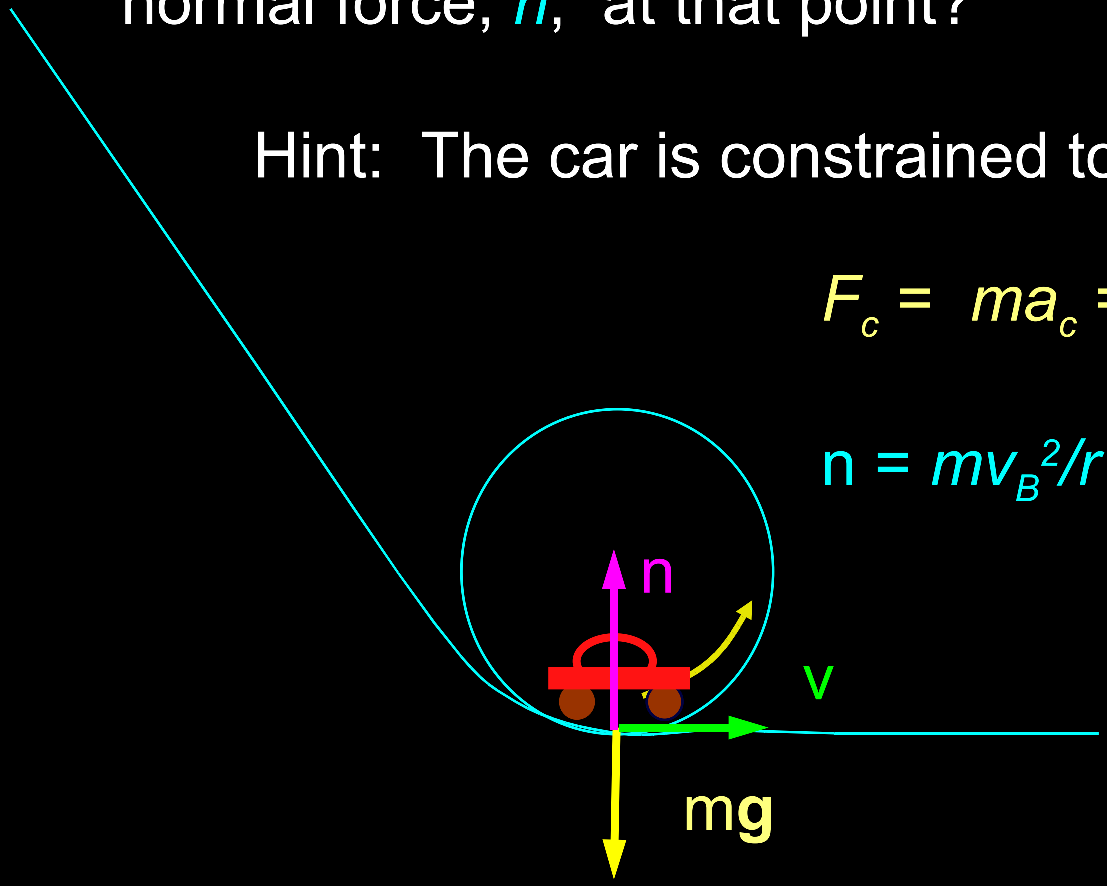
Loop-the-loop 3

The match box car is going to do a loop-the-loop. If the speed at the bottom is v_B, what is the normal force, n, at that point?

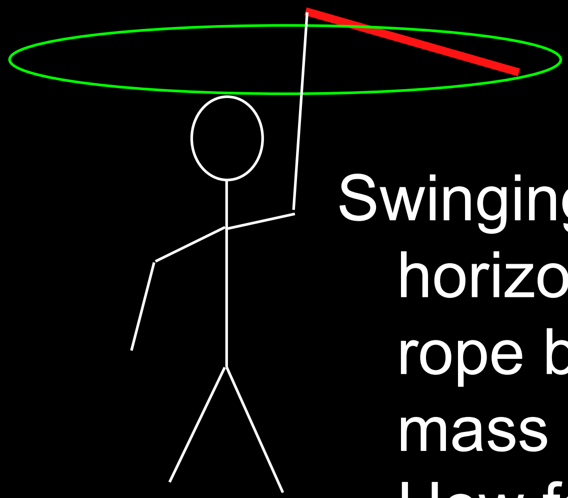
Hint: The car is constrained to the track.

