Chapter 13 notes:

Key issues for exam:

1. Right hand rule
2. Center of Mass
3. Torque
4. Moment of Inertia
5. Rotational Energy
6. Rotational Momentum

The explicit formulas

$$\vec{R}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{M} = X_{CM}\,\hat{i} + Y_{CM}\,\hat{j} + Z_{CM}\,\hat{k}$$

$$\tau_{TOT} = |r|\,|F_{Tang}| = |r|\,|F| \sin\phi$$

$$I \equiv \sum_{i=1}^{N} m_i r_i^2$$

$$K = \tfrac{1}{2} I \omega^2$$

$$\vec{L} \equiv I\vec{\omega} \qquad \Delta K = \tfrac{1}{2} I \left(\omega_f^2 - \omega_i^2\right) = W_{NET}$$

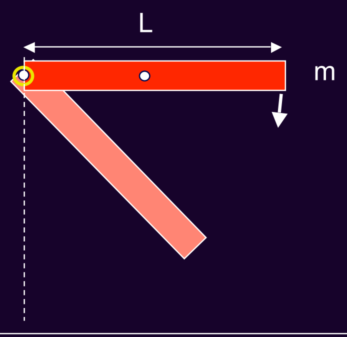
Example: Rotating Rod

- A uniform rod of length L=0.5 m and mass m=1 kg is free to rotate on a frictionless pin passing through one end as shown below. The rod is released from rest in the horizontal position. What is

  (A) its angular speed when it reaches the lowest point ?

  (B) its initial angular acceleration ?

  (C) its initial linear acceleration of its free end ?



Example: Rotating Rod

- A uniform rod of length L=0.5 m and mass m=1 kg is free to rotate …

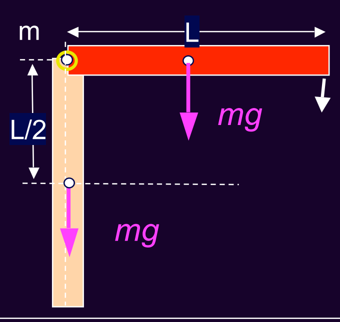
What is (A) its angular speed when it reaches the lowest point ?

1. For forces you need to locate the Center of Mass

Notice that the Center of Mass is at L/2 (halfway) and use the Work-Energy Theorem or Conservation of mechanical Energy

$$K_i + U_i = K_f + U_f$$

$$0 + mgh_{CM} = (1/2)\, I\, \omega^2 + 0$$



$I_{CM} = (m/12)\ L^2$ (from table)

$I_{end} = (m/3)\ L^2$

$$mgL / 2 = (1/2)\, (m\, (L/3)^2)\, \omega^2$$

$$9\, g / L = \omega^2$$

Example: Rotating Rod

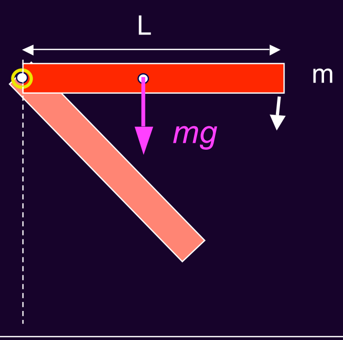
- A uniform rod of length L=0.5 m and mass m=1 kg is free to rotate …

What is (B) its initial angular acceleration ?

1. For forces you need to locate the Center of Mass

CM is at L/2 (halfway) and put in the Force on a FBD

$\Sigma \boldsymbol{F} = 0$ occurs only at the hinge



$\tau_z = I\ \alpha_z = r\ F \sin 90°$

at the center of mass

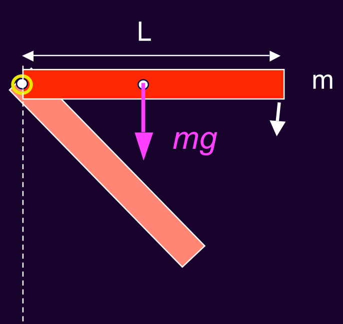
$(m/3)\ L^2\ \ \alpha_z = -(L/2)\ mg$ (CW)

$\alpha_z = -\ 3g\ /\ (2L)$

Example: Rotating Rod

- A uniform rod of length L=0.5 m and mass m=1 kg is free to rotate …

What is (C) initial linear acceleration of its free end ?

