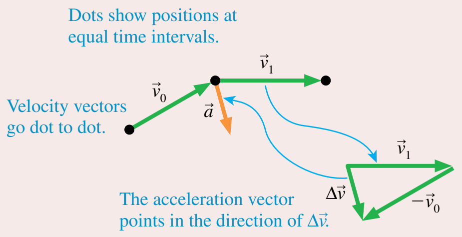
SUMMARY

The goal of Chapter 1 has been to introduce the fundamental concepts of motion.

GENERAL STRATEGY

Motion Diagrams

- Help visualize motion.
- Provide a tool for finding acceleration vectors.



► These are the average velocity and the average acceleration vectors.

Problem Solving

MODEL Make simplifying assumptions.

VISUALIZE Use:

- **Pictorial representation**
- **Physical representation**
- **Graphical representation**

SOLVE Use a **mathematical representation** to find numerical answers.

ASSESS Does the answer have the proper units? Does it make sense?

IMPORTANT CONCEPTS

The particle model represents a moving object as if all its mass were concentrated at a single point.

Position locates an object with respect to a chosen coordinate system. Change in position is called displacement.

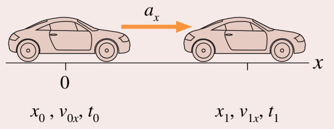
Velocity is the rate of change of the position vector $\vec{r}$.

Acceleration is the rate of change of the velocity vector $\vec{v}$. An object has an acceleration if it

- Changes speed and/or
- Changes direction.

Pictorial Representation

1. Sketch the situation.
2. Establish coordinates.
3. Define symbols.
4. List knowns.
5. Identify desired unknown.



Known

$x_0 = v_{0x} = t_0 = 0$

$a_x = 2\ \text{m/s}^2 \quad t_1 = 2\ \text{s}$

Find

x_1

APPLICATIONS

For **motion along a line:**

- Speeding up: $\vec{v}$ and $\vec{a}$ point in the same direction.
- Slowing down: $\vec{v}$ and $\vec{a}$ point in opposite directions.
- Constant speed: $\vec{a} = \vec{0}$.

Significant figures are reliably known digits. Three significant figures is the standard for this book. The number of significant figures for:

- **Multiplication, division, powers** is set by the value with the fewest significant figures.
- **Addition, subtraction** is set by the value with the smallest number of decimal places.

SUMMARY

The goal of Chapter 2 has been to learn how to solve problems about motion in a straight line.

GENERAL PRINCIPLES

Kinematics describes motion in terms of position, velocity, and acceleration.

General kinematic relationships are given **mathematically** by:

Instantaneous velocity $v_s = ds/dt$ = slope of position graph

Instantaneous acceleration $a_s = dv_s/dt$ = slope of velocity graph

Final position $s_f = s_i + \int_{t_i}^{t_f} v_s\,dt = s_i + \begin{cases}\text{area under the velocity curve}\\ \text{from } t_i \text{ to } t_f\end{cases}$

Final velocity $v_{fs} = v_{is} + \int_{t_i}^{t_f} a_s\,dt = v_{is} + \begin{cases}\text{area under the acceleration}\\ \text{curve from } t_i \text{ to } t_f\end{cases}$

The kinematic equations for **motion with constant acceleration:**

$$v_{fs} = v_{is} + a_s\Delta t$$

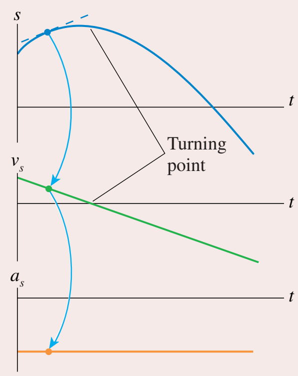
$$s_f = s_i + v_{is}\Delta t + \tfrac{1}{2}a_s(\Delta t)^2$$

$$v_{fs}^2 = v_{is}^2 + 2a_s\Delta s$$

IMPORTANT CONCEPTS

Position, velocity, and acceleration are related graphically.

- The slope of the position-versus-time graph is the value on the velocity graph.
- The slope of the velocity graph is the value on the acceleration graph.
- s is a maximum or minimum at a turning point, and $v_s = 0$.



Motion with constant acceleration is uniformly accelerated motion.

