Physics 207, Lecture 15, Oct. 24

Agenda: Chapter 11, Finish, Chapter 13, Just Start

- Chapter 11:
  - ❖ Variable forces
  - ❖ Conservative vs. Non-conservative forces
  - ❖ Power
  - ❖ Work & Potential Energy
- Start Chapter 13
  - ❖ Rotation
  - ❖ Torque

Assignment: For Monday read Chapter 13 carefully (you may skip the parallel axis theorem and vector cross products)

- MP Homework 7, Ch. 11, 5 problems, available today,

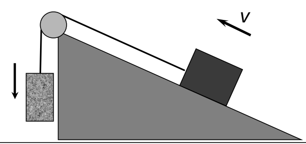
Due Wednesday at 4 PM

- MP Homework 6, Due tonight

Lecture 15, *Exercise 1*
Work in the presence of friction and non-contact forces

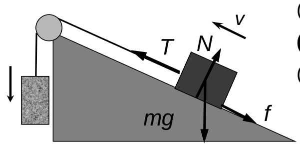
- A box is pulled up a rough ($\mu > 0$) incline by a rope-pulley-weight arrangement as shown below.
  - ❖ How many forces are doing work on the box ?
  - ❖ Of these which are positive and which are negative?
  - ❖ Use a Force Body Diagram
  - ❖ Compare force and path

A. 2
B. 3
C. 4



Lecture 15, *Exercise 1*
Work in the presence of friction and non-contact forces

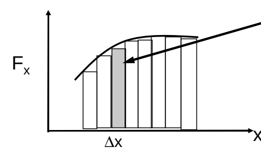
- A box is pulled up a rough ($\mu > 0$) incline by a rope-pulley-weight arrangement as shown below.
  - ❖ How many forces are doing work on the box ?
  - ❖ And which are positive and which are negative?
  - ❖ Use a Force Body Diagram



(A) *2*
(B) ***3*** is correct
(C) *4*

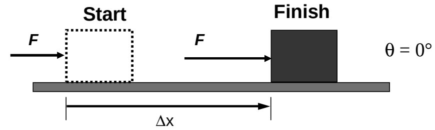
Work and Varying Forces (1D)

- Consider a varying force F(x)



Area = $F_x \Delta x$
F is **increasing**
Here $W = \mathbf{F} \cdot \Delta \mathbf{r}$
becomes $dW = F\,dx$

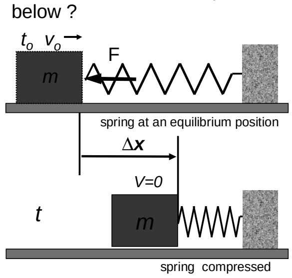
$$W = \int_{x_i}^{x_f} F(x)\,dx$$



Work is a scalar, the rub is that there is no time/position info on hand

Example: Work Kinetic-Energy Theorem

- How much will the spring compress (i.e. $\Delta \mathbf{x}$) to bring the object to a stop (i.e., $v = 0$) if the object is moving initially at a constant velocity (v_o) on frictionless surface as shown below ?



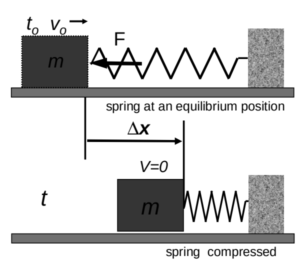
Notice that the spring force is opposite to the displacement.