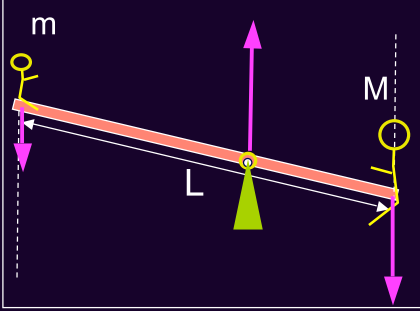


$a = \alpha\ L$

Two children on a see-saw

- Two children of mass 30 kg and 10 kg are sitting at the ends of a massless see-saw of length L= 4.0 m. The see-saw is at an angle θ = 30° from the horizontal as shown. The heavy child is 1.5 m from the pivot point.
- What is the net torque about this pivot?

r = 2.5 m
R = 1.5 m



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Two children on a see-saw

- Two children of mass 30 kg and 10 kg are sitting at the ends of a massless see-saw of length L= 4.0 m. The see-saw is at an angle θ = 30° from the horizontal as shown. The heavy child is 1.5 m from the pivot point.
- What is the net torque about this pivot?

$$\tau_z = I\,\alpha_z = -\,R\,Mg\,\sin 60^\circ + 0 + r\,mg\,\sin 60^\circ$$

$$\tau_z = -1.5 \times 30 \times 10 \times 0.83 + 2.5 \times 10 \times 10 \times 0.83$$

$$\tau_z = -170\ \text{N m}$$

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Two children on a see-saw

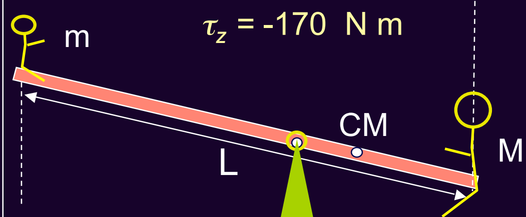
- Two children of mass 30 kg and 10 kg are sitting at the ends of a massless see-saw of length L= 4.0 m. The see-saw is at an angle θ = 30° from the horizontal as shown. The heavy child is 1.5 m from the pivot point.
- What is the net torque about the pivot using the center of mass?

If horizontal: x_{CM} = 10 x 0 + (30 x 4)/(10+30) = 3.0 m from left edge or 0.5 m to the right of the pivot

$\tau_z = I\,\alpha_z = -\,r'\,(M+m)\,g\,\sin 60° + 0$

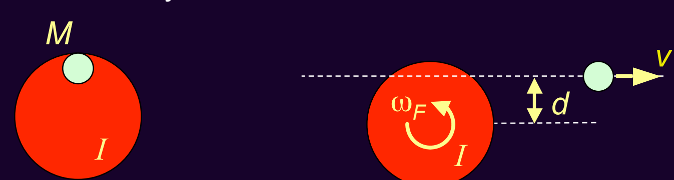
τ_z = -0.5 x (30+10) x 10 x 0.83

τ_z = -170 N m



Example: Throwing ball from stool

- A student sits on a stool, initially at rest, but which is free to rotate. The moment of inertia of the student plus the stool is I. They throw a heavy ball of mass *M* with speed *v* such that its velocity vector moves a distance *d* from the axis of rotation.
  - What is the angular speed ω_F of the student-stool system after they throw the ball ?



Top view: before after

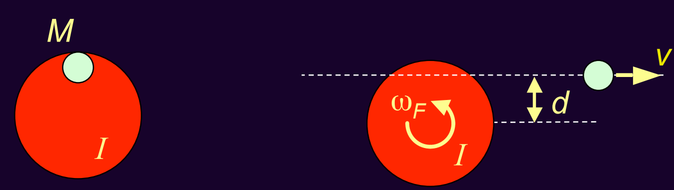
Example: Throwing ball from stool

- What is the angular speed ω_F of the student-stool system after they throw the ball ?
- Process: (1) Define system (2) Identify Conditions

(1) System: student, stool and ball (No Ext. torque, L is constant)

(2) Momentum is conserved

$$L_{init} = 0 = L_{final} = -m\,v\,d + I\,\omega_f$$



Top view: before   after

SUMMARY

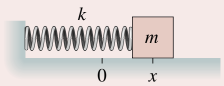
The goal of Chapter 14 has been to understand systems that oscillate with simple harmonic motion.

