"Professor Goddard does not know the relation between action and reaction and the need to have something better than a vacuum against which to react. He seems to lack the basic knowledge ladled out daily in high schools."

New York Times editorial, 1921, about Robert Goddard's revolutionary rocket work.

“Correction: It is now definitely established that a rocket can function in a vacuum. The ‘Times’ regrets the error.”


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Partial Survey Summary

- Lectures:
  - Too many slides that come too quickly
  - More problem solving on white board
  - Too much time spent on “interactive problems but, when used, not enough time spent on explanation
  - More demos

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Another example with friction and pulley

- Three 1 kg masses are connected by two strings as shown below. There is friction between the stacked masses but the table top is frictionless.
- Assume the pulleys are massless and frictionless.
- What is $T_1$?

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Impulse & Linear Momentum

- Transition from forces to conservation laws

Newton’s Laws $\rightarrow$ Conservation Laws

Conservation Laws $\rightarrow$ Newton’s Laws

They are different faces of the same physics phenomenon.

NOTE: We already have studied “impulse” and “momentum” but we have not explicitly named them as such

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Lecture 11, Example 1

A 2 kg cart initially at rest on frictionless horizontal surface is acted on by a 10 N horizontal force along the positive x-axis for 2 seconds what is the final velocity?

- $F$ is in the x-direction $F = ma$ so $a = F/m = 5 \text{ m/s}^2$
- $v = v_0 + at = 0 \text{ m/s} + 2 \times 5 \text{ m/s} = 10 \text{ m/s} (+x\text{-direction})$

What if the mass had been 4 kg?

What is the new final velocity?
Twice the mass

- Same force
- Same time
- Half the acceleration
- Half the velocity! (5 m/s)

Example 1

- Notice that the final velocity in this case is inversely proportional to the mass (i.e., if thrice the mass….one-third the velocity).
- It would seem that mass times the velocity always gives the same value. (Here is it always 20 kg m/s.)

Example 1

- There are many situations in which the product of “mass times velocity” is a constant and so we give a special name, “momentum” and associate it with a conservation law.
  
  (Units: kg m/s or N s)

  - A force applied for a certain period of time can be plotted and the area under this curve is called “impulse”

Example 1 with Action-Reaction

- Now the 10 N force from before is applied by person (me) and I happen to be standing on a frictionless surface as well and I am also initially at rest.
- What is the force on me and for how long?

Example 1 with Action-Reaction

- The 10 N force from before is applied by person (me) and I happen to be standing on a frictionless surface as well and I am also initially at rest.
- What is the force on me and for how long?
  - 10 N but in the –x direction
  - 2 seconds
- And what is my final velocity ($V$) if I’m mass “M”?
  - $V = a't = F/M t$ in the –x direction
Example 1 with Action-Reaction

- The 10 N force from before is applied by person (me) and I happen to be standing on a frictionless surface as well and I am also initially at rest.
- What is the force on me and for how long?
  - 10 N but in the \(-x\) direction
  - 2 seconds
- And what is my final velocity \((V)\) if I'm mass "M"?
  - \(V = \frac{F}{M} t = -20 \text{ kg m/s}\)
- And notice that the total momentum before and after (that of the cart and myself) remained zero.
- This is the essence of momentum conservation

Applications of Momentum Conservation

Radioactive decay:

\[ ^{238}\text{U} \rightarrow ^{234}\text{Th} + ^{4}\text{He} \]

Explosions

\(v_1 = 300 \text{ m/s} \quad m_1 = 5.00 \text{ kg} \)

Collisions

Impulse & Linear Momentum

- Definition: For a single particle, the momentum \(p\) is defined as:
  \[ p = mv \quad (p\text{ is a vector since } v\text{ is a vector}) \]
  So \(p_x = mv_x\) and so on (y and z directions)
- Newton's 2nd Law: \(F = ma\)

\[
\frac{d}{dt}p = \frac{d}{dt}(mv) \quad \Rightarrow \quad \frac{d}{dt}p = F = \frac{d}{dt}m \vec{v}
\]
- This is the most general statement of Newton's 2nd Law

Momentum Conservation

- Momentum conservation (recasts Newton's 2nd Law when \(F = 0\)) is a fundamentally important principle.
- A vector expression \((p_x, p_y, p_z)\)
  - And applicable in any situation in which there is NO net external force applied.

