Physics 207, Lecture 14, Oct. 22

Agenda: Finish Chapter 10, Chapter 11

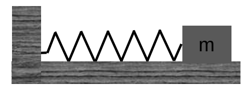
- Chapter 10: Energy
  - ❖ Energy diagrams
  - ❖ Springs
- Chapter 11: Work
  - ❖ Work and Net Work
  - ❖ Work and Kinetic Energy
  - ❖ Work and Potential Energy
  - ❖ Conservative and Non-conservative forces

Assignment:

- HW6 due Wednesday
- HW7 available soon
- Wednesday, Read Chapter 11

Force vs. Energy for a Hooke's Law spring

- F = - k (x – $x_{equilibrium}$)
- F = ma = m dv/dt
  = m (dv/dx dx/dt)
  = m dv/dx v
  = mv dv/dx
- So - k (x – $x_{equilibrium}$) dx = mv dv
- Let u = x – $x_{eq.}$ →



$$\int_{x_i}^{x_f} -ku\,du = \int_{v_i}^{v_f} mv\,dv$$

$$-\tfrac{1}{2}ku^2 \Big|_{x_i}^{x_f} = \tfrac{1}{2}mv^2 \Big|_{v_i}^{v_f}$$

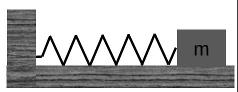
$$-\tfrac{1}{2}kx_f^2 + \tfrac{1}{2}kx_i^2 = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$\boxed{\tfrac{1}{2}kx_i^2 + \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}kx_f^2 + \tfrac{1}{2}mv_f^2}$$

Energy for a Hooke's Law spring

$$\boxed{\tfrac{1}{2}kx_i^2 + \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}kx_f^2 + \tfrac{1}{2}mv_f^2}$$

- Associate ½ kx^2 with the "potential energy" of the spring



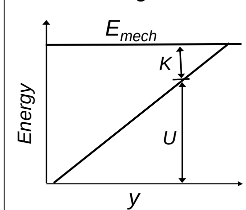
$$\boxed{U_{si} + K_i = U_{sf} + K_f}$$

- Perfect Hooke's Law springs are "conservative" so the mechanical energy is constant

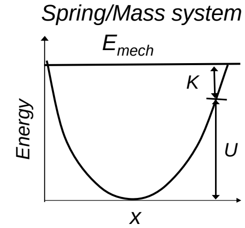
Energy diagrams

- In general:

Ball falling

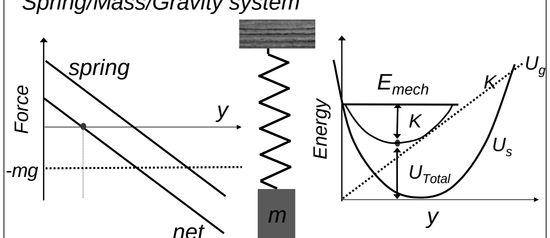


Spring/Mass system



Energy diagrams

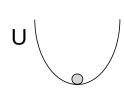
Spring/Mass/Gravity system



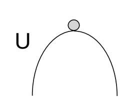
Notice: mass has maximum kinetic energy when the net force is zero (acceleration changes sign)

Equilibrium

- Example
  - ❖ Spring: F_x = 0 => dU / dx = 0 for x=0
  The spring is in equilibrium position
- In general: dU / dx = 0 → for ANY function establishes equilibrium



stable equilibrium



unstable equilibrium

Comment on Energy Conservation

- We have seen that the total kinetic energy of a system undergoing an inelastic collision is not conserved.
  - ❖ Mechanical energy is lost:
    - ➢Heat (friction)
    - ➢Bending of metal and deformation
- Kinetic energy is not conserved by these non-conservative forces occurring during the collision !
- Momentum along a specific direction **is** conserved when there are no external forces acting in this direction.
  - ❖ In general, easier to satisfy conservation of momentum than energy conservation.

