Physics 207, Lecture 12, Oct. 15

Agenda: Finish Chapter 9, start Chapter 10

- Chapter 9: Momentum & Impulse
  - ❖ Collisions
  - ❖ Momentum conservation in 2D
  - ❖ Impulse

Assignment:

- HW5 due Wednesday
- HW6 posted soon

Impulse & Linear Momentum

- **Transition from forces to conservation laws**

Newton's Laws → Conservation Laws
Conservation Laws → Newton's Laws

They are different faces of the same physics phenomenon for special "cases"

Lecture 12, *Exercise 1* Momentum Conservation

- Two balls of equal mass are thrown horizontally with the same initial velocity. They hit identical stationary boxes resting on a frictionless horizontal surface.
- The ball hitting box 1 bounces elastically back, while the ball hitting box 2 sticks.
  - ❖ In which case does the box ends up moving fastest ?
  - ❖ No external force then, vectorially, COM

A. Box 1
B. Box 2
C. Same

Lecture 12, *Exercise 1* Momentum Conservation

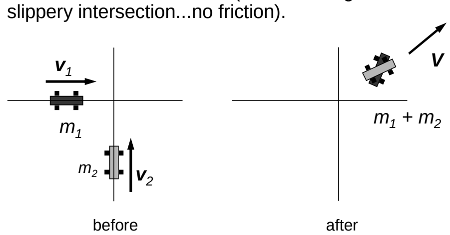
- Which box ends up moving fastest ?
- Examine the change in the momentum of the ball.

In the case of box 1 the balls momentum changes sign and so its net change is largest. Since momentum is conserved the box must have the largest velocity to compensate.

(A) **Box 1** (B) **Box 2** (C) **same**

A perfectly inelastic collision in 2-D

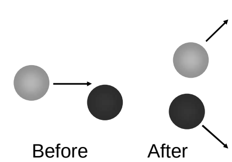
- Consider a collision in 2-D (cars crashing at a slippery intersection...no friction).



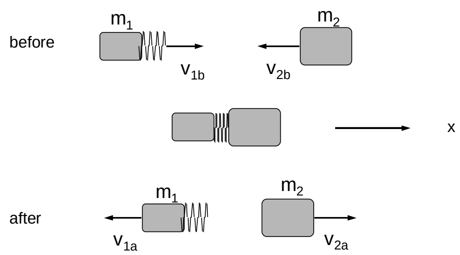
- If no external force momentum is conserved.
- Momentum is a vector so p_x, p_y and p_z

Elastic Collisions

- Elastic means that the objects do not stick.
- There are many more possible outcomes but, if no external force, then momentum will always be conserved
- Start with a 1-D problem.



Elastic Collision in 1-D



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Force and Impulse
(A variable force applied for a given time)

- Gravity: usually a constant force to an object
- Springs often provide a linear force (-*k* x) towards its equilibrium position (Chapter 10)
- Collisions often involve a varying force
  F(t):   0 → maximum → 0
- We can plot force vs time for a typical collision. The impulse, ***J***, of the force is a vector defined as the integral of the force during the time of the collision.

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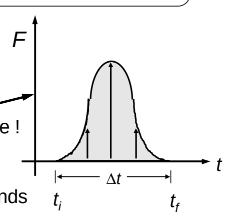
Force and Impulse
(A variable force applied for a given time)

- ***J*** reflects momentum transfer

$$\vec{J} = \int^{t} \vec{F}\, dt = \int^{t} (d\vec{p}/dt)dt = \int^{p} d\vec{p}$$

Impulse ***J*** = area under this curve !
(Transfer of momentum !)



Impulse has units of Newton-seconds

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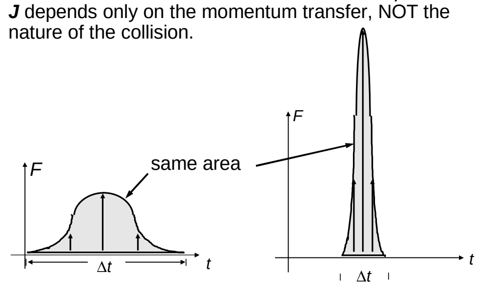
Force and Impulse

- Two different collisions can have the same impulse since ***J*** depends only on the momentum transfer, NOT the nature of the collision.