GENERAL PRINCIPLES

Dynamics

SHM occurs when a linear restoring force acts to return a system to an equilibrium position.

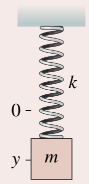
Horizontal spring



$(F_{net})_x = -kx$

Vertical spring

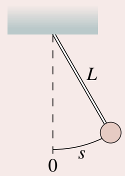
The origin is at the equilibrium position $\Delta L = mg/k$.



$(F_{net})_y = -ky$

$\omega = \sqrt{\dfrac{k}{m}}$ $\qquad T = 2\pi\sqrt{\dfrac{m}{k}}$

Pendulum

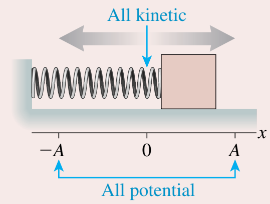


$(F_{net})_t = -\left(\dfrac{mg}{L}\right)s$

$\omega = \sqrt{\dfrac{g}{L}}$ $\qquad T = 2\pi\sqrt{\dfrac{L}{g}}$

Energy

If there is **no friction** or dissipation, kinetic and potential energy are alternately transformed into each other, but the total mechanical energy $E = K + U$ is conserved.

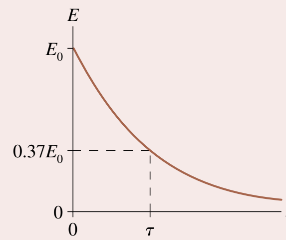


$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2$$
$$= \frac{1}{2}m(v_{max})^2$$
$$= \frac{1}{2}kA^2$$

In a **damped system,** the energy decays exponentially

$$E = E_0 e^{-t/\tau}$$

where τ is the time constant.



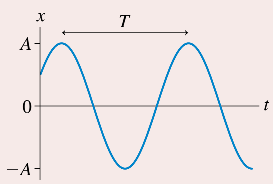
IMPORTANT CONCEPTS

Simple harmonic motion (SHM) is a sinusoidal oscillation with period T and amplitude A.

Frequency $f = \dfrac{1}{T}$

Angular frequency

$$\omega = 2\pi f = \frac{2\pi}{T}$$



Position $x(t) = A\cos(\omega t + \phi_0)$

$$= A\cos\left(\frac{2\pi t}{T} + \phi_0\right)$$

Velocity $v_x(t) = -v_{max}\sin(\omega t + \phi_0)$ with maximum speed $v_{max} = \omega A$

Acceleration $a_x = -\omega^2 x$

SHM is the projection onto the x-axis of **uniform circular motion.**

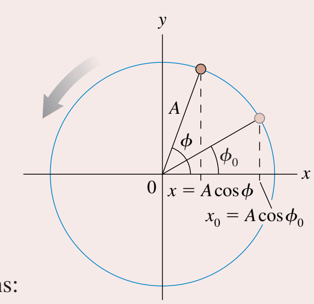
$\phi = \omega t + \phi_0$ is the phase

The position at time t is

$$x(t) = A\cos\phi$$
$$= A\cos(\omega t + \phi_0)$$

The phase constant ϕ_0 determines the initial conditions:

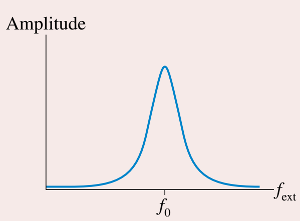
$x_0 = A\cos\phi_0 \qquad v_{0x} = -\omega A\sin\phi_0$



APPLICATIONS

Resonance

When a system is driven by a periodic external force, it responds with a large-amplitude oscillation if $f_{ext} \approx f_0$ where f_0 is the system's natural oscillation frequency, or **resonant frequency.**

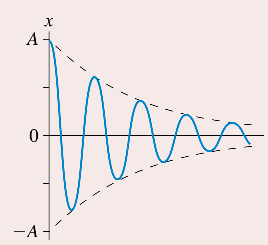


Damping

If there is a drag force $\vec{D} = -b\vec{v}$, where b is the damping constant, then (for lightly damped systems)

$$x(t) = Ae^{-bt/2m}\cos(\omega t + \phi_0)$$

The time constant for energy loss is $\tau = m/b$.