Chapter 11, Work

- Potential Energy (U)
- Kinetic Energy (K)
- Thermal Energy (E_{th} , new)

where $E_{sys} = E_{mech} + E_{th} = K + U + E_{th}$

- Any process which changes the potential or kinetic energy of a system is said to have done work W on that system

$$\Delta E_{sys} = W$$

W can be positive or negative depending on the direction of energy transfer

- Net work reflects changes in the kinetic energy

$$W_{net} = \Delta K$$

Examples of "Net" Work (W_{net})

$$\Delta K = W_{net}$$

- Pushing a box on a smooth floor with a constant force

Examples of No "Net" Work

$$\Delta K = W_{net}$$

- Pushing a box on a rough floor at constant speed
- Driving at constant speed in a horizontal circle
- Holding a book at constant height

This last statement reflects what we call the "system"

(Dropping a book is more complicated because it involves changes in U and K)

Changes in K with a constant F

- In one-D, from F = ma = m dv/dt = m dv/dx dx/dt to net work.

$$\int_{x_i}^{x_f} F_x \, dx = \int_{v_{xi}}^{v_{xf}} m v_x \, dv_x$$

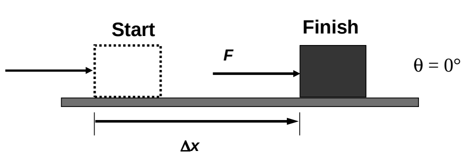
- F is constant

$$F_x \int_{x_i}^{x_f} dx = \int_{v_{xi}}^{v_{xf}} m v_x \, dv_x$$

$$F_x (x_f - x_i) = F_x \Delta x = \tfrac{1}{2} m v_{xf}^2 - \tfrac{1}{2} m v_{xi}^2 = \Delta K$$

Net Work: 1-D Example (constant force)

- A force ***F*** = *10 N* pushes a box across a **frictionless** floor for a distance Δx = *5 m*.



- (Net) Work is F **Δx** = 10 x 5 N m = 50 J
- 1 Nm is defined to be 1 Joule and this is a unit of energy
- Work reflects energy transfer

Units:

Force x Distance = Work

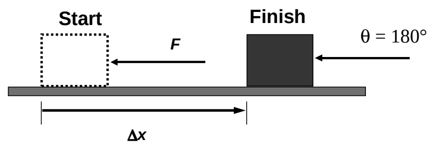
Newton x Meter = Joule

$[M][L] / [T]^2$ $[L]$ $[M][L]^2 / [T]^2$

mks	cgs	Other
N-m (Joule)	Dyne-cm (erg) = 10^{-7} J	BTU = 1054 J calorie= 4.184 J foot-lb = 1.356 J eV = 1.6×10^{-19} J

Net Work: 1-D 2nd Example (constant force)

- A force $\boldsymbol{F}$ = *10 N* is opposite the motion of a box across a **frictionless** floor for a distance $\Delta \boldsymbol{x}$ = *5 m*.



- (Net) Work is F $\Delta \boldsymbol{x}$ = -10 x 5 N m = -50 J
- **Work reflects energy transfer**

Work in 3D….

- x, y and z with constant F:

$$F_x(x_f - x_i) = F_x \Delta x = \tfrac{1}{2} m v_{xf}^2 - \tfrac{1}{2} m v_{xi}^2$$

$$F_y(y_f - y_i) = F_y \Delta y = \tfrac{1}{2} m v_{yf}^2 - \tfrac{1}{2} m v_{yi}^2$$

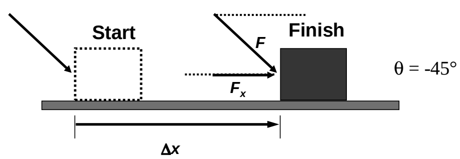
$$F_z(z_f - z_i) = F_z \Delta z = \tfrac{1}{2} m v_{zf}^2 - \tfrac{1}{2} m v_{zi}^2$$

$$F_x \Delta x + F_y \Delta y + F_z \Delta z + = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2 = \Delta K$$

with $v^2 = v_x^2 + v_y^2 + v_z^2$

Work: "2-D" Example (constant force)

