Physics 207, Lecture 13, Oct. 15

Agenda: Finish Chapter 10, start Chapter 11

- Chapter 10: Energy
  - ❖ Potential Energy (gravity, springs)
  - ❖ Kinetic energy
  - ❖ Mechanical Energy
  - ❖ Conservation of Energy
  - ❖ Start Chapter 11, Work

Assignment:

- HW5 due tonight
- HW6 available today
- Monday, finish reading chapter 11

Chapter 10: Energy

- Rearranging Newton's Laws gives (Fd *vs.* ½ mv^2 relationship)

$$-2mg\,(y_f - y_i) = m\,(v_{yf}^2 - v_{yi}^2)$$

- or → $\frac{1}{2} m v_{yi}^2 + mgy_i = \frac{1}{2} m v_{yf}^2 + mgy_f$
- and adding $\frac{1}{2} m v_{xi}^2 + \frac{1}{2} m v_{zi}^2$ and $\frac{1}{2} m v_{xf}^2 + \frac{1}{2} m v_{zf}^2$

$$\boxed{\tfrac{1}{2} m v_i^2 + mgy_i = \tfrac{1}{2} m v_f^2 + mgy_f}$$

- where $v_i^2 = v_{xi}^2 + v_{yi}^2 + v_{zi}^2$

$\frac{1}{2} m v^2$ terms are referred to as kinetic energy

Energy

- If only "conservative" forces are present, the total energy (sum of potential, U, and kinetic energies, K) of a system is <u>conserved</u>.

$$K \equiv \tfrac{1}{2} mv^2$$
$$U \equiv mgy$$

$$\boxed{E_{mech} = K + U = \textbf{constant}}$$

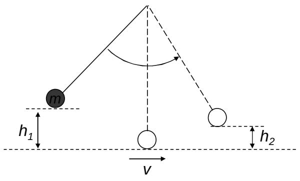
$$K_i + U_i = K_f + U_f$$

- *K* and *U* may change, but $E = K + U_{mech}$ remains constant.

E$_{mech}$ is called "mechanical energy"

Another example of a conservative system: The simple pendulum.

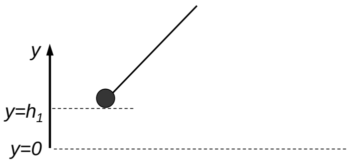
- Suppose we release a mass *m* from rest a distance h_1 above its lowest possible point.
  - ❖ What is the maximum speed of the mass and where does this happen ?
  - ❖ To what height h_2 does it rise on the other side ?



Example: The simple pendulum.

❖ What is the maximum speed of the mass and where does this happen ?

E = K + U = constant and so K is maximum when U is a minimum.



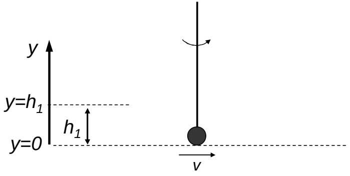
Example: The simple pendulum.

❖ What is the maximum speed of the mass and where does this happen ?

E = K + U = constant and so K is maximum when U is a minimum

E = mgh$_1$ at top

E = mgh$_1$ = ½ mv^2 at bottom of the swing

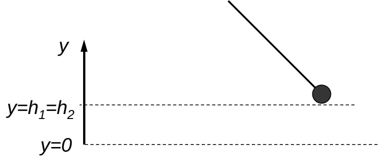


Example: The simple pendulum.

To what height *h2* does it rise on the other side?

E = K + U = constant and so when U is maximum again (when K = 0) it will be at its highest point.

E = $mgh_1 = mgh_2$ or $h_1 = h_2$



Lecture 13, *Exercise 1*
Conservation of Mechanical Energy

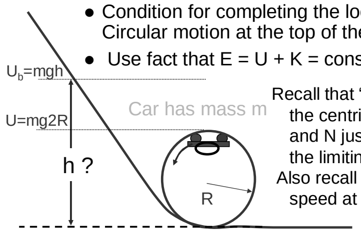
A block is shot up a frictionless 40° slope with initial velocity ***v***. It reaches a height ***h*** before sliding back down. The same block is shot with the <u>same</u> velocity up a frictionless 20° slope.

On this slope, the block reaches height

- ✗ A. 2h
- ✗ B. h
- ✗ C. h/2
- ✗ D. Greater than *h*, but we can't predict an exact value.
- ✗ E. Less than *h*, but we can't predict an exact value.