Momentum Conservation

- Many problems can be addressed through momentum conservation even if other physical quantities (e.g. mechanical energy) are not conserved

Collisions always conserve momentum if not acted upon by an external force
Lecture 11, Exercise 1
Momentum is a Vector (!) quantity
• A block slides down a frictionless ramp and then falls and lands in a cart which then rolls horizontally without friction
• After the block leaves the ramp is momentum conserved?

A. Yes
B. No
C. Yes & No
D. Too little information given

Answer: Yes (x-direction) and No (y-direction)

Inelastic collision in 1-D: Example 2
• A block of mass $M$ is initially at rest on a frictionless horizontal surface. A bullet of mass $m$ is fired at the block with a muzzle velocity (speed) $v$. The bullet lodges in the block, and the block ends up with a speed $V$. In terms of $m$, $M$, and $V$:

What is the momentum of the bullet with speed $v$?

Key question: Is $x$-momentum conserved?

Before: $mv + M0 = (m + M)V$

After: $mV + M0 = (m + M)V$

$\text{Key question: Is } x\text{-momentum conserved?}$

$mV = 0$

$Lecture 11, Example 2
Inelastic Collision in 1-D with numbers

Do not try this at home!

Before: A 4000 kg bus, twice the mass of the car, moving at 30 m/s impacts the car at rest.

What is the final speed after impact if they move together?
**Physics 207 – Lecture 11**

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**Lecture 11, Exercise 2**

**Momentum Conservation**

- Two balls of equal mass are thrown horizontally with the same initial velocity. They hit identical stationary boxes resting on a frictionless horizontal surface.
- The ball hitting box 1 bounces elastically back, while the ball hitting box 2 sticks.
- Which box ends up moving fastest?

- A. Box 1
- B. Box 2
- C. same

---

**Elastic Collisions**

- Elastic means that the objects do not stick.
- There are many more possible outcomes but, if no external force, then momentum will always be conserved.
- Start with a 1-D problem.

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**Force and Impulse**

**(A variable force applied for a given time)**

- Gravity: usually a constant force to an object
- Springs often provides a linear force (-kx) towards its equilibrium position
- Collisions often involve a varying force $F(t): 0 \rightarrow \text{maximum} \rightarrow 0$
- We can plot force vs time for a typical collision. The impulse, $J$, of the force is a vector defined as the integral of the force during the time of the collision.

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**A perfectly inelastic collision in 2-D**

- Consider a collision in 2-D (cars crashing at a slippery intersection...no friction).

---

**Elastic Collisions in 1-D**

- Before
- After

---

**Force and Impulse**

**(A variable force applied for a given time)**

- $J$ reflects momentum transfer
  
  \[ J = \int F \, dt = \int (dp/dt) \, dt = \int \delta p \]

---

**Impulse**

- Impulse $J$ = area under this curve!
- Impulse has units of Newton-seconds
Force and Impulse

- Two different collisions can have the same impulse since \( J \) depends only on the momentum transfer, NOT the nature of the collision.

\[
\text{Average Force and Impulse}
\]

Lecture 11, Exercise 3

Force & Impulse

- Two boxes, one heavier than the other, are initially at rest on a horizontal frictionless surface. The same constant force \( F \) acts on each one for exactly 1 second.

Which box has the most momentum after the force acts?

- A. heavier
- B. lighter
- C. same
- D. can’t tell

Back of the envelope calculation

\[
\overline{J} = \int F \, dt = \overline{F} \Delta t
\]

1. \( m_{\text{arm}} = 7 \text{ kg} \)
2. \( v_{\text{arm}} = 7 \text{ m/s} \)
3. Impact time \( \Delta t = 0.01 \text{ s} \)

- Impulse: \( J = \Delta p = m_{\text{arm}} v_{\text{arm}} = 49 \text{ kg m/s} \)
- \( F \approx J/\Delta t = 4900 \text{ N} \)

(1) \( m_{\text{head}} = 6 \text{ kg} \)

\[
\text{Impact force} = F = \frac{F}{m_{\text{head}}} \approx 800 \text{ m/s}^2 \approx 80 \text{ g} \\
\text{Enough to cause unconsciousness} \sim 40\% \text{ of fatal blow}
\]
During "collision" with a tree, nominally

\[ a_{\text{head}} \approx 600 - 1500 \text{ g}!! \]

**How do they survive?**

- Jaw muscles act as shock absorbers
- Straight head trajectory reduces damaging rotations (rotational motion is very problematic)

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**Physics 207, Lecture 11, Oct. 10**

**Assignment:**

- Read through Chapter 10
- MP HW5 available now, due Wednesday 10/17, 11:59 PM