$$F_c = ma_c = mv_B^2/r = n - mg$$

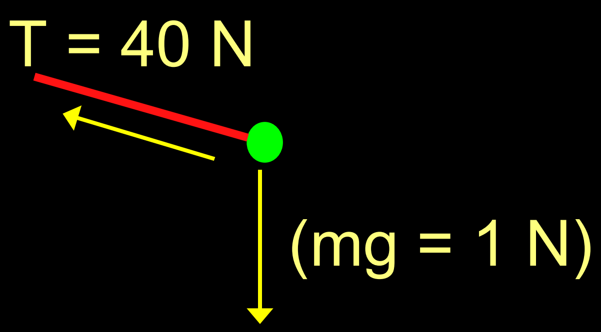
$$n = mv_B^2/r + mg$$



Example Problem



Swinging around a ball on a rope in a “nearly” horizontal circle over your head. Eventually the rope breaks. If the rope breaks at **64 N**, the ball’s mass is **0.10 kg** and the rope is **0.10 m**
How fast is the ball going when the rope breaks?
(neglect g)



$$F_c = m\, v_t^2 / r$$

$$v_t = (r\, F_c / m)^{1/2}$$

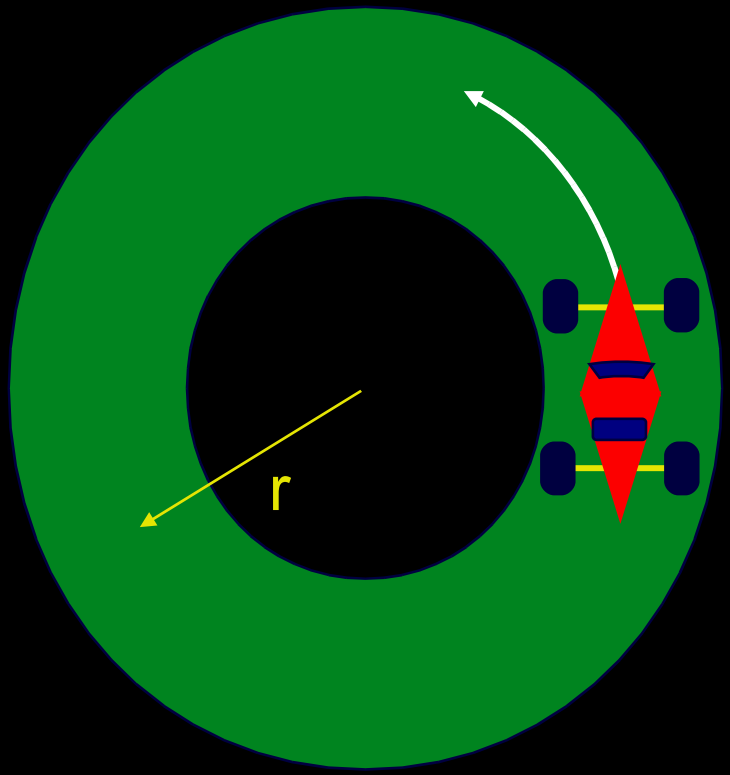
$$v_t = (0.10 \times 64 / 0.10)^{1/2} \text{ m/s}$$

$$v_t = 8 \text{ m/s}$$

Lecture 8, *Example*
Circular Motion Forces with Friction
(recall $ma_c = m\,v^2 / r \qquad F_f \le \mu_s\, n$)

- How fast can the race car go?

  (How fast can it round a corner with this radius of curvature?)



$m_{car} = 1600$ kg

$\mu_S = 0.5$ for tire/road

$r = 80$ m

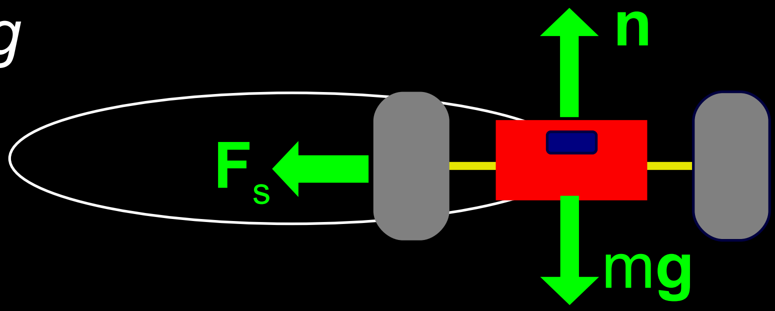
$g = 10$ m/s^2

Lecture 8, *Example*

- Only one force, that of friction, is in the horizontal direction

x-dir: $F_c = ma_c = m\,v^2 / r = F_s = \mu_s\, n$ (at maximum)