For the mass m, work is negative

For the spring, *work is positive*

Example: Work Kinetic-Energy Theorem

- How much will the spring compress (i.e. $\Delta \mathbf{x} = x_f - x_i$) to bring the object to a stop (i.e., $v = 0$) if the object is moving initially at a constant velocity (v_o) on frictionless surface as shown below ?



$$W_{\text{box}} = \int_{x_i}^{x_f} F(x)\,dx$$

$$W_{\text{box}} = \int_{x_i}^{x_f} -kx\,dx$$

$$W_{\text{box}} = -\tfrac{1}{2} kx^2 \Big|_{x_i}^{x_f}$$

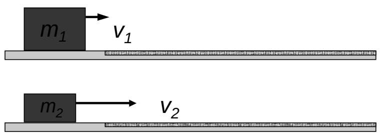
$$W_{\text{box}} = -\tfrac{1}{2} k\,\Delta x^2 = \Delta K$$

$$-\tfrac{1}{2} k\,\Delta x^2 = \tfrac{1}{2} m 0^2 - \tfrac{1}{2} m v_0^2$$

Lecture 15, *Example*
Work & Friction

- Two blocks having mass m_1 and m_2 where $\boldsymbol{m_1 > m_2}$. They are sliding on a frictionless floor and have the <u>same kinetic energy</u> when they encounter a long rough stretch (i.e. $\mu > 0$) which slows them down to a stop.
- Which one will go farther before stopping?
- **Hint:** How much work does friction do on each block ?

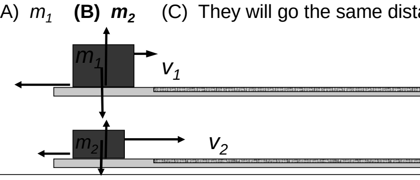
(A) m_1 (B) m_2 (C) They will go the same distance



Lecture 15, *Example*
Work & Friction

- $W = F\,d = -\mu\,N\,d = -\mu\,mg\,d = \Delta K = 0 - \frac{1}{2}\,mv^2$
- $-\mu\,m_1 g\,d_1 = -\mu\,m_2 g\,d_2 \rightarrow d_1 / d_2 = m_2 / m_1$

(A) m_1 **(B) m_2** (C) They will go the same distance



Work & Power:

- Power is the rate at which work is done.

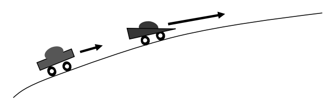
Average Power:	Instantaneous Power:	Units (SI) are Watts (W):
$\bar{P} = \frac{W}{\Delta t}$	$P = \frac{dW}{dt}$	1 W = 1 J / 1s

Example 1 :

- A person of mass 80.0 kg walks up to 3rd floor (12.0m). If he/she climbs in 20.0 sec what is the average power used.
- P_{avg} = F h / t = mgh / t = 80.0 x 9.80 x 12.0 / 20.0 W
- P = 470. W

Work & Power:

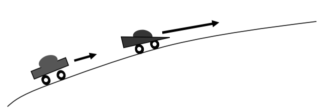
- Two cars go up a hill, a Corvette and a ordinary Chevy Malibu. Both cars have the same mass.
- Assuming identical friction, both engines do the same amount of work to get up the hill.
- Are the cars essentially the same ?
- **NO.** The Corvette can get up the hill quicker
- It has a more **powerful** engine.



Work & Power:

- Instantaneous Power is, $P = \frac{dW}{dt}$
- If force constant, $W = F\,\Delta x = F\,(v_0\,t + \frac{1}{2}\,at^2)$

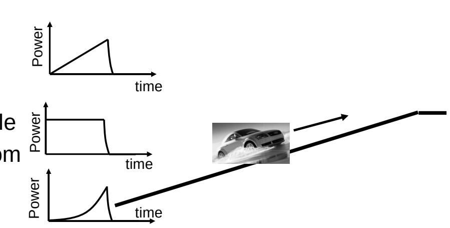
and $P = dW/dt = F\,(v_0 + at)$



Lecture 15, *Exercise 2*
Work & Power

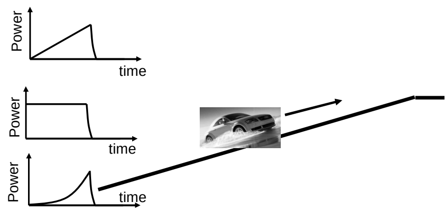
- Starting from rest, a car drives up a hill at constant acceleration and then suddenly stops at the top. The instantaneous power delivered by the engine during this drive looks like which of the following,