- A force $\boldsymbol{F}$ = *10 N* pushes a box across a **frictionless** floor for a distance $\Delta \boldsymbol{x}$ = *5 m* and $\Delta \boldsymbol{y}$ = *0 m*



- (Net) Work is F_x $\Delta \boldsymbol{x}$ = F cos(-45°) = 50 x 0.71 Nm = 35 J
- Work reflects energy transfer

Scalar Product (or Dot Product)

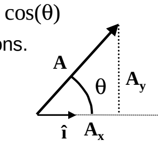
$\mathbf{A} \cdot \mathbf{B} \equiv |\mathbf{A}|\,|\mathbf{B}| \cos(\theta)$

- Useful for performing projections.

$\mathbf{A} \bullet \hat{\imath} = A_x$

$\hat{\imath} \bullet \hat{\imath} = 1$

$\hat{\imath} \bullet \mathbf{j} = 0$



- Calculation can be made in terms of components.

$\mathbf{A} \bullet \mathbf{B} = (A_x)(B_x) + (A_y)(B_y) + (A_z)(B_z)$

Calculation also in terms of magnitudes and relative angles.

$\mathbf{A} \bullet \mathbf{B} \equiv |A|\,|B| \cos\theta$

You choose the way that works best for you!

Scalar Product (or Dot Product)

Compare:

$\mathbf{A} \bullet \mathbf{B} = (A_x)(B_x) + (A_y)(B_y) + (A_z)(B_z)$

with

$F_x \Delta x + F_y \Delta y + F_z \Delta z = \Delta K$

Notice:

$\mathbf{F} \bullet \Delta\mathbf{r} = (F_x)(\Delta x) + (F_y)(\Delta z) + (F_z)(\Delta z)$

So here

$\mathbf{F} \bullet \Delta\mathbf{r} = \Delta K = W_{net}$

More generally a Force acting over a Distance does work

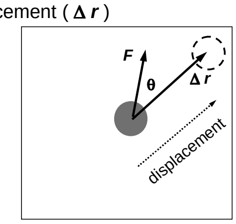
Definition of Work, The basics

Ingredients: Force ($\boldsymbol{F}$), displacement ($\Delta \boldsymbol{r}$)

Work, *W*, of a constant force $\boldsymbol{F}$ acting through a displacement $\Delta \boldsymbol{r}$ is:

$W = \boldsymbol{F} \cdot \Delta \boldsymbol{r}$ (Work is a scalar)

"Scalar or Dot Product"

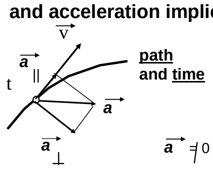


Work tells you something about <u>what happened on the path</u>!

Did something do work on you? Did you do work on something?

Simplest case (**no** frictional forces and **no** non-contact forces) Did your **speed** change?

Remember that a path evolves with time and acceleration implies a force acting on an object



$\vec{a} = \vec{a}_{\parallel} + \vec{a}_{\perp}$

$\vec{a} = \vec{a}_{tang} + \vec{a}_{radial}$

$\vec{F} = \vec{F}_{tang} + \vec{F}_{radial}$

$\vec{a} \neq 0$

Two possible options:

Change in the magnitude of $\vec{v}$ $\quad \vec{a}_{\parallel} \neq 0$

Change in the direction of $\vec{v}$ $\quad \vec{a}_{\perp} \neq 0$

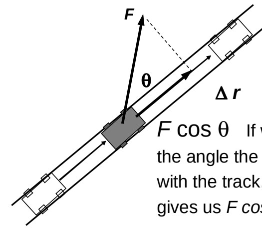
- A tangetial force is the important one for work!
  - ❖ How long (time dependence) gives the kinematics
  - ❖ The distance over which this $force_{Tang}$ is applied: Work

Definition of Work...

