Physics 207, Lecture 8, Oct. 1

Agenda:

- Chapter 7 (Circular Motion, Dynamics III)
  - ❖ Uniform and non-uniform circular motion
- Next time: Problem Solving and Review for MidTerm I

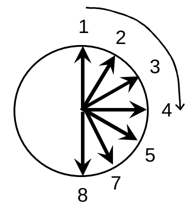
Assignment: (NOTE special time!)

- MP Problem Set 4 due Oct. 3,Wednesday, 4 PM
- MidTerm Thurs., Oct. 4, Chapters 1-6 & 7 (lite), 90 minutes, 7:15-8:45 PM

Rooms: B102 & B130 in **Van Vleck**.

Rotation requires a new lexicon

- Period
- Frequency
- Angular frequency
- Angular position
- Angular velocity



Period (T): The time required to sweep out one full revolution, 360° or 2π radians

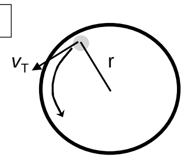
Frequency (f): 1/T, number of cycles per unit time

Angular frequency or velocity (ω): $2\pi f$, number of radians traced out per unit time

Relating rotation motion to linear velocity

- Assume a particle moves at constant tangential speed v_T around a circle of radius r.
- Distance = velocity X time
- Once around… $2\pi r = v_T$ X T or
  rearranging $(2\pi/T)\ r = v_T$

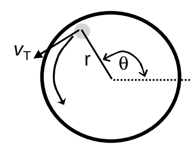
$\boxed{\omega\, r = v_T}$



Note: UCM is uniform circular motion

Angular displacement and velocity

- Arc traversed $s = \theta\, r$
  In time Δt then $\Delta s = \Delta\theta\, r$
  so $\Delta s / \Delta t = (\Delta\theta / \Delta t)\, r$
  In the limit $\Delta t \rightarrow 0$
  one gets
  $ds/dt = d\theta/dt\ \ r$
  $v_T = \omega\ \ r$



$$\boxed{\begin{array}{l}\omega = d\theta/dt \\ \text{and if } \omega \text{ is constant} \\ \theta = \theta + \omega\, t\end{array}}$$

Counter-clockwise is positive, clockwise is negative

And we note v_{radial} (or v_r) and v_z (are both zero if UCM)

And if ω is increasing…

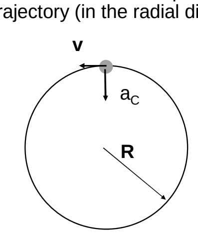
- Then angular acceleration [i.e., $\alpha(t)$]
  and $d\omega/dt \neq 0$
- If $d\omega/dt$ is constant or just α (either positive or negative)

$$\boxed{\begin{array}{l}\theta = \theta_0 + \omega_0\, t + \tfrac{1}{2}\,\alpha\, t^2 \\ \\ \omega = \omega_0 + \alpha\, t\end{array}}$$

- Many analogies to linear motion but it isn't one to one
- Example: if constant angular velocity, there is radial acceleration.

Newton's Laws and Circular Motion (Chapter 7)

Uniform circular motion involves only changes in the direction of the velocity vector and the associated acceleration must be perpendicular to any point on the trajectory (in the radial direction). Quantitatively (see text)



Centripetal Acceleration
$a_C = v_T^2/R$

Circular motion involves continuous radial acceleration and this means a central force.

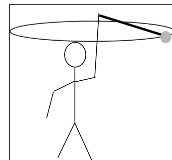
$F_C = ma_C = mv_T^2/R = m\omega^2\ R$

No central force…no UCM!

Circular Motion Demo….

- We don't have a hoop but a string.
- UCM enables high accelerations (g's) in a small space
- Comment: In automobile accidents involving rotation severe injury or death can occur even at modest speeds.

[In physics speed doesn't kill….acceleration (i.e., force) does.]



Applications

- Mass based separations:
  - Centrifuges
  - Mass Spectroscopy

How many *g*'s?