Uniform motion is motion with constant velocity and zero acceleration.

$$s_f = s_i + v_s\Delta t$$

APPLICATIONS

The sign of v_s indicates the direction of motion.

- $v_s > 0$ is motion to the right or up.
- $v_s < 0$ is motion to the left or down.

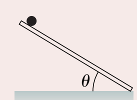
The sign of a_s indicates which way $\vec{a}$ points, *not* whether the object is speeding up or slowing down.

- $a_s > 0$ if $\vec{a}$ points to the right or up.
- $a_s < 0$ if $\vec{a}$ points to the left or down.
- The direction of $\vec{a}$ is found with a motion diagram.

An object is **speeding up** if and only if v_s and a_s have the same sign. An object is **slowing down** if and only if v_s and a_s have opposite signs.

Free fall is constant-acceleration motion with $a_y = -g = -9.80\ \text{m/s}^2$.

Motion on an inclined plane has $a_s = \pm g\sin\theta$. The sign depends on the direction of the tilt.

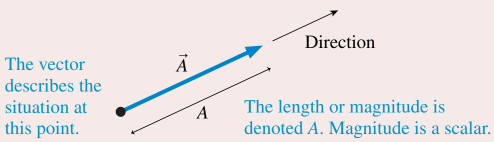


SUMMARY

The goal of Chapter 3 has been to learn how vectors are represented and used.

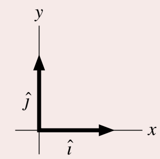
GENERAL PRINCIPLES

A vector is a quantity described by both a magnitude and a direction.



Unit Vectors

Unit vectors have magnitude 1 and no units. Unit vectors $\hat{\imath}$ and $\hat{\jmath}$ define the directions of the x- and y-axes.



USING VECTORS

Components

The component vectors are parallel to the x- and y-axes.

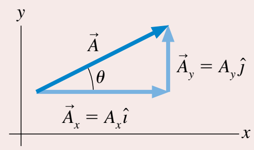
$$\vec{A} = \vec{A}_x + \vec{A}_y = A_x\hat{\imath} + A_y\hat{\jmath}$$

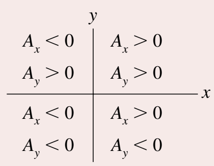
In the figure at the right, for example:

$$A_x = A\cos\theta \qquad A = \sqrt{A_x^2 + A_y^2}$$

$$A_y = A\sin\theta \qquad \theta = \tan^{-1}(A_y/A_x)$$

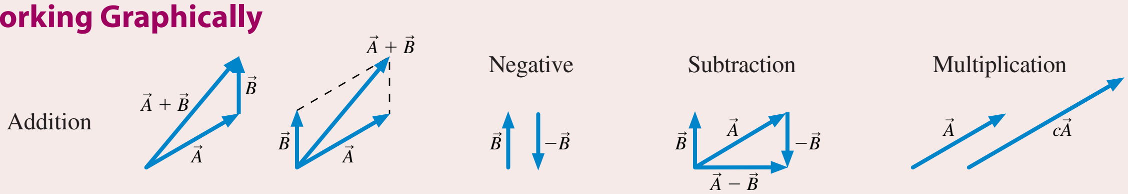
► Minus signs need to be included if the vector points down or left.





The components A_x and A_y are the magnitudes of the component vectors $\vec{A}_x$ and $\vec{A}_y$ *and* a plus or minus sign to show whether the component vector points toward the positive end or the negative end of the axis.

Working Graphically



Working Algebraically

Vector calculations are done component by component.

$$\vec{C} = 2\vec{A} + \vec{B} \quad \text{means} \quad \begin{cases} C_x = 2A_x + B_x \\ C_y = 2A_y + B_y \end{cases}$$

The magnitude of $\vec{C}$ is then $C = \sqrt{C_x^2 + C_y^2}$ and its direction is found using $\tan^{-1}$.

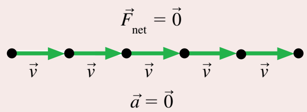
SUMMARY

The goal of Chapter 4 has been to learn how force and motion are connected.

GENERAL PRINCIPLES

Newton's First Law

An object at rest will remain at rest, or an object that is moving will continue to move in a straight line with constant velocity, if and only if the net force on the object is zero.