A. Top
B. Middle
C. Bottom



Lecture 15, *Exercise 2*
Work & Power

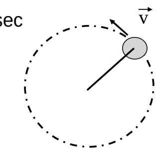
- $P = dW / dt$ & $W = F\,d = (\mu\, mg \cos\theta - mg \sin\theta)\, d$
  and $d = ½\, a\, t^2$ (constant accelation)
  So $W = F\, ½\, a\, t^2$ → $P = F\, a\, t = F\, v$
- **(A)**
- (B)
- (C)



Lecture 15, *Exercise 3*
Power for Circular Motion

- I swing a sling shot over my head. The tension in the rope keeps the shot moving in a circle. How much power must be provided by me, through the rope tension, to keep the shot in circular motion ?

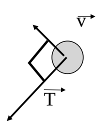
  Note that: Rope Length = 1m
  Shot Mass = 1 kg
  Angular frequency = 2 rad / sec



- A. 16 J/s
- B. 8 J/s
- C. 4 J/s
- D. 0 J/s

Lecture 15, *Exercise 3*
Power for Circular Motion

- Note that the string expends no power (because it does no work).
- By the work / kinetic energy theorem, work done equals change in kinetic energy.
- $K = 1/2\, mv^2$, thus since $|v|$ doesn't change, neither does K.
- A force perpendicular to the direction of motion does not change speed, $|v|$, and so does no work.
- Answer is **(D)**



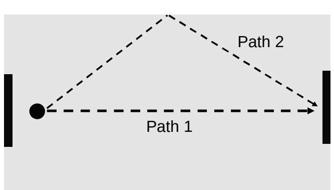
Non-conservative Forces :

- If the work done does not depend on the path taken, the force involved is said to be conservative.
- If the work done does depend on the path taken, the force involved is said to be non-conservative.
- An example of a non-conservative force is friction:
- Pushing a box across the floor, the amount of work that is done by friction depends on the path taken.

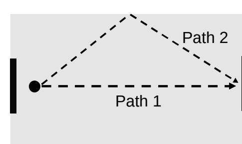
Work done is proportional to the length of the path !

A Non-Conservative Force, Friction

- Looking down on an air-hockey table with no air flowing ($\mu > 0$).
- Now compare two paths in which the puck starts out with the same speed ($K_1 = K_2$) .



A Non-Conservative Force



Since $path_2$ distance $>path_1$ distance the puck will be traveling slower at the end of path 2.

Work done by a non-conservative force irreversibly removes energy out of the "system".

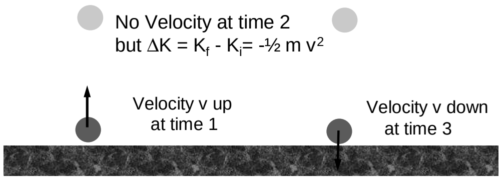
Here $W_{NC} = E_{final} - E_{initial} < 0$

Potential Energy

- What <u>is</u> "Potential Energy" ?

  It is a way of effecting energy transfer in a system so that it can be "recovered" (i.e. transferred out) at a later time or place.

- Example: Throwing a ball up a height h above the ground.



At times 1 and 3 the ball will have the same K and U

Compare work with changes in potential energy

- Consider the ball moving up to height *h*

  (from time 1 to time 2)

- How does this relate to the potential energy?

