"Professor Goddard does not know the relation between action and reaction and the need to have something better than a vacuum against which to react. He seems to lack the basic knowledge ladled out daily in high schools."

New York Times editorial, 1921, about Robert Goddard's revolutionary rocket work.

"Correction: It is now definitely established that a rocket can function in a vacuum.
The 'Times' regrets the error."

New York Times editorial, July 1969.

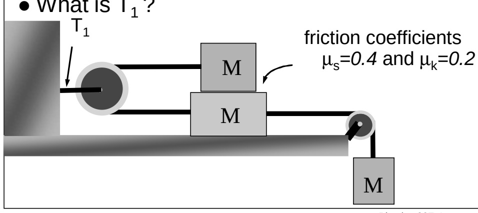
Partial Survey Summary

Lecture

- Too many slides that come too quickly
- More problem solving on white board
- Too much time spent on "interactive problems but, when used, not enough time spent on explanation
- More demos

Another example with friction and pulley

- Three 1 kg masses are connected by two strings as shown below. There is friction between the stacked masses but the table top is frictionless.
- Assume the pulleys are massless and frictionless.
- What is T_1 ?



Physics 207, Lecture 11, Oct. 10

Agenda:

- Chapter 9: Momentum & Impulse
  - ❖ Momentum conservation
  - ❖ Collisions
  - ❖ Impulse

Assignment:

- Read through Chapter 10
- MP HW5 available now, due Wednesday 10/17, 11:59 PM

Impulse & Linear Momentum

- **Transition from forces to conservation laws**

Newton's Laws → Conservation Laws
Conservation Laws → Newton's Laws

They are different faces of the same physics phenomenon.

NOTE: We already have studied "impulse" and "momentum" but we have not explicitly named them as such

Lecture 11, Example 1

A 2 kg cart initially at rest on frictionless horizontal surface is acted on by a 10 N horizontal force along the positive x-axis for 2 seconds what is the final velocity?

- F is in the x-direction F = ma so a = F/m = 5 m/s²
- $v = v_0 + a\,t$ = 0 m/s + 2 x 5 m/s = 10 m/s (+x-direction)

What if the mass had been 4 kg?
What is the new final velocity?

Twice the mass

- Same force
- Same time
- Half the acceleration
- Half the velocity ! (5 m/s)

Example 1

- Notice that the final velocity in this case is inversely proportional to the mass (i.e., if thrice the mass….one-third the velocity).
- It would seems that mass times the velocity always gives the same value. (Here is it always 20 kg m/s.)

Example 1

- There many situations in which the product of "mass times velocity" is a constant and so we give a special name, "momentum" and associate it with a conservation law.

  (Units: kg m/s or N s)
- A force applied for a certain period of time can be plotted and the area under this curve is called "impulse"

Example 1 with Action-Reaction

- Now the 10 N force from before is applied by person (me) and I happen to be standing on a frictionless surface as well and I am also initially at rest.
- What is the force on me and for how long ?

Example 1 with Action-Reaction

- The 10 N force from before is applied by person (me) and I happen to be standing on a frictionless surface as well and I am also initially at rest.
- What is the force on me and for how long ?
- 10 N but in the –x direction
- 2 seconds

Example 1 with Action-Reaction

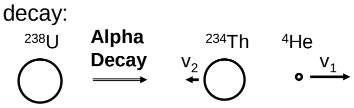
- The 10 N force from before is applied by person (me) and I happen to be standing on a frictionless surface as well and I am also initially at rest.
- What is the force on me and for how long ?
- 10 N but in the –x direction
- 2 seconds
- And what is my final velocity (V) if I'm mass "M" ?
- $V = a' t = F/M t$ in the –x direction

Example 1 with Action-Reaction

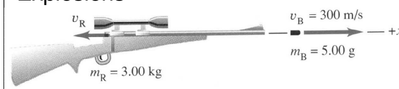
- The 10 N force from before is applied by person (me) and I happen to be standing on a frictionless surface as well and I am also initially at rest.
- What is the force on me and for how long ?
- 10 N but in the –x direction
- 2 seconds
- And what is my final velocity (V) if I'm mass "M" ?
- V = F/M t in the –x direction
- but rearranging gives MV = Ft = -20 kg m/s
- And notice that the total momentum before and after (that of the cart and myself) remained zero.
- This is the essence of momentum conservation

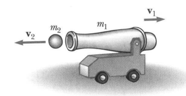
Applications of Momentum Conservation

Radioactive decay:

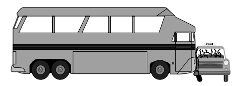


Explosions





Collisions



Impulse & Linear Momentum

- **Definition:** For a single particle, the momentum ***p*** is defined as:

$$\boldsymbol{p} \equiv m\boldsymbol{v}$$ (***p*** is a vector since ***v*** is a vector)

So $p_x = mv_x$ and so on (y and z directions)

- Newton's 2nd Law: $\boldsymbol{F} = m\boldsymbol{a}$

$$= m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) \quad \Rightarrow \quad \vec{F} = \frac{d\vec{p}}{dt}$$

- This is the most general statement of Newton's 2nd Law

Momentum Conservation

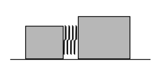
$$\boldsymbol{F}_{EXT} = \frac{d\boldsymbol{P}}{dt} \Rightarrow \frac{d\boldsymbol{P}}{dt} = 0 \Leftarrow \boldsymbol{F}_{EXT} = 0$$

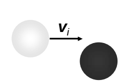
- Momentum conservation (recasts Newton's 2nd Law when **F** = 0) is a fundamentally important principle.
- A vector expression (P_x, P_y and P_z) .
  - ❖ And applicable in any situation in which there is NO net external force applied.

