Physics 207 – Lecture 15

**Lecture 15, Exercise 1**
Work in the presence of friction and non-contact forces
- A box is pulled up a rough (μ > 0) incline by a rope-pulley-weight arrangement as shown below.
  - How many forces are doing work on the box?
  - Of these which are positive and which are negative?
  - Use a Force Body Diagram
  - Compare force and path

**Example: Work Kinetic-Energy Theorem**
- How much will the spring compress (i.e., Δx) to bring the object to a stop (i.e., v = 0) if the object is moving initially at a constant velocity (v₀) on frictionless surface as shown below?

**Example: Work and Varying Forces (1D)**
- Consider a varying force F(x)
  - Area = F_x * Δx
  - F is increasing
  - Here W = \( \int F(x) \, dx \)
  - Work is kinetic
  - \( \text{Start} \)
  - \( \text{Finish} \)
  - \( \theta = 0^\circ \)
  - Work is a scalar, the rub is that there is no time/position info on hand

**Example: Work Kinetic-Energy Theorem**
- How much will the spring compress (i.e., Δx = x_f - x_i) to bring the object to a stop (i.e., v = 0) if the object is moving initially at a constant velocity (v₀) on frictionless surface as shown below?

- Notice that the spring force is opposite to the displacement.
- For the mass m, work is negative
- For the spring, work is positive

- \( W_{\text{box}} = \frac{1}{2} k \Delta x^2 \)
- \( W_{\text{box}} = \frac{1}{2} k \Delta x^2 \quad (\text{finite}) \)
- \( W_{\text{box}} = \frac{1}{2} k \Delta x^2 = \Delta K \)
- \( \int F(x) \, dx \)
- \( \int -kx \, dx \)
- \( \int \frac{1}{2} k \Delta x^2 \, dx \)
- \( \int \frac{1}{2} k x^2 \, dx \)
- \( \frac{1}{2} k \Delta x^2 = \Delta K \)
- \( \frac{1}{2} k \Delta x^2 = \frac{1}{2} m v_0^2 \)
Lecture 15, Example

Work & Friction

- Two blocks having mass $m_1$ and $m_2$ where $m_1 > m_2$
  They are sliding on a frictionless floor and have the same kinetic energy when they encounter a long rough stretch (i.e. $\mu > 0$) which slows them down to a stop.
- Which one will go farther before stopping?
- Hint: How much work does friction do on each block?

(A) $m_1$  (B) $m_2$  (C) They will go the same distance

$W = F d = - \mu N d = - \mu mg d = \Delta K = 0 - \frac{1}{2} mv^2$

- $\mu m_1 g d_1 = - \mu m_2 g d_2 \rightarrow d_1 / d_2 = m_2 / m_1$
(A) $m_1$  (B) $m_2$  (C) They will go the same distance

Work & Power:
- Power is the rate at which work is done.

\[
P = \frac{W}{\Delta t} \quad \text{Instantaneous Power:} \quad P = \frac{dW}{dt}
\]

Units (SI) are Watts (W):

\[
1 \text{ W} = 1 \text{ J} / 1\text{s}
\]

Example 1:
- A person of mass 80.0 kg walks up to 3rd floor (12.0m).
  If he/she climbs in 20.0 sec what is the average power used.

\[
P_{avg} = F h / t = mgh / t = 80.0 \times 9.80 \times 12.0 / 20.0 \text{ W}
\]

\[
P = 470. \text{ W}
\]

Work & Power:
- Instantaneous Power is,

\[
P = \frac{dW}{dt}
\]

- If force constant, $W = F \Delta x = F (v_0 t + \frac{1}{2} at^2)$
  and $P = dW/dt = F (v_0 + at)$

Exercise 2

A. Top  B. Middle  C. Bottom

Starting from rest, a car drives up a hill at constant acceleration and then suddenly stops at the top.

The instantaneous power delivered by the engine during this drive looks like which of the following,
Work & Power

\[ P = \frac{dW}{dt} \quad \text{and} \quad W = F \cdot d = (\mu mg \cos \theta - mg \sin \theta) \cdot d \]

So \[ W = F \cdot \frac{1}{2} a t^2 \quad \Rightarrow \quad P = F \cdot a \cdot t = F \cdot v \]

(A) \[ \begin{array}{c}
\text{Power} \\
\text{time}
\end{array} \]

(B) \[ \begin{array}{c}
\text{Power} \\
\text{time}
\end{array} \]

(C) \[ \begin{array}{c}
\text{Power} \\
\text{time}
\end{array} \]

Power for Circular Motion

- I swing a slingshot over my head. The tension in the rope keeps the shot moving in a circle. How much power must be provided by me, through the rope tension, to keep the shot in circular motion?