The first law tells us that no "cause" is needed for motion. Uniform motion is the "natural state" of an object.

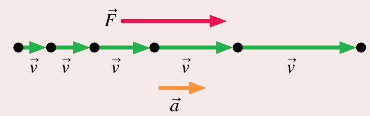
Newton's laws are valid only in inertial reference frames.

Newton's Second Law

An object with mass m will undergo acceleration

$$\vec{a} = \frac{1}{m}\vec{F}_{net}$$

where $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$ is the vector sum of all the individual forces acting on the object.



The second law tells us that a net force causes an object to accelerate. This is the connection

IMPORTANT CONCEPTS

Acceleration is the link to kinematics.

From a, find v and x.
From v and x, find a.

$\vec{a} = \vec{0}$ is the condition for equilibrium.

Static equilibrium if $\vec{v} = \vec{0}$.
Dynamic equilibrium if $\vec{v}$ = constant.

Equilibrium occurs if and only if $\vec{F}_{net} = \vec{0}$.

Mass is the resistance of an object to acceleration. It is an intrinsic property of an object.

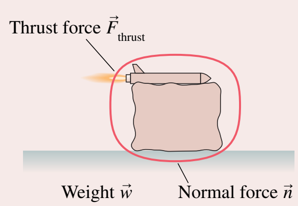
Force is a push or a pull on an object.

- Force is a vector, with a magnitude and a direction.
- Force requires an agent.
- Force is either a contact force or a long-range force.

KEY SKILLS

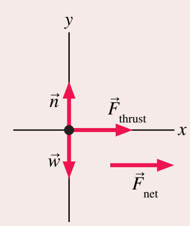
Identifying Forces

Forces are identified by locating the points where the environment touches the system. These are points where contact forces are exerted. In addition, objects with mass feel a long-range weight force.



Free-Body Diagrams

A free-body diagram represents the object as a particle at the origin of a coordinate system. Force vectors are drawn with their tails on the particle. The net force vector is drawn beside the diagram.



SUMMARY

The goal of Chapter 5 has been to learn how to solve problems about motion in a straight line.

GENERAL STRATEGY

All examples in this chapter follow a four-part strategy. You'll become a better problem solver if you adhere to it as you do the homework problems. The *Dynamics Worksheets* will help you structure your work in this way.

Equilibrium Problems

Object at rest or moving with constant velocity.

MODEL Make simplifying assumptions.

VISUALIZE

- **Physical representation:** Forces and free-body diagram
- **Pictorial representation:** Translate words to symbols.

SOLVE Use Newton's first law

$$\vec{F}_{net} = \sum_i \vec{F}_i = \vec{0}$$

"Read" the vectors from the free-body diagram.

ASSESS Is the result reasonable?

Go back and forth between representations as needed.

Dynamics Problems

Object accelerating.

MODEL Make simplifying assumptions.

VISUALIZE

- **Pictorial representation:** Sketch to define situation. Translate words to symbols.
- **Physical representation:** Forces and free-body diagram

SOLVE Use Newton's second law

$$\vec{F}_{net} = \sum_i \vec{F}_i = m\vec{a}$$

"Read" the vectors from the free-body diagram.
Use kinematics to find velocities and positions.

ASSESS Is the result reasonable?

IMPORTANT CONCEPTS

Specific information about three important forces:

Weight $\vec{w} = (mg, \text{downwards})$

Friction $\vec{f}_s = (0 \text{ to } \mu_s n, \text{direction as necessary to prevent motion})$

$\vec{f}_k = (\mu_k n, \text{direction opposite the motion})$

$\vec{f}_r = (\mu_r n, \text{direction opposite the motion})$

Drag $\vec{D} \approx (\frac{1}{4}Av^2, \text{direction opposite the motion})$

Newton's laws are vector expressions. You must write them out by components:

$$(F_{net})_x = \sum F_x = ma_x \text{ or } 0$$

$$(F_{net})_y = \sum F_y = ma_y \text{ or } 0$$

APPLICATIONS

Apparent weight is the magnitude of the contact force supporting an object. It is what a scale would read, and it is your sensation of weight. It equals your true weight $w = mg$ only when $a = 0$.

$$w_{app} = w\left(1 + \frac{a_y}{g}\right)$$

Terminal speed is $v_{term} \approx \sqrt{\frac{4mg}{A}}$

SUMMARY

The goal of Chapter 6 has been to learn to solve problems about motion in a plane.