Lecture 13, *Example*
The Loop-the-Loop … again

- To complete the loop the loop, how high do we have to let the release the car?
- Condition for completing the loop the loop: Circular motion at the top of the loop ($a_c = v^2 / R$)
- Use fact that E = U + K = constant !



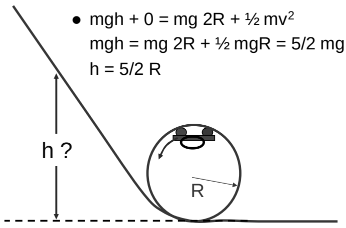
Recall that "g" is the source of the centripetal acceleration and N just goes to zero is the limiting case.
Also recall the minimum speed at the top is $\boxed{v = \sqrt{gR}}$

(A) 2R (B) 3R (C) 5/2 R (D) $2^{3/2}$ R

Lecture 13, *Example*
The Loop-the-Loop … again

- Use E = K + U = constant
- mgh + 0 = mg 2R + ½ mv^2
  mgh = mg 2R + ½ mgR = 5/2 mgR
  h = 5/2 R

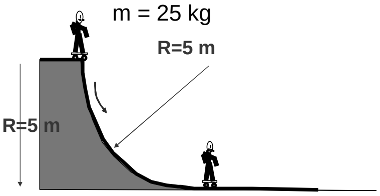
$\boxed{v = \sqrt{gR}}$



(A) 2R (B) 3R **5/2 R** (D) $2^{3/2}$ R

Lecture 13, *Example*
Skateboard

- What speed will the skateboarder reach at bottom of the hill if there is no friction and the skeateboarder starts at rest?
- Assume we can treat the skateboarder as "point"
- Zero of gravitational potential energy is at bottom of the hill



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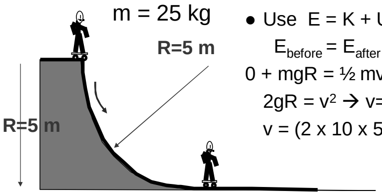


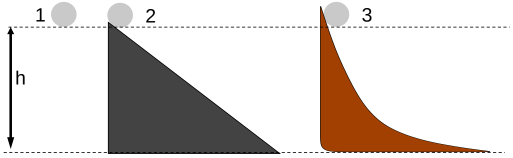
- Use E = K + U = constant
  $E_{before} = E_{after}$
  0 + mgR = ½ mv^2 + 0
  $2gR = v^2 \rightarrow v = (2gR)^{1/2}$
  $v = (2 \times 10 \times 5)^{1/2} = 10$ m/s

Potential Energy, Energy Transfer and Path

- A ball of mass m, initially at rest, is released and follows three difference paths. All surfaces are frictionless

1. The ball is dropped
2. The ball slides down a straight incline
3. The ball slides down a curved incline

After traveling a vertical distance h, how do the three speeds compare?



(A) 1 > 2 > 3 (B) 3 > 2 > 1 (C) 3 = 2 = 1 (D) Can't tell

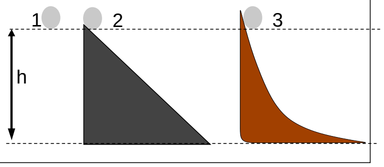
Lecture 13, *Exercise 2*
Potential Energy, Energy Transfer and Path

- A ball of mass m, initially at rest, is released and follows three difference paths. All surfaces are frictionless

1. The ball is dropped
2. The ball slides down a straight incline
3. The ball slides down a curved incline

After traveling a vertical distance h, how do the speeds compare?

✗ A. 1 > 2 > 3
✗ B. 3 > 2 > 1
✗ C. 3 = 2 = 1
✗ D. Can't tell



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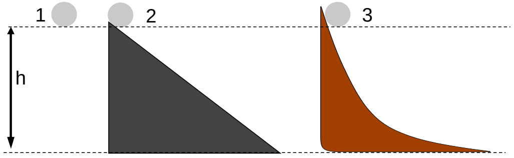


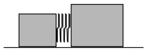
(A) 1 > 2 > 3 (B) 3 > 2 > 1 **(C) 3 = 2 = 1** (D) Can't tell

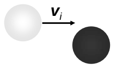
Elastic vs. Inelastic Collisions

- A collision is said to be *elastic* when energy as well as momentum is conserved before and after the collision.

$$K_{before} = K_{after}$$

  - ❖ Carts colliding with a perfect spring, billiard balls, etc.