y-dir: $ma = 0 = n - mg \quad n = mg$



$v = (\mu_s\; m\, g\, r / m\,)^{1/2}$

$v = (\mu_s\; g\, r\;)^{1/2} = (0.5 \times\; 10 \times 80)^{1/2}$

$v = 20$ m/s

$m_{car} = 1600$ kg

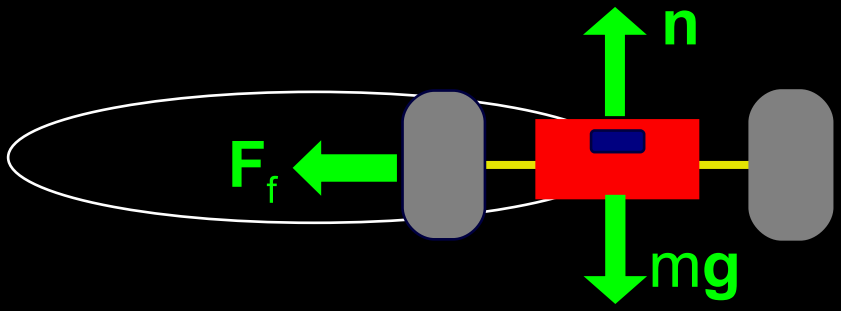
$\mu_S\; = 0.5$ for tire/road

$r\; = 80$ m

$g\; = 10$ m/s^2

Banked Curves

In the previous scenario, we drew the following free body diagram for a race car going around a curve on a flat track.



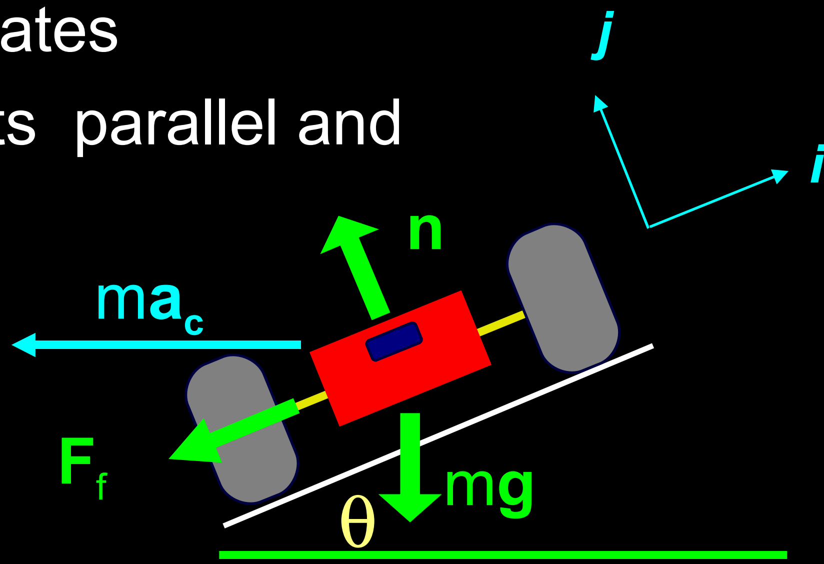
What differs on a banked curve?

Banked Curves

Free Body Diagram for a banked curve.

Use rotated x-y coordinates

Resolve into components parallel and perpendicular to bank

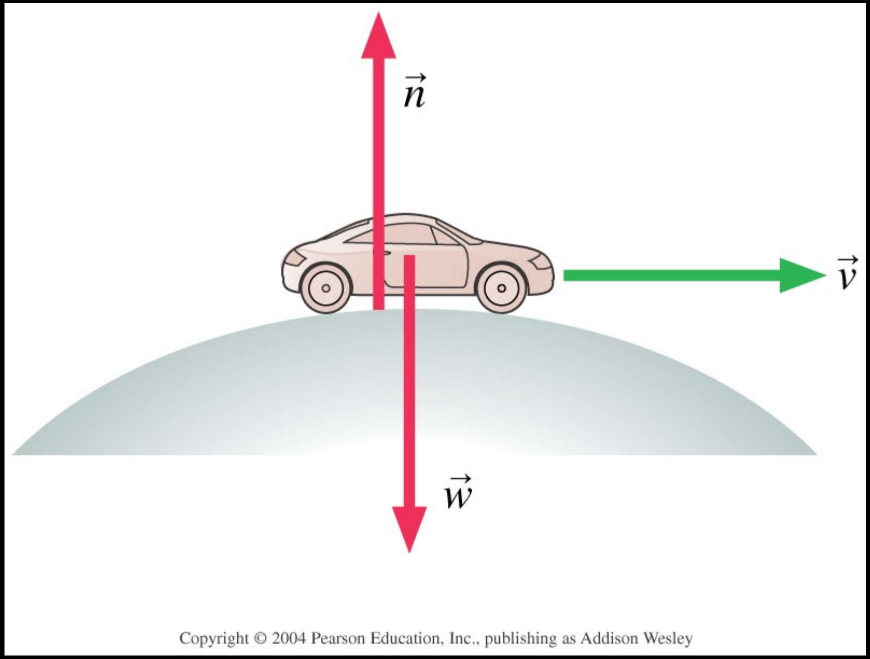


For very small banking angles, one can approximate that F_f is parallel to ma_c. This is equivalent to the small angle approximation $\sin\theta = \tan\theta$, but very effective at pushing the car toward the center of the curve!!