SUMMARY

The goal of Chapter 15 has been to understand macroscopic systems that flow or deform.

GENERAL PRINCIPLES

Fluid Statics

Gases

- Freely moving particles
- Compressible
- Pressure primarily thermal
- Pressure constant in a laboratory-size container

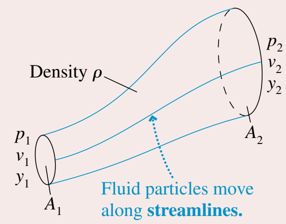
Liquids

- Loosely bound particles
- Incompressible
- Pressure primarily gravitational
- Hydrostatic pressure at depth d is $p = p_0 + \rho g d$

Fluid Dynamics

Ideal-fluid model

- Incompressible
- Smooth, laminar flow
- Nonviscous
- Irrotational



Equation of continuity

$$v_1 A_1 = v_2 A_2$$

Bernoulli's equation

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

Bernoulli's equation is a statement of energy conservation.

IMPORTANT CONCEPTS

Density $\rho = m/V$, where m is mass and V is volume.

Pressure $p = F/A$, where F is the magnitude of the fluid force and A is the area on which the force acts.

- Exists at all points in a fluid
- Pushes equally in all directions
- Constant along a horizontal line
- Gauge pressure $p_g = p - 1$ atm

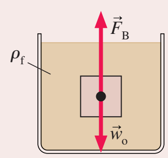
APPLICATIONS

Buoyancy is the upward force of a fluid on an object.

Archimedes' principle

The magnitude of the buoyant force equals the weight of the fluid displaced by the object.

Sink	$\rho_{avg} > \rho_f$	$F_B < w_o$
Rise to surface	$\rho_{avg} < \rho_f$	$F_B > w_o$
Neutrally buoyant	$\rho_{avg} = \rho_f$	$F_B = w_o$

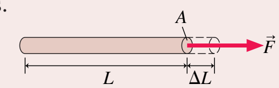


Elasticity describes the deformation of solids and liquids under stress.

Linear stretch and compression:

$$(F/A) = Y(\Delta L/L)$$

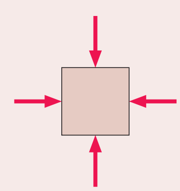
Tensile stress — Young's modulus — Strain



Volume compression:

$$p = -B(\Delta V/V)$$

Bulk modulus — Volume strain



SUMMARY

The goal of Chapter 16 has been to learn the characteristics of macroscopic systems.

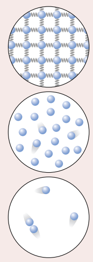
GENERAL PRINCIPLES

Three Phases of Matter

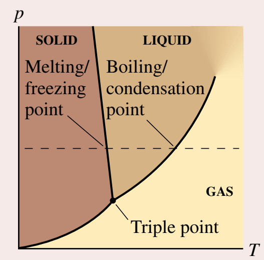
Solid Rigid, definite shape. Nearly incompressible.

Liquid Molecules loosely held together by molecular bonds, but able to move around. Nearly incompressible.

Gas Molecules move freely through space. Compressible.



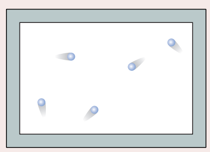
The different phases exist for different conditions of temperature T and pressure p. The boundaries separating the regions of a phase diagram are lines of phase equilibrium. Any amounts of the two phases can coexist in equilibrium. The **triple point** is the one value of temperature and pressure at which all three phases can coexist in equilibrium.



IMPORTANT CONCEPTS

Ideal-Gas Model

- Atoms and molecules are small, hard spheres that travel freely through space except for occasional collisions with each other or the walls.
- The molecules have a distribution of speeds.
- The model is valid when the density is low and the temperature well above the condensation point.