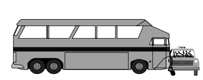
Momentum Conservation

- Many problems can be addressed through momentum conservation even if other physical quantities (e.g. mechanical energy) are not conserved

Collisions alway conserve momentum if not acted upon by an external force

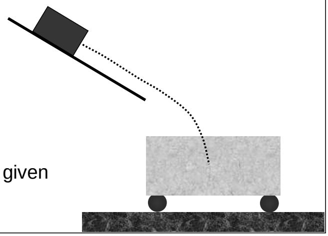






Lecture 11, Exercise 1
Momentum is a Vector (!) quantity

- A block slides down a frictionless ramp and then falls and lands in a cart which then rolls horizontally without friction
- **After** the block leaves the ramp is momentum conserved?



A. Yes
B. No
C. Yes & No
D. Too little information given

Lecture 11, Exercise 1
Momentum is a Vector (!) quantity

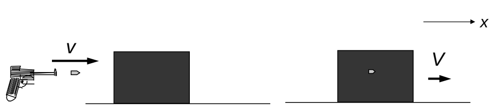
- This is a really hard problems because once the block leaves the rail there is no net external force in the x-direction but there are lots of external forces in the y-direction. First gravity acts on the block and then the earth acts on the cart + block. So if the cart and block comprise the "system" then momentum is conserved in the x-direction motion and NOT conserved with respect to the y-direction motion. The rain and in the boat homework problem is best understood in this context.

- Answer: Yes (x-direction) and No (y-direction)

Inelastic collision in 1-D: Example 2

- A block of mass *M* is initially at rest on a frictionless horizontal surface. A bullet of mass *m* is fired at the block with a muzzle velocity (speed) *v*. The bullet lodges in the block, and the block ends up with a speed *V*. In terms of *m, M*, and *V* :

What is the momentum of the bullet with speed *v* ?



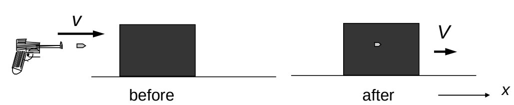
Inelastic collision in 1-D: Example 2

What is the momentum of the bullet with speed *v* ? $m\vec{\mathrm{v}}$

❖ Key question: Is x-momentum conserved ?

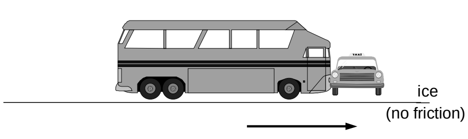
Before ***After***

$$m\mathrm{v} + \mathrm{M}\,0 = (m+M)\mathrm{V}$$



Lecture 11, Example 2
Inelastic Collision in 1-D with numbers

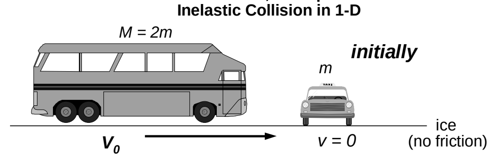
Do not try this at home!



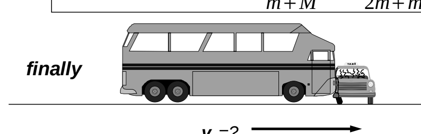
Before: A 4000 kg bus, twice the mass of the car, moving at 30 m/s impacts the car at rest.

What is the final speed after impact if they move together?

Lecture 12, Example 2
Inelastic Collision in 1-D



$$M\mathrm{V}_0 = (m+M)\mathrm{V} \quad \text{or} \quad \mathrm{V} = \frac{M}{m+M}\mathrm{V}_0 = \frac{2m}{2m+m}\mathrm{V}_0$$



2V$_o$/3 = 20 m/s

Lecture 11, *Exercise 2*
Momentum Conservation

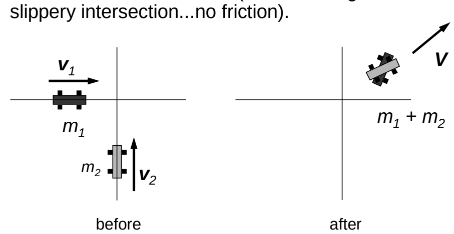
- Two balls of equal mass are thrown horizontally with the same initial velocity. They hit identical stationary boxes resting on a frictionless horizontal surface.
- The ball hitting box 1 bounces elastically back, while the ball hitting box 2 sticks.

❖ Which box ends up moving fastest ?