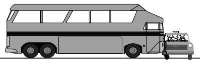




- A collision is said to be *inelastic* when energy is not conserved before and after the collision, but momentum is conserved.

$$K_{before} \neq K_{after}$$

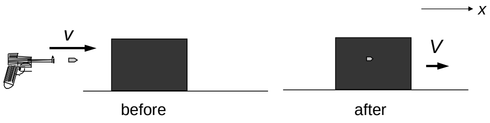
  - ❖ Car crashes, collisions where objects stick together, etc.



Inelastic collision in 1-D: Example 1

- A block of mass *M* is initially at rest on a frictionless horizontal surface. A bullet of mass *m* is fired at the block with a muzzle velocity (speed) *v*. The bullet lodges in the block, and the block ends up with a speed *V.*

  - ❖ What is the initial energy of the system ?
  - ❖ What is the final energy of the system ?
  - ❖ Is energy conserved?

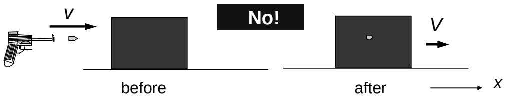


Inelastic collision in 1-D: Example 1

What is the momentum of the bullet with speed *v* ? $m\vec{v}$

- ❖ What is the initial energy of the system ? $\frac{1}{2}m\vec{v}\bullet\vec{v} = \frac{1}{2}mv^2$
- ❖ What is the final energy of the system ? $\frac{1}{2}(m+M)V^2$
- ❖ Is momentum conserved (yes)? $mv + M\,0 = (m+M)V$
- ❖ Is energy conserved? Examine E_{before}-E_{after}

$$\frac{1}{2}mv^2 - \frac{1}{2}[(m+M)V]V = \frac{1}{2}mv^2 - \frac{1}{2}(mv)\frac{m}{m+M}v = \frac{1}{2}mv^2\left(1-\frac{m}{m+M}\right)$$

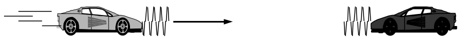


Example – Fully Elastic Collision

- Suppose I have 2 identical bumper cars.
- One is motionless and the other is approaching it with velocity v_1. If they collide elastically, what is the final velocity of each car ?

Identical means $m_1 = m_2 = m$

Initially $v_{Green} = v_1$ and $v_{Red} = 0$



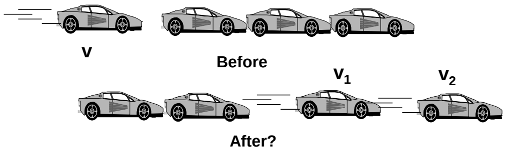
- COM → $mv_1 + 0 = mv_{1f} + mv_{2f}$ → $v_1 = v_{1f} + v_{2f}$
- COE → $\frac{1}{2} mv_1^2 = \frac{1}{2} mv_{1f}^2 + \frac{1}{2} mv_{2f}^2$ → $v_1^2 = v_{1f}^2 + v_{2f}^2$
- $v_1^2 = (v_{1f} + v_{2f})^2 = v_{1f}^2 + 2v_{1f}v_{2f} + v_{2f}^2$ → $2\, v_{1f}\, v_{2f} = 0$
- Soln 1: $v_{1f} = 0$ and $v_{2f} = v_1$ Soln 2: $v_{2f} = 0$ and $v_{1f} = v_1$

Lecture 13, *Exercise for home*
Elastic Collisions

- I have a line of 3 bumper cars all touching. A fourth car smashes into the others from behind. Is it possible to satisfy both conservation of energy and momentum if two cars are moving after the collision?

All masses are identical, elastic collision.

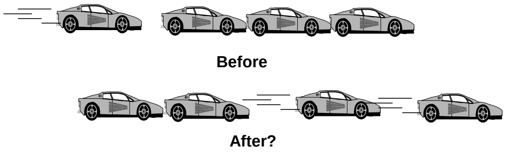
(A) Yes (B) No (C) Only in one special case



Lecture 13, *Exercise for home*
Elastic Collisions

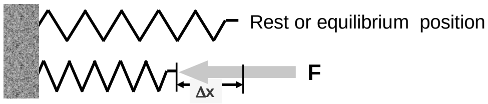
- COM → $mv = mv_1 + mv_2$ so $v = v_1 + v_2$
- COE → $\frac{1}{2} mv^2 = \frac{1}{2} mv_1^2 + \frac{1}{2} mv_2^2$
- $v^2 = (v_1 + v_2)^2 = v_1^2 + v_2^2$ → $v_1 v_2 = 0$

(A) Yes **(B) No** (C) Only in one special case



Variable force devices: Hooke's Law Springs

- Springs are everywhere, (probe microscopes, DNA, an effective interaction between atoms)