Navigating a hill

Knight concept exercise: A car is rolling over the top of a hill at speed *v*. At this instant,

- $n > w$.
- $n = w$.
- $n < w$.
- We can't tell about *n* without knowing *v*.



At what speed does the car lose contact?

This occurs when the normal force goes to zero or, equivalently, when all the weight is used to achieve circular motion.

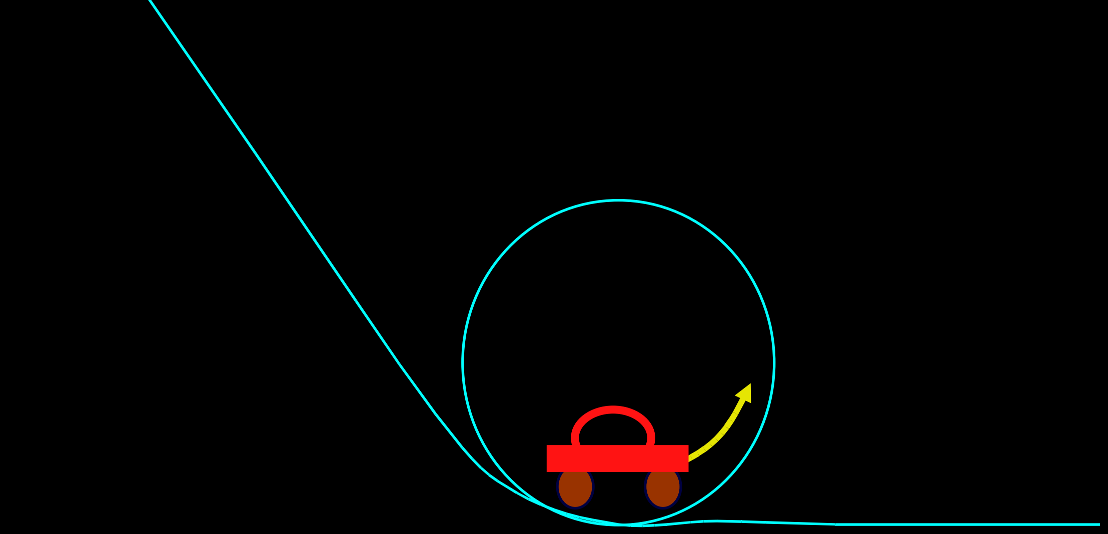
$F_c = mg = m\,v^2/r \rightarrow \quad v = (gr)^{1/2}$ ½ (just like an object in orbit)

Note this approach can also be used to estimate the maximum walking speed.

Loop-the-loop 2

Once again the car is going to execute a loop-the-loop. What must be its minimum speed at the bottom so that it can make the loop successfully?

This is a difficult problem to solve using just forces. We will skip it now and revisit it using energy considerations later on…



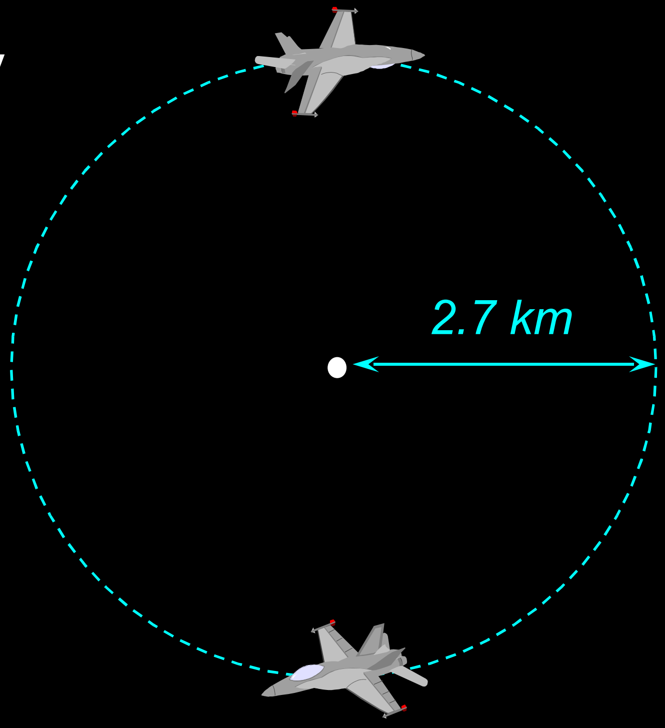
Lecture 8, *Exercise*

- When a pilot executes a loop-the-loop (figure on the right) the aircraft moves in a vertical circle of radius R=2.70 km at a constant speed of v=225 m/s. Is the force exerted by the seat on the pilot:

(A) Larger

(B) Same

(C) Smaller



than the pilot's weight (mg) at

(I) the bottom and

(II) at the top of the loop?