Δ*t* big, ***F*** small   Δ*t* small, ***F*** big

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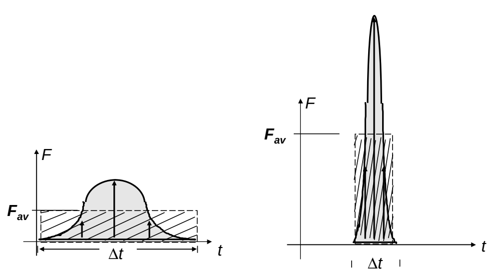
Average Force and Impulse



Δ*t* big, F_{av} small   Δ*t* small, F_{av} big

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Example from last time

A 2 kg cart initially at rest on frictionless horizontal surface is acted on by a 10 N horizontal force along the positive x-axis for 2 seconds what is the final velocity?

- F is in the x-direction F = ma   so   a = F/m = 5 m/s²
- $v = v_0 + a\,t$ = 0 m/s + 2 x 5 m/s = 10 m/s (+x-direction)
- but mv = F t   [which is the area with respect to F(t) curve]

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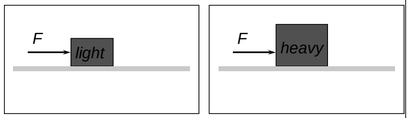
Lecture 12, *Exercise 2*
Force & Impulse

- Two boxes, one heavier than the other, are initially at rest on a horizontal frictionless surface. The same constant force *F* acts on each one for exactly 1 second.

Which box has the most momentum after the force acts ?



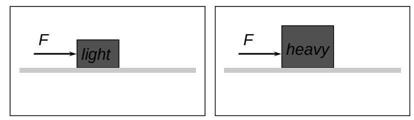
A. heavier
B. lighter
C. same
D. can't tell

Lecture 12, *Exercise 2*
Force & Impulse

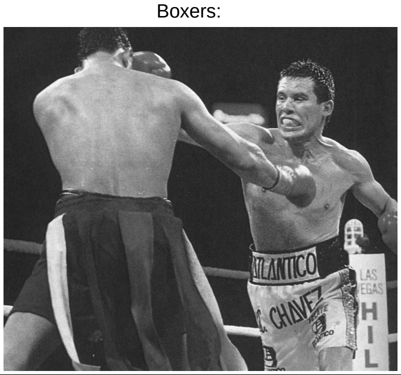
- Two boxes, one heavier than the other, are initially at rest on a horizontal frictionless surface. The same constant force *F* acts on each one for exactly 1 second.

Which box has the most momentum after the force acts ?

(A) heavier (B) lighter **(C) same**



Boxers:



Back of the envelope calculation

$$\vec{J} = \int^{t} \vec{F}\, dt = \vec{F}_{\text{avg}}\, \Delta t$$

(1) m_{arm}~ 7 kg (2) **v_{arm}**~7 m/s (3) **Impact time Δt** ~ 0.01 s

→ **Impulse** ***J* = Δp** ~ m_{arm} v_{arm} ~ 49 kg m/s

→ **F ~ J/Δt ~ 4900 N**

(1) m_{head} ~ 6 kg

→ **a_{head} = F / m_{head} ~ 800 m/s^2 ~ 80 g !**

- **Enough to cause unconsciousness ~ 40% of fatal blow**

Woodpeckers

During "collision" with a tree
a_{head} ~ 600 - 1500 g

How do they survive?

- Jaw muscles act as shock absorbers
- Straight head trajectory reduces damaging rotations (rotational motion is very problematic)

Chapter 10: Energy

- What do we mean by an isolated system ?
- What do we mean by a conservative force ?
- If a force acting on an object act for a period of time then we have an Impulse → change (transfer) of momentum
- What if we consider this force acting over a distance:
  Can we identify another useful quantity?

