When more than one wave is present, the displacement of the medium is the sum of the displacements due to each individual wave. Informal statement: waves add up.

The superposition of 2 or more waves is called interference.

**Constructive interference:**
- These two waves are in phase.
- Their crests are aligned.
- Their superposition produces a wave with amplitude $2a$.

**Destructive interference:**
- These two waves are out of phase.
- The crests of one are aligned with the troughs of the other.
- Their superposition produces a wave with zero amplitude.
Interference: space and time

Is this a point of constructive or destructive interference?

What do we need to do to make the sound from these two speakers interfere constructively?

Two waves traveling in opposite direction interfere with each other. 
If the conditions are right, their interference generates a standing wave.
A standing wave does not propagate in space, it "stands" in place.
A standing wave has nodes and antinodes:

The amplitude of the oscillation $A$ changes with position: $A(x)$

$D_R = a \sin(kx - \omega t)$ The outer curve is the amplitude function $A(x) = 2\sin(kx)$

$D_L = a \sin(kx + \omega t)$ $k$ = wave number $= 2\pi/\lambda$. 

$$
\begin{align*}
D_R &= a \sin(kx - \omega t) \\
D_L &= a \sin(kx + \omega t)
\end{align*}
$$
standing waves on a string

...if the conditions are right

Fundamental frequency \( f_1 = \frac{v}{2L} \)

A standing wave can exist on a string only if its frequency is one of the values given by:

\[ \lambda_m = \frac{2L}{m} \quad f_m = m \frac{v}{2L} = mf_1, \quad m = 1, 2, 3, \ldots \]

Remember that \( v = \sqrt{\frac{T_s}{\mu_s}} = \sqrt{\frac{T_s L}{m_s}} \)

thus \( f_1 = \frac{1}{2} \sqrt{\frac{T_s L}{m_s}} = \frac{1}{2} \sqrt{\frac{T_s}{m_s L}} \)

The STRING family

The STRING family is named for the gut, wire, or nylon cords that are stretched over the instruments. Striking, bowing, plucking, or strumming the strings produces musical sounds that are amplified acoustically or electronically.

\[ f_1 = \frac{v}{2L} = \frac{1}{2} \sqrt{\frac{T_s}{m_s L}} \]
An open end of a pipe must be an antinode, a closed end a node.
The blue curves are displacement oscillations. Pressure makes opposite nodes and antinodes

<table>
<thead>
<tr>
<th>Open-open pipe</th>
<th>Closed-closed pipe</th>
<th>Open-closed pipe</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m = 1 )</td>
<td>( m = 1 )</td>
<td>( m = 1 )</td>
</tr>
<tr>
<td>( m = 2 )</td>
<td>( m = 2 )</td>
<td>( m = 3 )</td>
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<tr>
<td>( m = 3 )</td>
<td>( m = 3 )</td>
<td>( m = 5 )</td>
</tr>
</tbody>
</table>

Standing waves in a pipe

- \( \lambda_m = \frac{2L}{m} \)
- \( f_m = \frac{v}{2L} = mf_1 \)
- \( m = 1, 2, 3, 4, ... \)

Standing waves in pipe instruments

Wind Instruments
- Woodwinds
  - bassoon
  - flute
  - clarinet
  - oboe
  - pipe organ

Brass
- trumpet
- tuba
- French horn

- Most open pipes
- Open because conical
- Open but with "fictitious" open fundamental
- Warm up before tuning

\( f_1 = \frac{v}{2L} \)
\( f_1 = \frac{v}{4L} \)

\( v = v(T) \)
Beats are loud-soft-loud-soft modulations of intensity. They occur when two waves of slightly different frequency are superimposed.

The beats between waves of frequencies $f_1$ and $f_2$ have frequency $f_{\text{beat}} = f_1 - f_2$.

The medium oscillates rapidly at frequency $f_{\text{avg}}$.

The amplitude is slowly modulated as $2a \cos(\omega_{\text{mod}} t)$. 

$D$

-2a

0

2a

Loud Soft Loud Soft Loud Soft Loud

t