A. Box 1
B. Box 2
C. same

A perfectly inelastic collision in 2-D

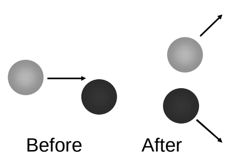
- Consider a collision in 2-D (cars crashing at a slippery intersection...no friction).



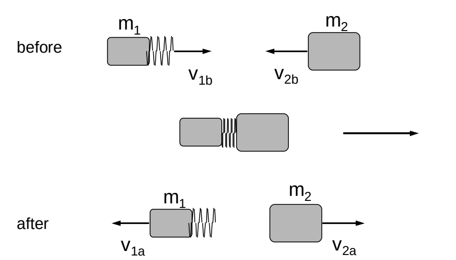
- If no external force momentum is conserved.
- Momentum is a vector so p_x, p_y and p_z

Elastic Collisions

- Elastic means that the objects do not stick.
- There are many more possible outcomes but, if no external force, then momentum will always be conserved
- Start with a 1-D problem.



Elastic Collision in 1-D



Force and Impulse
(A variable force applied for a given time)

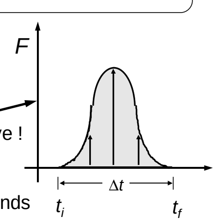
- Gravity: usually a constant force to an object
- Springs often provides a linear force ($-k$ x) towards its equilibrium position
- Collisions often involve a varying force
  F(t): 0 → maximum → 0
- We can plot force vs time for a typical collision. The impulse, ***J***, of the force is a vector defined as the integral of the force during the time of the collision.

Force and Impulse
(A variable force applied for a given time)

- ***J*** reflects momentum transfer

$$\vec{J} = \int^{t} \vec{F}\ dt = \int^{t} (d\vec{p}/dt)dt = \int^{p} d\vec{p}$$

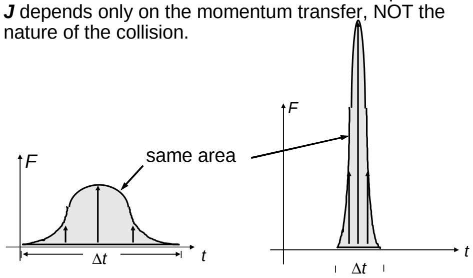
Impulse ***J*** = area under this curve !
(Transfer of momentum !)



Impulse has units of Newton-seconds

Force and Impulse

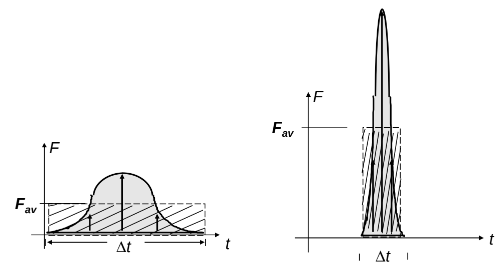
- Two different collisions can have the same impulse since ***J*** depends only on the momentum transfer, NOT the nature of the collision.



Δt big, ***F*** small

Δt small, ***F*** big

Average Force and Impulse



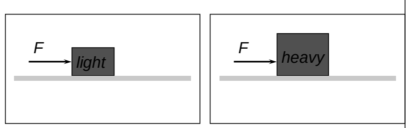
Δt big, F_{av} small

Δt small, F_{av} big

Lecture 11, *Exercise 3*
Force & Impulse

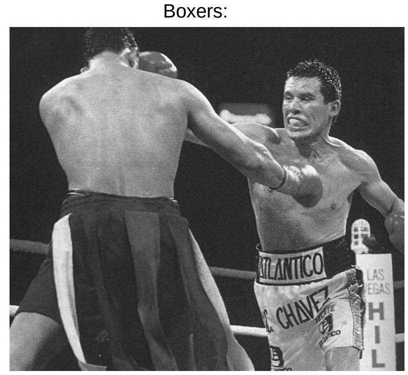
- Two boxes, one heavier than the other, are initially at rest on a horizontal frictionless surface. The same constant force *F* acts on each one for exactly 1 second.

Which box has the most momentum after the force acts ?



A. heavier
B. lighter
C. same
D. can't tell

Boxers:



Back of the envelope calculation

$$\vec{J} = \int^{t} \vec{F}\, dt = \vec{F}_{\text{avg}}\, \Delta t$$

(1) m_{arm}~ 7 kg (2) **v_{arm}**~7 m/s (3) **Impact time Δt** ~ 0.01 s

→ **Impulse** **$J = \Delta p$** ~ $m_{arm}\, v_{arm}$ ~ 49 kg m/s

→ **F ~ J/Δt ~ 4900 N**

(1) m_{head} ~ 6 kg

→ **$a_{head} = F / m_{head}$ ~ 800 m/s² ~ 80 g !**

- **Enough to cause unconsciousness ~ 40% of fatal blow**

Woodpeckers:

During "collision" with a tree, nominally

$$\mathbf{a_{head} \sim 600 - 1500\ g!!}$$

How do they survive?

- Jaw muscles act as shock absorbers
- Straight head trajectory reduces damaging rotations (rotational motion is very problematic)

Physics 207, Lecture 11, Oct. 10

Assignment:

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