Energy

- ❖ $F_y = m\,a_y$ and let the force be constant
- ❖ $y(t) = y_0 + v_{y0}\,t + ½\,a_y\,t^2 \rightarrow \Delta y = y(t)-y_0 = v_{y0}\,t + ½\,a_y\,t^2$
- ❖ $v_y(t) = v_{y0} + a_y\,t \quad \rightarrow \quad t = (v_y - v_{y0}) / a_y = \Delta v_y / a_y$
- ❖ So $\Delta y = v_{y0}\,\Delta v_y / a_y + ½\,a_y\,(\Delta v_y/a_y)^2$

  $= (v_y\,v_{y0} - v_{y0}^2) / a_y + ½\,(v_y^2 - 2v_y\,v_{y0} + v_{y0}^2) / a_y$
- ❖ $2\,a_y\,\Delta y = (v_y^2 - v_{y0}^2)$
- ❖ Finally: $m a_y\,\Delta y = ½\,m\,(v_y^2 - v_{y0}^2)$

If falling: $-mg\,\Delta y = ½\,m\,(v_y^2 - v_{y0}^2)$

Energy

$$-mg\,\Delta y = ½\,m\,(v_y^2 - v_{y0}^2)$$

$$-mg\,(y_f - y_i) = ½\,m\,(v_{yf}^2 - v_{yi}^2)$$

A relationship between *y displacement* and y speed

Rearranging

$$½\,m\,v_{yi}^2 + mgy_i = ½\,m\,v_{yf}^2 + mgy_f$$

We associate mgy with the “gravitational potential energy”

Energy

- Notice that if we only consider gravity as the external force then
- then the x and z velocities remain constant
- To $½\,m\,v_{yi}^2 + mgy_i = ½\,m\,v_{yf}^2 + mgy_f$
- Add $½\,m\,v_{xi}^2 + ½\,m\,v_{zi}^2$ and $½\,m\,v_{xf}^2 + ½\,m\,v_{zf}^2$

$$½\,m\,v_i^2 + mgy_i = ½\,m\,v_f^2 + mgy_f$$

- where $v_i^2 = v_{xi}^2 + v_{yi}^2 + v_{zi}^2$

$½\,m\,v^2$ terms are referred to as kinetic energy

Energy

- If only “conservative” forces are present, the total energy (sum of potential, U, and kinetic energies, K) of a system is conserved.

$E_{mech} = K + U$

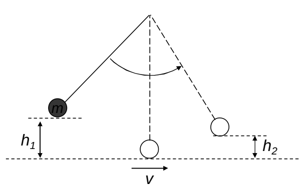
$E_{mech} = K + U =$ ***constant***

- K and U may change, but $E = K + U_{mech}$ remains a fixed value.

E_{mech} is called “mechanical energy”

Another example of a conservative system:
The simple pendulum.

- Suppose we release a mass m from rest a distance h_1 above its lowest possible point.
  - ❖ What is the maximum speed of the mass and where does this happen ?
  - ❖ To what height h_2 does it rise on the other side ?



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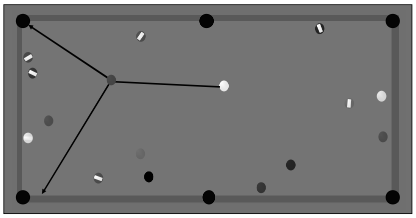
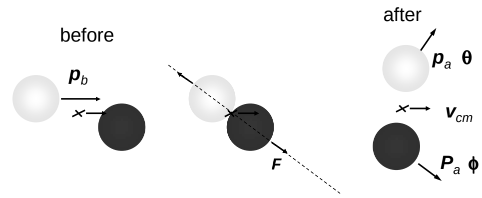
Example of 2-D Elastic collisions: Billiards

- If all we are given is the initial velocity of the cue ball, we don't have enough information to solve for the exact paths after the collision. But we can learn some useful things...



Billiards

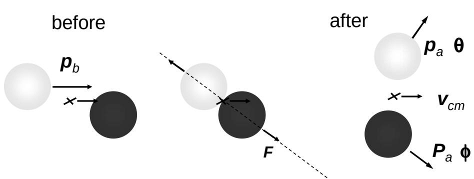
- Consider the case where one ball is initially at rest.



The final direction of the red ball will depend on where the balls hit.

Billiards: All that really matters is conservation of energy and momentum

- COE: $\frac{1}{2} m v_b^2 = \frac{1}{2} m v_a^2 + \frac{1}{2} m V_a^2$
- x-dir COM: $m v_b = m v_a \cos\theta + m V_b \cos\phi$
- y-dir COM: $0 = m v_a \sin\theta + m V_b \sin\phi$



- The final directions are separated by 90°: $\theta - \phi = 90°$