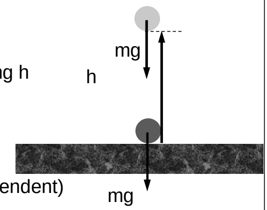
Work done by the Earth's gravity on the ball)

$W = \mathbf{F} \bullet \Delta \mathbf{x} = mg\,(y_f - y_i) = -mg\,h$

$\Delta U = U_f - U_i = mg\,h - mg\,0 = mg\,h$

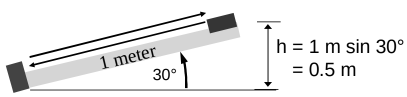
$\Delta U = -W$

This is a general result for all conservative forces (path independent)



Lecture 15, *Example* Work Done by Gravity

- An frictionless track is at an angle of 30° with respect to the horizontal. A cart (mass 1 kg) is released from rest. It slides 1 meter downwards along the track bounces and then slides upwards to its original position.
- How much total work is done by gravity on the cart when it reaches its original position? ($g = 10\ m/s^2$)



(A) 5 J (B) 10 J (C) 20 J (D) 0 J

Conservative Forces and Potential Energy

- So we can also describe work and changes in potential energy (for conservative forces)

  $\Delta U = -W$

- Recalling

  $W = F_x \Delta x$

- Combining these two,

  $\Delta U = -F_x \Delta x$

- Letting small quantities go to infinitesimals,

  $dU = -F_x\, dx$

- Or,

  $F_x = -dU / dx$

Examples of the U - F relationship

- Remember the spring,
  - $U(x) = \frac{1}{2} kx^2$
- Calculate the derivative
  - $F_x = -dU / dx$
  - $F_x = -d\,(\frac{1}{2} kx^2) / dx$
  - $F_x = -\frac{1}{2}\,k\,(2x)$
  - $F_x = -k\,x$

Main concepts

Work (W) of a constant force $\boldsymbol{F}$ acting through a displacement $\boldsymbol{\Delta r}$ is:

$$W = \boldsymbol{F} \cdot \boldsymbol{\Delta r} = F\,\Delta r \cos\theta = F_{\text{along path}}\,\boldsymbol{\Delta r}$$

Work (net) Kinetic-Energy Theorem:

$$\boxed{W_{net} = \Delta K} = K_2 - K_1 = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

Work-potential energy relationship:

$$W = -\Delta U$$

Work done reflects change in system energy (ΔE_{sys}, U, K & E_{th})

Important Definitions

- **Conservative Forces** - Forces for which the work done does not depend on the path taken, but only the initial and final position (no loss).
- **Potential Energy** - describes the amount of work that can potentially be done by one object on another under the influence of a conservative force
  - ❖ $W = -\Delta U$

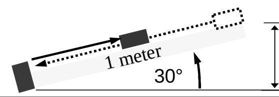
Only differences in potential energy matter.

Lecture 15, *Exercise 4*
Work/Energy for Non-Conservative Forces

- The air track is once again at an angle of 30° with respect to horizontal. The cart (with mass 1.0 kg) is released 1.0 meter from the bottom and hits the bumper at a speed, v_1. This time the vacuum/ air generator breaks half-way through and the air stops. The cart only bounces up half as high as where it started.
- How much work did friction do on the cart ?($g=10\ m/s^2$)

Notice the cart only bounces to a height of 0.25 m

- ✗ A. 2.5 J
- ✗ B. 5.0 J
- ✗ C. 10. J
- ✗ D. -2.5 J
- ✗ E. -5.0 J
- ✗ F. -10. J



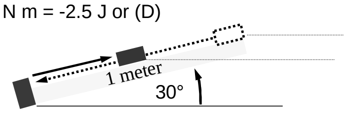
Lecture 15, *Exercise 4*
Work/Energy for Non-Conservative Forces

- How much work did friction do on the cart ? ($g=10\ m/s^2$)

  $W = F\,\Delta x$ is not easy to do…
- Work done (W) is equal to the change in the energy of the system (just U and/or K). $E_{final} - E_{initial}$ and is < 0. ($E = U+K$)

  Use $W = U_{final} - U_{init} = mg\,(h_f - h_i) = -\,mg \sin 30° 0.5\ m$

  $W = -2.5\ N\ m = -2.5\ J$ or (D)



(A) 2.5 J (B) 5 J (C) 10 J **(D) –2.5 J** (E) –5 J (F) –10 J

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