- Only the component of ***F*** along the path (i.e. "displacement") does work.

<u>The vector dot product does that automatically.</u>

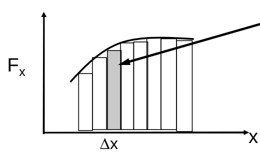
- ❖ Example: Train on a track.



F cos θ If we know the angle the force makes with the track, the dot product gives us *F cos θ* and Δr

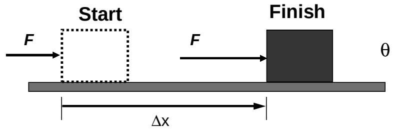
Work and Varying Forces (1D)

- Consider a varying force F(x)



Area = $F_x \Delta x$
F is **increasing**
Here $W = \mathbf{F} \cdot \Delta \mathbf{r}$
becomes dW = F dx

$$W = \int_{x_i}^{x_f} F(x)\,dx$$

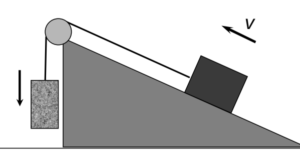


Work is a scalar, the rub is that there is no time/position info on hand

Lecture 14, *Exercise 1*
Work in the presence of friction and non-contact forces

- A box is pulled up a rough (μ > 0) incline by a rope-pulley-weight arrangement as shown below.
  - ❖ How many forces are doing work on the box ?
  - ❖ Of these which are positive and which are negative?
  - ❖ Use a Force Body Diagram
  - ❖ Compare force and path

A. 2
B. 3
C. 4



Work Kinetic-Energy Theorem:

{***Net Work*** done on object}

=

{***change*** in ***kinetic energy*** of object}

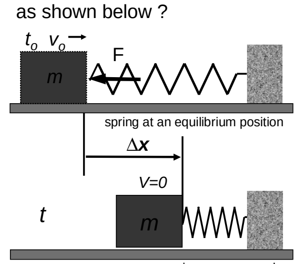
$$W_{net} = \Delta K$$

$$= K_2 - K_1 \qquad \text{(final – initial)}$$

$$= \frac{1}{2} m v_2^{\,2} - \frac{1}{2} m v_1^{\,2}$$

Example: Work Kinetic-Energy Theorem

- How much will the spring compress (i.e. $\Delta \boldsymbol{x}$) to bring the object to a stop (i.e., *v = 0*) if the object is moving initially at a constant velocity (v_o) on frictionless surface as shown below ?



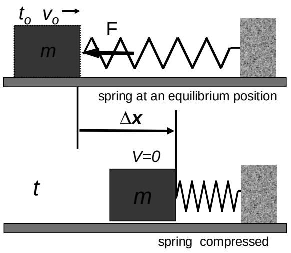
Notice that the spring force is opposite to the displacement.

For the mass m, work is negative

For the spring, *work is positive*

Example: Work Kinetic-Energy Theorem

- How much will the spring compress (i.e. $\Delta x = x_f - x_i$) to bring the object to a stop (i.e., $v = 0$) if the object is moving initially at a constant velocity (v_o) on frictionless surface as shown below ?



$$W_{box} = \int_{x_i}^{x_f} F(x)\, dx$$

$$W_{box} = \int_{x_i}^{x_f} -kx\, dx$$

$$W_{box} = -\tfrac{1}{2} kx^2 \Big|_{x_i}^{x_f}$$

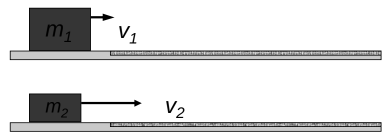
$$W_{box} = -\tfrac{1}{2} k\, \Delta x^2 = \Delta K$$

$$-\tfrac{1}{2} k\, \Delta x^2 = \tfrac{1}{2} m 0^2 - \tfrac{1}{2} m v_0^2$$

Lecture 14, *Example*
Work & Friction

- Two blocks having mass m_1 and m_2 where $m_1 > m_2$. They are sliding on a frictionless floor and have the same kinetic energy when they encounter a long rough stretch (i.e. $\mu > 0$) which slows them down to a stop.
- Which one will go farther before stopping?
- **Hint:** How much work does friction do on each block ?

