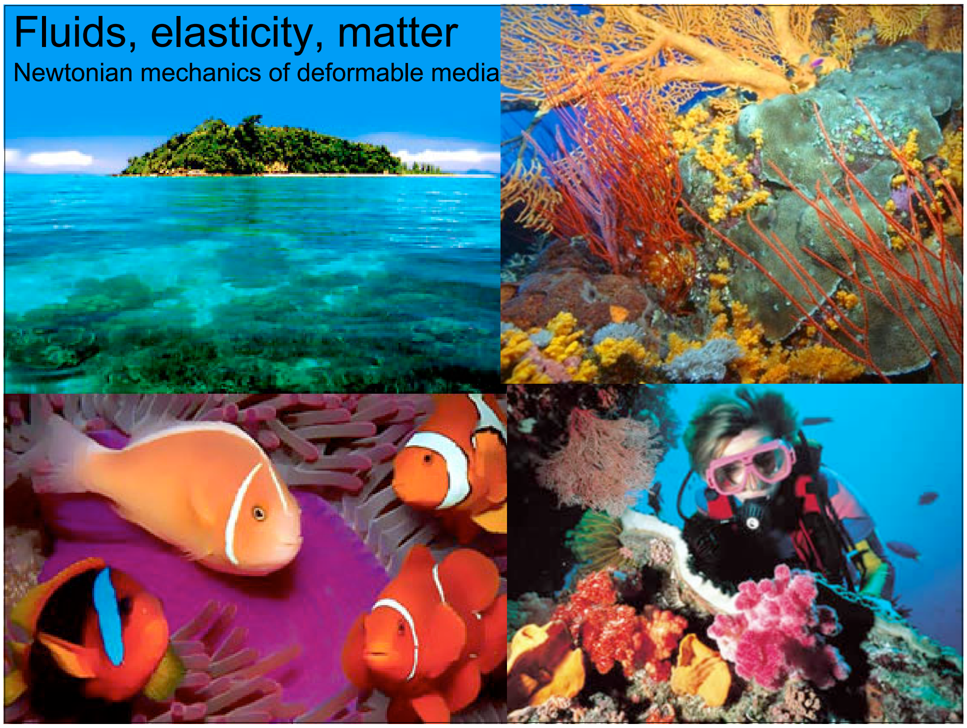
Fluids, elasticity, matter

Newtonian mechanics of deformable media



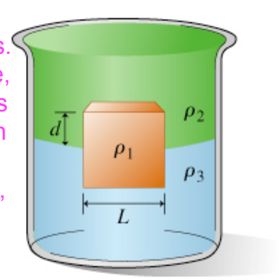
Fluids: a problem

A beaker contains a thick layer of oil (shown in green) of density ρ_2 floating on water (shown in blue), which has density ρ_3. A cubical block of wood of density ρ_1 with side length L is gently lowered into the beaker, so as not to disturb the layers of liquid, until it floats peacefully between the layers, as shown in the figure.

What is the distance d between the top of the wood cube (after it has come to rest) and the interface between oil and water?

Hint: After the wood block has come to rest, it is in static equilibrium. Thus, the magnitude of the buoyant force (directed upward) must exactly equal the magnitude of the gravitational force (directed downward). The buoyant force will depend on the quantity d that you are trying to find.

The total buoyant force has two contributions, one from each of the two different fluids. To find the total buoyant force, imagine that the wood block is divided into two pieces, one in oil and one in water. Apply Archimedes' principle to each, and add the two buoyant forces to find the total force.



$F_{oil} = \rho_2\, g\, (L^2 d)$

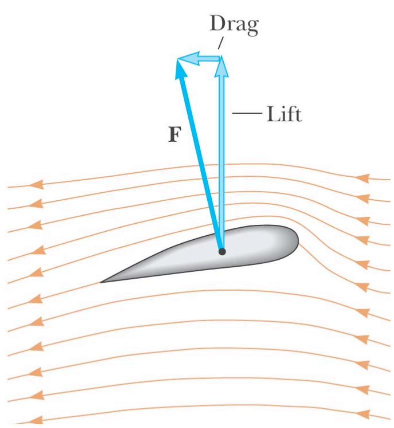
$F_{water} = \rho_3\, g\, (L^2 (L-d))$

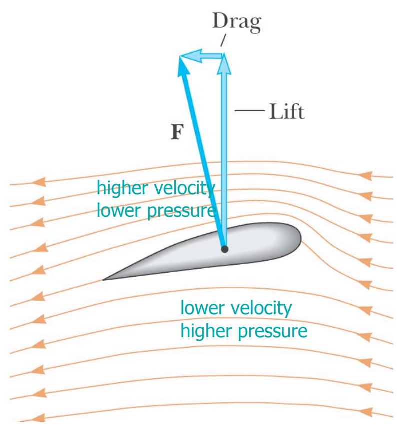
$w = \rho_1\, g\, L^3$

$w = F_{oil} + F_{water}$

DC

Fluids: how are airplanes kept aloft?