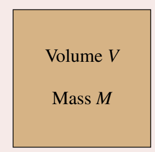
Counting atoms and moles

A macroscopic sample of matter consists of N atoms (or molecules), each of mass m (the **atomic** or **molecular mass**):

$$N = \frac{M}{m}$$

Alternatively, we can state that the sample consists of n **moles**

$$n = \frac{N}{N_A} \quad \text{or} \quad \frac{M(\text{in grams})}{M_{mol}}$$



$N_A = 6.02 \times 10^{23}$ mol^{-1} is **Avogadro's number.**

The numerical value of the molar mass M_{mol}, in g/mol, equals the numerical value of the atomic or molecular mass m in u. The atomic or molecular mass m, in atomic mass units u, is well approximated by the **atomic mass number** A.

$$1 \text{ u} = 1.661 \times 10^{-27} \text{ kg}$$

The **number density** of the sample is $\frac{N}{V}$.

Ideal-Gas Law

The **state variables** of an ideal gas are related by the ideal-gas law

$$pV = nRT \quad \text{or} \quad pV = Nk_BT$$

where $R = 8.31$ J/mol K is the universal gas constant and $k_B = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant.

p, V, and T *must* be in SI units of Pa, m^3, and K. For a gas in a sealed container, with constant n:

$$\frac{p_2V_2}{T_2} = \frac{p_1V_1}{T_1}$$

APPLICATIONS

Temperature scales

$$T_F = \frac{9}{5}T_C + 32° \qquad T_K = T_C + 273$$

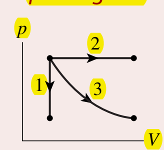
The Kelvin temperature scale is based on:

- Absolute zero at $T_0 = 0$ K
- The triple point of water at $T_3 = 273.16$ K

Three basic gas processes

1. **Isochoric,** or constant volume
2. **Isobaric,** or constant pressure
3. **Isothermal,** or constant temperature

pV diagram



SUMMARY

The goal of Chapter 17 has been to expand our understanding of energy and to develop the first law of thermodynamics as a general statement of energy conservation.

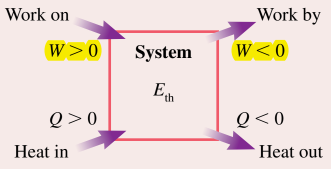
GENERAL PRINCIPLES

First Law of Thermodynamics

$\Delta E_{th} = W + Q$

The first law is a general statement of energy conservation.

Work W and heat Q depend on the process by which the system is changed.



The change in the system depends only on the total energy exchanged $W + Q$, not on the process.

Energy

Thermal energy E_{th} Microscopic energy of moving molecules and stretched molecular bonds. ΔE_{th} depends on the initial/final states but is independent of the process.

Work W Energy transferred to the system by forces in a mechanical interaction.

Heat Q Energy transferred to the system via atomic-level collisions when there is a temperature difference. A thermal interaction.

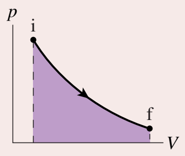
IMPORTANT CONCEPTS

The work done on a gas is

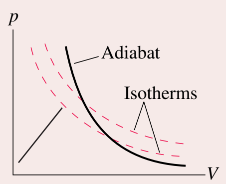
$$W = -\int_{V_i}^{V_f} p\,dV$$

540

$$= -(\text{area under the } pV \text{ curve})$$



An adiabatic process is one for which $Q = 0$. Gases move along an **adiabat** for which $pV^\gamma = \text{constant}$, where $\gamma = C_P/C_V$ is the **specific heat ratio.** An adiabatic process changes the temperature of the gas without heating it.



Calorimetry When two or more systems interact thermally, they come to a common final temperature determined by

$$Q_{net} = Q_1 + Q_2 + \ldots = 0$$

The heat of transformation L is the energy needed to cause 1 kg of substance to undergo a phase change

$$Q = \pm ML$$

The specific heat c of a substance is the energy needed to raise the temperature of 1 kg by 1 K.

$$Q = Mc\Delta T$$

The molar specific heat C is the energy needed to raise the temperature of 1 mol by 1 K.

$$Q = nC\Delta T$$

The molar specific heat of gases depends on the *process* by which the temperature is changed:

C_V = molar specific heat at **constant volume.**

C_P = molar specific heat at **constant pressure.**

$C_P = C_V + R$, where R is the universal gas constant.