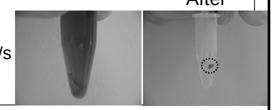
$a_c = v^2 / r$ and $f = 10^4$ rpm is typical with $r = 0.1$ m

and $v = \omega r = 2\pi f r$

$v = (2\pi \times 10^4 / 60) \times 0.1$ m/s $= 100$ m/s

$a_c = 1 \times 10^4 / 0.1$ m/s^2 = 10 000 g's

Before After



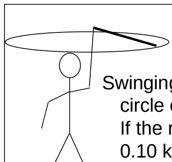
Benchmarks with respect to humans

Some Typical g-Forces

- 1 g Standing
- 1.2 g Normal elevator acceleration (up).
- 1.5-2g Walking down stairs.
- 2-3 g Hopping down stairs.
- 1.5 g Commercial airliner during takeoff run.
- 2 g Commercial airliner at rotation
- 3.5 g Maximum acceleration in amusement park rides (design guidelines).
- 4 g Indy cars in the second turn at Disney World (side and down force).
- 4+ g Carrier based aircraft launch.
- 10 g Threshold for blackout during violent maneuvers in high performance aircraft.
- 11 g Alan Shepard in his historic sub orbital Mercury flight experience a maximum force of 11 g.
- 20 g The Colonel Stapp experiments on acceleration in rocket sleds indicated that in the 10 to 20 g range there was the possibility of injury because of organs moving inside the body. Beyond 20 g they concluded that there was the potential for death due to internal injuries. Their experiments were limited to 20 g.
- 30 g The design maximum for sleds used to test dummies with commercial restraint and air bag systems is 30 g.

A bad day at the lab….

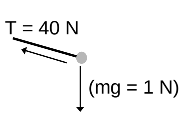
- In 1998, a Cornell campus laboratory was seriously damaged when the rotor of an ultracentrifuge failed while in use. Flying metal fragments damaged walls, the ceiling and other equipment.
- Description of the Cornell Accident -- On December 16, 1998, milk samples were running in a Beckman. L2-65B ultracentrifuge using a large aluminum rotor. The rotor had been used for this procedure many times before. Approximately one hour into the operation, the rotor failed due to excessive mechanical stress caused by the "g" forces of the high rotation speed. The subsequent explosion completely destroyed the centrifuge. The safety shielding in the unit did not contain all the metal fragments. The half inch thick sliding steel door on top of the unit buckled allowing fragments, including the steel rotor top, to escape. Fragments ruined a nearby refrigerator and an ultra_cold freezer in addition to making holes in the walls and ceiling. The unit itself was propelled sideways and damaged cabinets and shelving that contained over a hundred containers of chemicals. Sliding cabinet doors prevented the containers from falling to the floor and breaking. A shock wave from the accident shattered all four windows in the room. The shock wave also destroyed the control system for an incubator and shook an interior wall



Example Problem

Swinging a ball a rope around in a "near" horizontal circle over my head. Eventually the rope breaks. If the rope breaks at 64 N, the ball's mass is 0.10 kg and the rope is 0.10 m (neglect g)

How fast is the ball going when the rope breaks?



$F_c = m v_T^2 / r$

$v_T = (r F_c / m)^{1/2}$

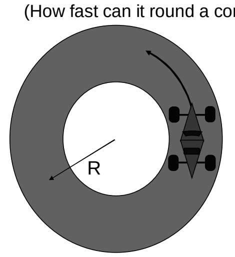
$v_T = (0.10 \times 64 / 0.10)^{1/2}$ m/s

$v_T = 8$ m/s

Lecture 7, *Example*
Circular Motion Forces with Friction
(recall $ma_C = m v^2 / R$ $F_f \le \mu_s N$)

- How fast can the race car go ?

(How fast can it round a corner with this radius of curvature?)



m_{car} = 1600 kg

μ_S = 0.5 for tire/road

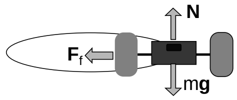
R = 80 m

g = 10 m/s^2

Lecture 7, *Example*
(1^{st} Draw A Free Body Diagram)

- Only one force, that of friction, is in the horizontal direction

x-dir: $F_C = ma_C = m\,v^2 / R = F_f = \mu_s N$ (at maximum)
y-dir: $ma = 0 = N - mg$



$v = (\mu_s\ m\,gR / m\,)^{1/2}$

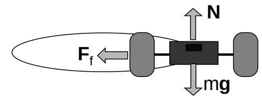
$v = (\mu_s\ g\,R\ )^{1/2} = (0.5 \times\ 10 \times 80)^{1/2}$

$v = 20$ m/s

What if there is a banked curve ?

Banked Corners

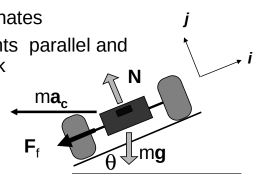
In the previous scenario, we drew the following free body diagram for a race car going around a curve on a flat track.



What differs on a banked curve ?

