Physics 207 – Lecture 12

Agenda: Finish Chapter 10, start Chapter 11
- Chapter 10: Energy
  - Potential Energy (gravity, springs)
  - Kinetic energy
  - Mechanical Energy
  - Conservation of Energy
- Start Chapter 11, Work

Assignment:
- HW5 due tonight
- HW6 available today
- Monday, finish reading chapter 11

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**Chapter 10: Energy**

- Rearranging Newton’s Laws gives (Fd vs. \( \frac{1}{2} mv^2 \) relationship)
  \[-2mg (y_f - y_i) = m (v_{yi}^2 - v_{yi}^2)\]
- or \( \frac{1}{2} m v_{yi}^2 + mgy_i = \frac{1}{2} m v_{yi}^2 + mgy_f \)
- and adding \( \frac{1}{2} m v_{xi}^2 + \frac{1}{2} m v_{zi}^2 \) and \( \frac{1}{2} m v_{xf}^2 + \frac{1}{2} m v_{zf}^2 \)

\[
\frac{1}{2} m v_i^2 + mgy_i = \frac{1}{2} m v_f^2 + mgy_f
\]

where \( v_i^2 = v_{xi}^2 + v_{yi}^2 + v_{zi}^2 \)

\( \frac{1}{2} m v^2 \) terms are referred to as kinetic energy

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**Energy**

- If only “conservative” forces are present, the total energy (sum of potential, \( U \), and kinetic energies, \( K \)) of a system is conserved.

\[
K \equiv \frac{1}{2} mv^2 \\
U \equiv mgy
\]

\[
E_{\text{mech}} = K + U = \text{constant}
\]

- \( K_i + U_i = K_f + U_f \)

- \( K \) and \( U \) may change, but \( E = K + U_{\text{mech}} \) remains constant.

\( E_{\text{mech}} \) is called “mechanical energy”

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**Another example of a conservative system: The simple pendulum.**

- Suppose we release a mass \( m \) from rest a distance \( h_1 \) above its lowest possible point.
  - What is the maximum speed of the mass and where does this happen?
  - To what height \( h_2 \) does it rise on the other side?

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**Example: The simple pendulum.**

- What is the maximum speed of the mass and where does this happen?
  - \( E = K + U = \text{constant} \) and so \( K \) is maximum when \( U \) is a minimum.

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**Example: The simple pendulum.**

- What is the maximum speed of the mass and where does this happen?
  - \( E = K + U = \text{constant} \) and so \( K \) is maximum when \( U \) is a minimum.

\[
E = mgh, \text{ at top} \\
E = mgh_1 = \frac{1}{2} mv^2, \text{ at bottom of the swing}
\]
Example: The simple pendulum.
To what height \( h_2 \) does it rise on the other side?

\[
E = K + U = \text{constant}
\]

Again (when \( K = 0 \)) it will be at its highest point.

\[
E = mg h_1 = mg h_2
\]

\[ h_1 = h_2 \]

\[ y = 0 \]

Exercise 1

Conservation of Mechanical Energy

A block is shot up a frictionless 40° slope with initial velocity \( v \).

It reaches a height \( h \) before sliding back down. The same block is shot with the same velocity up a frictionless 20° slope.

On this slope, the block reaches height

\[ \begin{align*}
& A. \ 2h \\
& B. \ h \\
& C. \ h/2 \\
& D. \ \text{Greater than} \ h, \ \text{but we can't predict an exact value.} \\
& E. \ \text{Less than} \ h, \ \text{but we can't predict an exact value.}
\end{align*} \]

The Loop

- To complete the loop the loop, how high do we have to let the release the car?
- Condition for completing the loop: Circular motion at the top of the loop \( \left( \frac{v^2}{R} \right) \)
- Use fact that \( E = U + K = \text{constant} \)

Use \( E = K + U = \text{constant} \)

\[
mgh + 0 = mg(2R) + \frac{1}{2}mv^2
\]

\[
h = \frac{5}{2} R
\]

(A) \( 2R \)  (B) \( 3R \)  (C) \( 5/2 \) \( R \)  (D) \( 2^{1/2} R \)

The Loop-the-Loop ... again

What speed will the skateboarder reach at bottom of the hill if there is no friction and the skateboarder starts at rest?

Assume we can treat the skateboarder as “point”

Zero of gravitational potential energy is at bottom of the hill

\[ E_{\text{before}} = E_{\text{after}} \]

\[ 0 = mgR + \frac{1}{2}mv^2 + 0 \]

\[ 2gR = v^2 \rightarrow v = (2gR)^{1/2} \]

\[ v = (2 \times 10 \times 5)^{1/2} = 10 \text{ m/s} \]
Potential Energy, Energy Transfer and Path

A ball of mass m, initially at rest, is released and follows three different paths. All surfaces are frictionless.
1. The ball is dropped
2. The ball slides down a straight incline
3. The ball slides down a curved incline

After traveling a vertical distance h, how do the three speeds compare?