SUMMARY OF BASIC GAS PROCESSES

Process	Definition	Stays constant	Work	Heat
Isochoric	$\Delta V = 0$	V and p/T	$W = 0$	$Q = nC_V\Delta T$
Isobaric	$\Delta p = 0$	p and V/T	$W = -p\Delta V$	$Q = nC_P\Delta T$
Isothermal	$\Delta T = 0$	T and pV	$W = -nRT\ln(V_f/V_i)$	$\Delta E_{th} = 0$
Adiabatic	$Q = 0$	pV^γ	$W = \Delta E_{th}$	$Q = 0$
All gas processes		Ideal-gas law	$pV = nRT$	
		First law	$\Delta E_{th} = W + Q = nC_V\Delta T$	

SUMMARY

The goal of Chapter 18 has been to understand the properties of a macroscopic system in terms of the microscopic behavior of its molecules.

GENERAL PRINCIPLES

Kinetic theory, the **micro/macro connection,** relates the macroscopic properties of a system to the motion and collisions of its atoms and molecules.

The Equipartition Theorem

Tells us how collisions distribute the energy in the system. The energy stored in each mode of the system (each **degree of freedom**) is $\frac{1}{2}Nk_BT$ or, in terms of moles, $\frac{1}{2}nRT$.

The Second Law of Thermodynamics

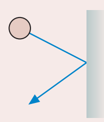
Tells us how collisions move a system toward equilibrium. The entropy of an isolated system can only increase or, in equilibrium, stay the same.

- Order turns into disorder and randomness.
- Systems run down.
- Heat energy is transferred spontaneously from the hotter to the colder system, never from colder to hotter.

IMPORTANT CONCEPTS

Pressure is due to the force of the molecules colliding with the walls.

$$p = \frac{1}{3}\frac{N}{V}m{v_{rms}}^2 = \frac{2}{3}\frac{N}{V}\epsilon_{avg}$$

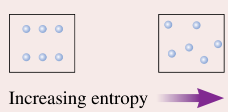


The average translational kinetic energy of a molecule is

$\epsilon_{avg} = \frac{3}{2}k_BT$. The temperature of the gas $T = \frac{2}{3k_B}\epsilon_{avg}$

measures the average translational kinetic energy.

Entropy measures the probability that a macroscopic state will occur or, equivalently, the amount of disorder in a system.

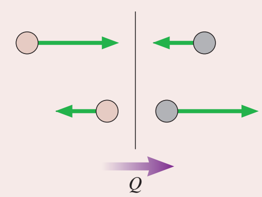


The thermal energy of a system is

E_{th} = translational kinetic energy + rotational kinetic energy + vibrational energy

- **Monatomic gas** $E_{th} = \frac{3}{2}Nk_BT = \frac{3}{2}nRT$
- **Diatomic gas** $E_{th} = \frac{5}{2}Nk_BT = \frac{5}{2}nRT$
- **Elemental solid** $E_{th} = 3Nk_BT = 3nRT$

Heat is energy transferred via collisions from more-energetic molecules on one side to less-energetic molecules on the other. Equilibrium is reached when $(\epsilon_1)_{avg} = (\epsilon_2)_{avg}$, which implies $T_{1f} = T_{2f}$.



APPLICATIONS

The root-mean-square speed v_{rms} is the square root of the average of the squares of the molecular speeds:

$$v_{rms} = \sqrt{(v^2)_{avg}}$$

For molecules of mass m at temperature T,

$$v_{rms} = \sqrt{\frac{3k_BT}{m}}$$

Molar specific heats can be predicted from the thermal energy because $\Delta E_{th} = nC\Delta T$.

- **Monatomic gas** $C_V = \frac{3}{2}R$
- **Diatomic gas** $C_V = \frac{5}{2}R$
- **Elemental solid** $C = 3R$