Banked Corners

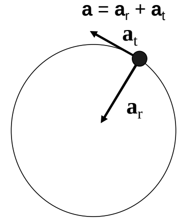
Free Body Diagram for a banked curve.
Use rotated x-y coordinates
Resolve into components parallel and perpendicular to bank



For very small banking angles, one can approximate that $\mathbf{F}_f$ is parallel to m**a**. This is equivalent to the small angle approximation sin θ = tan θ.

Non uniform Circular Motion

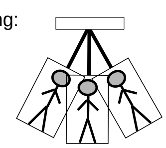
Earlier we saw that for an object moving along a curved pathwith non uniform speed then
$\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t$ (radial and tangential, a_θ)



What are F_r and F_t ?
ma_r and ma_t

Lecture 7, *Example*
Gravity, Normal Forces etc.

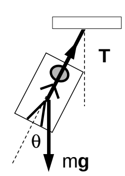
Consider a person on a swing:

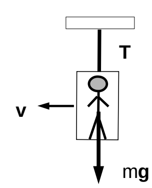


Active Figure

When is the tension on the rope largest ? And at that point is it :
(A) greater than
(B) the same as
(C) less than
the force due to gravity acting on the person

Lecture 7, *Example*
Gravity, Normal Forces etc.





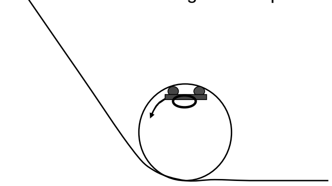
$F_c = m\,0^2 / r = 0 = T - mg\cos\theta$
$F_T = m\,aT = mg\sin\theta$

$F_c = m\,a_c = m\,v^2 / r = T - mg$
$T = mg + m\,v^2 / r$

At the bottom of the swings and is it (A) greater than the force due to gravity acting on the woman

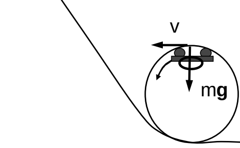
Loop-the-loop 1

A match box car is going to do a loop-the-loop of radius r.
What must be its minimum speed, v, at the top so that it can manage the loop successfully ?



Loop-the-loop 1

To navigate the top of the circle its tangential velocity, v, must be such that its centripetal acceleration at least equals the force due to gravity. At this point N, the normal force, goes to zero.



$F_c = - ma = - mg = - mv^2/r$

$v = (gr)^{1/2}$

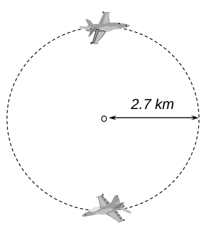
Lecture 7, *Exercise*

- When a pilot executes a loop-the-loop (as in figure on the right) the aircraft moves in a vertical circle of radius R=2.70 km at a constant speed of v=225 m/s. Is the force exerted by the seat on the pilot:

  (A) Larger
  (B) Same
  (C) Smaller

  then pilot's weight (mg) at

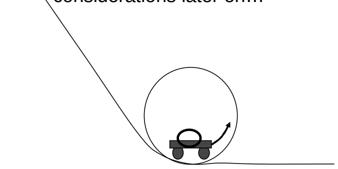
  (I) the bottom and
  (II) at the top of the loop.



Loop-the-loop 2

Once again the the box car is going to execute a loop-the-loop. What must be its minimum speed at the bottom so that it can make the loop successfully?

This is a difficult problem to solve using just forces. We will skip it now and revisit it using energy considerations later on…



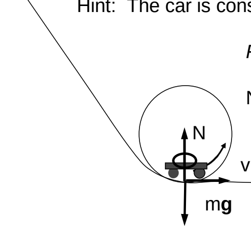
Loop-the-loop 3

The match box car is going to do a loop the loop. If the speed at the bottom is v_B, what is the normal force, N, at that point?

Hint: The car is constrained to the track.

$F_c = ma = mv_B^2/r = N - mg$

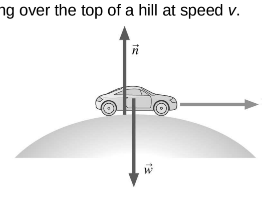
$N = mv_B^2/r + mg$



Navigating a hill

Knight concept exercise: A car is rolling over the top of a hill at speed v. At this instant,

A. $n > w$.
B. $n = w$.
C. $n < w$.
D. We can't tell about n without knowing v.



At what speed do we lose contact?

This occurs when the normal force goes to zero or, equivalently, when all the weight is used to achieve circular motion.

$F_c = mg = m v^2 /R \rightarrow v = (g/R)^{½}$ (just like an object in orbit)

Note this approach can also be used to estimate the maximum walking speed.

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