(A) 1 > 2 > 3  (B) 3 > 2 > 1  (C) 3 = 2 = 1  (D) Can’t tell

Elastic vs. Inelastic Collisions

A collision is said to be elastic when energy as well as momentum is conserved before and after the collision.

\[ K_{\text{before}} = K_{\text{after}} \]

- Carts colliding with a perfect spring, billiard balls, etc.

A collision is said to be inelastic when energy is not conserved before and after the collision, but momentum is conserved.

\[ K_{\text{before}} \neq K_{\text{after}} \]

- Car crashes, collisions where objects stick together, etc.

Inelastic collision in 1-D: Example 1

A block of mass M is initially at rest on a frictionless horizontal surface. A bullet of mass m is fired at the block with a muzzle velocity (speed) v. The bullet lodges in the block, and the block ends up with a speed V.

- What is the initial energy of the system?
- What is the final energy of the system?
- Is energy conserved?

\[
\begin{align*}
K_{\text{before}} &= \frac{1}{2}mv^2 \\
K_{\text{after}} &= \frac{1}{2}mV^2 + \frac{1}{2}MV^2 \\
\end{align*}
\]

\[
\text{Examine } E_{\text{before}} = E_{\text{after}}
\]

\[
\frac{1}{2}mv^2 = \frac{1}{2}mV^2 + \frac{1}{2}MV^2
\]

No!
Example – Fully Elastic Collision

- Suppose I have 2 identical bumper cars.
- One is motionless and the other is approaching it with velocity $v_1$. If they collide elastically, what is the final velocity of each car?

Identical means $m_1 = m_2 = m$

Initially $v_{Green} = v_1$ and $v_{Red} = 0$

- COM: $mv_1 + 0 = mv_{1f} + mv_{2f}$
  - $v_1 = v_{1f} + v_{2f}$
- COE: $\frac{1}{2} mv_1^2 = \frac{1}{2} mv_{1f}^2 + \frac{1}{2} mv_{2f}^2$
  - $v_1^2 = v_{1f}^2 + v_{2f}^2$

$v_1^2 = (v_{1f} + v_{2f})^2 = v_{1f}^2 + 2v_{1f}v_{2f} + v_{2f}^2 
\rightarrow 2v_{1f}v_{2f} = 0$

- Soln 1: $v_{1f} = 0$ and $v_{2f} = v_1$
- Soln 2: $v_{2f} = 0$ and $v_{1f} = v_1$

Lecture 13, Exercise for home
Elastic Collisions

- I have a line of 3 bumper cars all touching. A fourth car smashes into the others from behind. Is it possible to satisfy both conservation of energy and momentum if two cars are moving after the collision?

All masses are identical, elastic collision.

(A) Yes  (B) No  (C) Only in one special case

Variable force devices: Hooke’s Law Springs

- Springs are everywhere, (probe microscopes, DNA, an effective interaction between atoms)
- In this spring, the magnitude of the force increases as the spring is further compressed (a displacement).
- Hooke’s Law,
  \[ F_s = -k \Delta x \]
  $\Delta x$ is the amount the spring is stretched or compressed from it resting position.

Hooke’s Law Spring

- For a spring we know that $F_s = -kx$.

What are the units for the constant $k$?

(A) $\text{kg m}^2 / \text{s}^2$  (B) $\text{kg m} / \text{s}^2$  (C) $\text{kg} / \text{m}$  (D) $\text{kg} / \text{s}^2$

$F$ is in kg m/s$^2$ and dividing by m gives kg/s$^2$ or N/m
Lecture 13, Exercise 2
Hooke's Law

What is the spring constant "k"?

(A) 50 N/m  (B) 100 N/m  (C) 400 N/m  (D) 500 N/m

ΣF = 0 = F_s - mg = k Δx - mg
Use k = mg/Δx = 5 N / 0.01 m

F = mg

F-x relation for a foot arch:

<table>
<thead>
<tr>
<th>Force (N)</th>
<th>Displacement (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>20</td>
<td>8</td>
</tr>
</tbody>
</table>

F-x relation for a single DNA molecule

<table>
<thead>
<tr>
<th>Force (pN)</th>
<th>Extension (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td>5</td>
<td>600</td>
</tr>
<tr>
<td>10</td>
<td>1200</td>
</tr>
<tr>
<td>20</td>
<td>1400</td>
</tr>
</tbody>
</table>

Measurement technique: optical tweezers

Lecture 13, Oct. 15

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  - Mechanical Energy
  - Conservation of Energy
  - Chapter 11, Work

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