Note: density of flow lines reflects velocity, not density. We are assuming an incompressible fluid.

but……. this is not enough!

Bernoulli only works in closed systems, and air in the sky really is not a closed system

Simple Bernoulli calculation

- **Boeing 747-400**
- Dimensions:
  - Length: 231 ft 10 inches
  - Wingspan: 211 ft 5 in
  - Height: 63 ft 8 in
- Weight:
  - Empty: 4×10^5 lbs
  - Cargo: 2.5×10^5 lbs
  - Passengers+fuel: 1.5×10^5 lbs
  - Full at takeoff: 8×10^5 lbs
- Performance:
  - Cruising Speed: 583 mph
  - Range: 7,230 miles

$r (v_2^2 - v_1^2) / 2 = P_1 – P_2 = DP$

Let v_2 = 220.0 m/s v_2 = 210 m/s

So DP = 3×10^3 Pa = 0.03 atm

or 0.5 lbs/in^2

http://www.geocities.com/galemcraig/

Using Bernoulli's equation and a surface area of 200 ft x 15 ft you *only* produce an upward lift of 2×10^4 lbs

too low by a factor of 40!

Airplanes are kept aloft because the wings scatter off air molecules, and scatter more downward than upward

Elasticity

Elasticity, 1912, by Umberto Boccioni, futurist painter

Elasticity

Describes the deformation of solids and liquids under stress.

Linear stretch and compression

$$\underbrace{\frac{F}{A}}_{\text{tensile stress}} = \overbrace{Y}^{\text{Young's modulus}} \underbrace{\frac{\Delta L}{L}}_{\text{strain}}$$

Volume compression

$$\frac{F}{A} = \underbrace{p}_{\text{pressure}} = -\overbrace{B}^{\text{Bulk modulus}} \underbrace{\frac{\Delta V}{V}}_{\text{volume strain}}$$

tools

thermodynamics: a macroscopic description of matter

3 Phases of matter

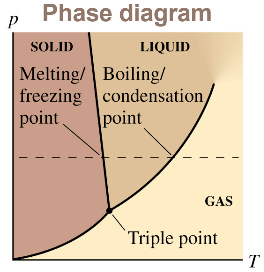
solid: rigid, definite shape. Nearly incompressible.

liquid: molecules held together by bonds, but able to flow. Nearly incompressible.

gas: molecules move freely. Compressible.

All 3 phases exist at different p,T conditions

Phase diagram



Triple point of water: **p = 0.006 atm**
T = 0.01°C

Triple point of CO_2: **p = 5 atm**
T = -56°C

demo: geyser & collapsing tank

ideal gas

- Atoms and molecules are **small**, **hard spheres** traveling freely through space. Occasionally they collide with each other or with the walls.
- The molecules have a **distribution of speeds**
- The model is valid when the **density is low** and **the temperature is high**, well above the condensation point

$$pV = nRT \quad \text{or} \quad pV = Nk_BT$$

R = 8.31 J/mol K universal gas constant
n = number of moles

k_B = 1.38 x 10^{-23} J/K Boltzmann's constant
N = 6.02 x 10^{23} molecules/mol Avogadro number

p, V, T must be in SI units: Pa, m^3, K

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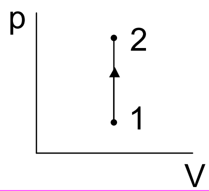
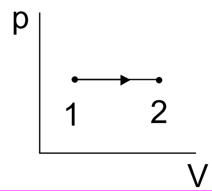
ideal gas

Isochoric process: $V = const$



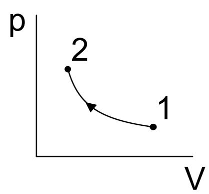
$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

Isobaric process: $p = const$



$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Isothermal process: $T = const$



$$p_1 V_1 = p_2 V_2$$

$$\frac{pV}{T} = const$$

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air bubble rising

A diver produces an air bubble underwater, where the absolute pressure is **p_1 = 3.5 atm**.
The bubble rises to the surface, where the pressure is **p_2 = 1 atm**.
The water temperatures at the bottom and the surface are, respectively,
T_1 = 4°C, T_2 = 23°C

What is the ratio **V_2/V_1** of the volume of the bubble as it reaches the surface (V_2) to its volume at the bottom (V_1)?

V_2/V_1 = 3.74

Is it safe for the diver to ascend while holding his breath?

No. Air in the lungs would expand, and he could have a lung rupture.
This is addition to "the bends", or decompression sickness, which is due to the pressure-dependent solubility of gas (in air, mostly nitrogen). At depth and at higher pressure N_2 is more soluble in blood. As divers ascend, N_2 dissolved in their blood stream becomes gaseous again and forms N_2 bubbles in blood vessels, which in turn can obstruct blood flow, and therefore provoke pain and in some cases even strokes or deaths.
Fortunately, this only happens when diving deeper than 30 m (100 feet).

The diver in this question only went down 25 meters. How do we know that?

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