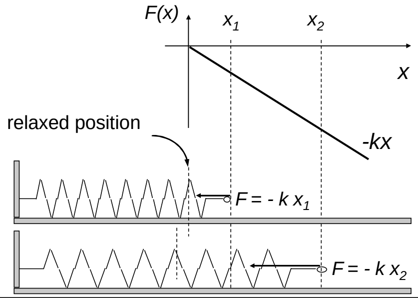
- In this spring, the magnitude of the force increases as the spring is further compressed (a displacement).
- Hooke's Law,

$$F_s = -k\, \Delta x$$

Δx is the amount the spring is stretched or compressed from it resting position.

Hooke's Law Spring

- For a spring we know that $F_x = -kx$.



Lecture 13, *Example*
Hooke's Law

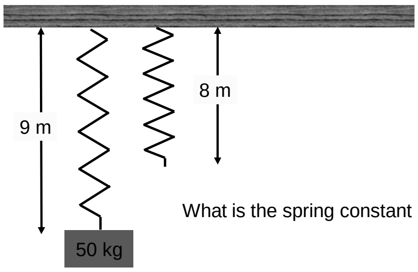
- Remembering Hooke's Law, $F_x = -k\, \Delta x$

What are the units for the constant k ?

(A) $\frac{\text{kg m}^2}{\text{s}^2}$ (B) $\frac{\text{kg m}}{\text{s}^2}$ (C) $\frac{\text{kg}}{\text{s}^2}$ (D) $\frac{\text{kg}^2\ \text{m}}{\text{s}^2}$

F is in kg m/s^2 and dividing by m gives kg/s^2 or N/m

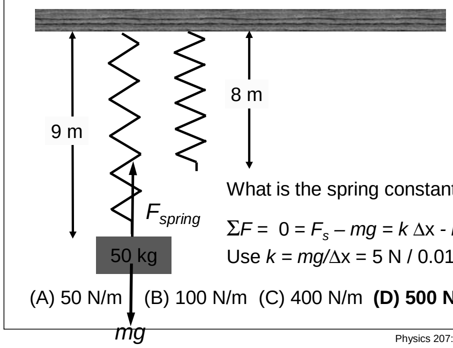
Lecture 13, *Exercise 2*
Hooke's Law



What is the spring constant "k" ?

(A) 50 N/m (B) 100 N/m (C) 400 N/m (D) 500 N/m

Lecture 10, *Exercise 2*
Hooke's Law



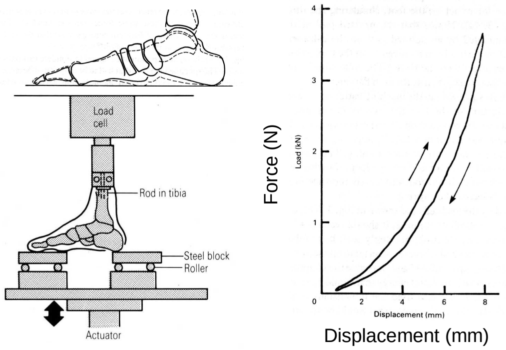
What is the spring constant "k" ?

$\Sigma F = 0 = F_s - mg = k\,\Delta x - mg$

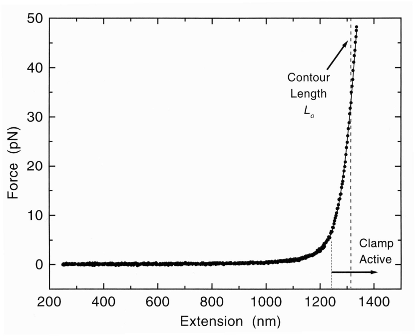
Use $k = mg/\Delta x$ = 5 N / 0.01 m

(A) 50 N/m (B) 100 N/m (C) 400 N/m **(D) 500 N/m**

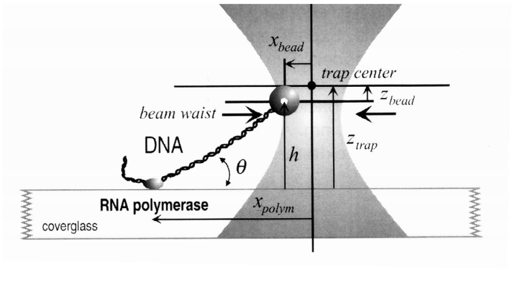
F-x relation for a foot arch:



F-x relation for a single DNA molecule



Measurement technique: optical tweezers



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