Note that:
- Rope Length = 1 m
- Shot Mass = 1 kg
- Angular frequency = 2 rad/sec

- A. 16 J/s
- B. 8 J/s
- C. 4 J/s
- D. 0 J/s

Non-conservative Forces:

- If the work done does not depend on the path taken, the force involved is said to be conservative.
- If the work done does depend on the path taken, the force involved is said to be non-conservative.
- An example of a non-conservative force is friction:
  - Pushing a box across the floor, the amount of work that is done by friction depends on the path taken.
  - Work done is proportional to the length of the path!

A Non-Conservative Force, Friction

- Looking down on an air-hockey table with no air flowing (\(\mu > 0\)).
- Now compare two paths in which the puck starts out with the same speed (\(K_1 = K_2\)).

Since \(\text{path}_2\) distance > \(\text{path}_1\) distance, the puck will be traveling slower at the end of \(\text{path}_2\).

Work done by a non-conservative force irreversibly removes energy out of the "system".

Here \(W_{\text{NC}} = E_{\text{final}} - E_{\text{initial}} < 0\).
Potential Energy

- What is “Potential Energy”? It is a way of effecting energy transfer in a system so that it can be “recovered” (i.e. transferred out) at a later time or place.
- Example: Throwing a ball up a height $h$ above the ground.

At times 1 and 3 the ball will have the same $K$ and $U$

Compare work with changes in potential energy

Consider the ball moving up to height $h$ (from time 1 to time 2)

$W = F \cdot \Delta x = mg (y_f - y_i) = -mg h$

$\Delta U = U_f - U_i = mg h - mg 0 = mg h$

$\Delta U = -W$

This is a general result for all conservative forces (path independent)

Lecture 15, Example

Work Done by Gravity

- An frictionless track is at an angle of 30° with respect to the horizontal. A cart (mass 1 kg) is released from rest. It slides 1 meter downwards along the track bounces and then slides upwards to its original position.
- How much total work is done by gravity on the cart when it reaches its original position? ($g = 10 \text{ m/s}^2$)

Work done by the Earth’s gravity on the ball)

$W = F \cdot \Delta x = mg (y_f - y_i) = -mg h$

$\Delta U = U_f - U_i = mg h - mg 0 = mg h$

$\Delta U = -W$

This is a general result for all conservative forces (path independent)

Examples of the $U - F$ relationship

- Remember the spring,
  $U(x) = \frac{1}{2} kx^2$

- Calculate the derivative
  $F_s = - \frac{dU}{dx}$
  $F_s = - (\frac{1}{2} kx^2) / dx$
  $F_s = - \frac{1}{2} k$ (2x)
  $F_s = - kx$

Main concepts

Work ($W$) of a constant force $F$ acting through a displacement $\Delta r$ is:

$W = F \cdot \Delta r = F \Delta r \cos \theta = F_{\text{along path}} \Delta r$

Work (net) Kinetic-Energy Theorem:

$W_{\text{net}} = \Delta K = K_f - K_i = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2$

Work-potential energy relationship:

$W = -\Delta U$

Work done reflects change in system energy ($\Delta E_{\text{sys}}, U, K \& E_i$)
Important Definitions

- **Conservative Forces** - Forces for which the work done does not depend on the path taken, but only the initial and final position (no loss).

- **Potential Energy** - describes the amount of work that can potentially be done by one object on another under the influence of a conservative force.

  \[ W = -\Delta U \]

  Only differences in potential energy matter.

**Lecture 15, Exercise 4**

**Work/Energy for Non-Conservative Forces**

- The air track is once again at an angle of 30° with respect to horizontal. The cart (with mass 1.0 kg) is released 1.0 meter from the bottom and hits the bumper at a speed, \( v_1 \). This time the vacuum/air generator breaks halfway through and the air stops. The cart only bounces up half as high as where it started.

- How much work did friction do on the cart? \((g=10 \text{ m/s}^2)\)

  Notice the cart only bounces to a height of 0.25 m

  \[ h = 1 \text{ m sin } 30° = 0.5 \text{ m} \]

**Physics 207, Lecture 15, Oct. 24**

**Agenda:** Chapter 11, Finish

**Assignment:** For Monday read Chapter 13 carefully (you may skip the parallel axis theorem and vector cross products).

- MP Homework 7, Ch. 11, 5 problems, available today, Due Wednesday at 4 PM
- MP Homework 6, Due tonight