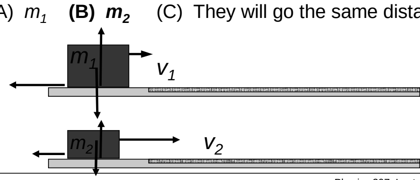
(A) m_1 (B) m_2 (C) They will go the same distance



Lecture 14, *Example*
Work & Friction

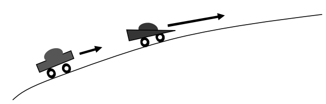
- $W = F\, d = -\mu N\, d = -\mu\, mg\, d = \Delta K = 0 - \tfrac{1}{2} mv^2$
- $-\mu\, m_1 g\, d_1 = -\mu\, m_2 g\, d_2 \rightarrow d_1 / d_2 = m_2 / m_1$

(A) m_1 **(B) m_2** (C) They will go the same distance



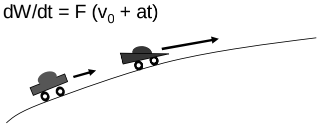
Work & Power:

- Two cars go up a hill, a Corvette and a ordinary Chevy Malibu. Both cars have the same mass.
- Assuming identical friction, both engines do the same amount of work to get up the hill.
- Are the cars essentially the same ?
- **NO.** The Corvette can get up the hill quicker
- It has a more **powerful** engine.



Work & Power:

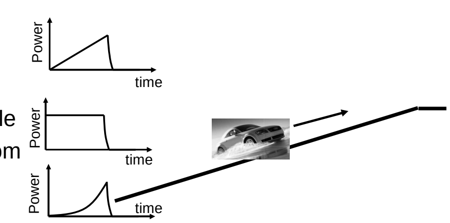
- Power is the rate at which work is done.
- Average Power is, $\bar{P} = \frac{W}{\Delta t}$
- Instantaneous Power is, $P = \frac{dW}{dt}$
- If force constant, $W = F\, \Delta x = F\, (v_0\, t + \tfrac{1}{2} at^2)$ and $P = dW/dt = F\, (v_0 + at)$



Lecture 14, *Exercise 2*
Work & Power

- Starting from rest, a car drives up a hill at constant acceleration and then suddenly stops at the top. The instantaneous power delivered by the engine during this drive looks like which of the following,

A. Top
B. Middle
C. Bottom



Work & Power:

- Power is the rate at which work is done.

Average Power:	Instantaneous Power:	Units (SI) are Watts (W):
$\bar{P} = \frac{W}{\Delta t}$	$P = \frac{dW}{dt}$	1 W = 1 J / 1s

Example 1 :

- A person of mass 80.0 kg walks up to 3rd floor (12.0m). If he/she climbs in 20.0 sec what is the average power used.
- P_{avg} = F h / t = mgh / t = 80.0 x 9.80 x 12.0 / 20.0 W
- P = 470. W

Lecture 14, Oct. 22

- On Wednesday, Finish Chapter 11 (Potential Energy and Work), Start Chapter 13

Assignment:

- HW6 due Wednesday
- HW7 available soon
- Wednesday, read chapter 13

Non-conservative Forces :

- If the work done does not depend on the path taken, the force involved is said to be conservative.
- If the work done does depend on the path taken, the force involved is said to be non-conservative.
- An example of a non-conservative force is friction:
- Pushing a box across the floor, the amount of work that is done by friction depends on the path taken.
  - Work done is proportional to the length of the path !