GENERAL PRINCIPLES

Galilean Principle of Relativity

Newton's laws of motion are valid in all inertial reference frames.

Newton's Second Law

Expressed in x- and y-component form:

$$(F_{\text{net}})_x = \sum F_x = ma_x$$

$$(F_{\text{net}})_y = \sum F_y = ma_y$$

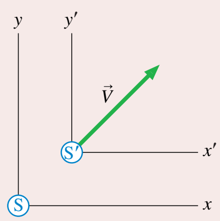
IMPORTANT CONCEPTS

Relative motion

Inertial reference frames move relative to each other with constant velocity $\vec{V}$. Measurements of position and velocity measured in frame S are related to measurements in frame S′ by the Galilean transformations

$$x' = x - V_x t \qquad v'_x = v_x - V_x$$

$$y' = y - V_y t \qquad v'_y = v_y - V_y$$



The instantaneous velocity

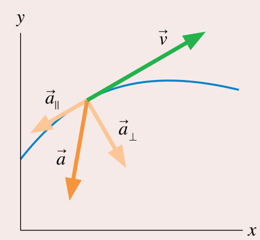
$$\vec{v} = d\vec{r}/dt,$$

is a vector tangent to the trajectory.

The instantaneous acceleration is

$$\vec{a} = d\vec{v}/dt$$

$\vec{a}_\parallel$, the component of $\vec{a}$ parallel to $\vec{v}$, is responsible for change of *speed*. $\vec{a}_\perp$, the component of $\vec{a}$ perpendicular to $\vec{v}$, is responsible for change of *direction*.



APPLICATIONS

Kinematics in two dimensions

If $\vec{a}$ is constant, then the x- and y-components of motion are independent of each other. For a particle that starts from initial position $\vec{r}_i$ and velocity $\vec{v}_i$, its position and velocity at a final point f are

$$x_f = x_i + v_{ix}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

$$y_f = y_i + v_{iy}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

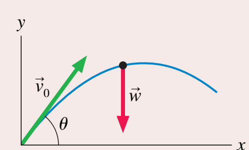
$$v_{fx} = v_{ix} + a_x\Delta t$$

$$v_{fy} = v_{iy} + a_y\Delta t$$

Projectile motion

Projectile motion occurs if the only force on the object is its weight.

- Uniform motion in the horizontal direction with $v_{0x} = v_0\cos\theta$.
- Free-fall motion in the vertical direction with $a_y = -g$ and $v_{0y} = v_0\sin\theta$.
- The combined motion is a parabola.
- The x and y kinematic equations have the *same* value for Δt.



SUMMARY

The goal of Chapter 7 has been to learn to solve problems about motion in a circle.

GENERAL PRINCIPLES

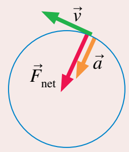
Newton's Second Law

Expressed in *rtz*-component form:

$$(F_{net})_r = \sum F_r = ma_r = \frac{mv^2}{r} = m\omega^2 r \qquad (F_{net})_t = \sum F_t = \begin{cases} 0 & \text{uniform motion} \\ ma_t & \text{nonuniform motion} \end{cases} \qquad (F_{net})_z = \sum F_z = 0$$

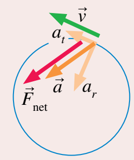
Uniform Circular Motion

- v is constant.
- $\vec{F}_{net}$ points toward the center of the circle.
- The **centripetal acceleration** $\vec{a}$ points toward the center of the circle. It changes the particle's direction but not its speed.



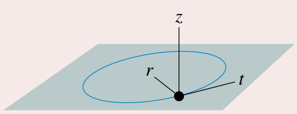
Nonuniform Circular Motion

- v changes.
- $\vec{a}$ is parallel to $\vec{F}_{net}$.
- The radial component a_r changes the particle's direction.
- The tangential component a_t changes the particle's speed.



IMPORTANT CONCEPTS

rtz-coordinates



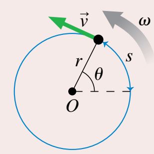
Angular position

$\theta = s/r$

Angular velocity

$\omega = d\theta/dt$

$v_t = \omega r$



APPLICATIONS

Circular motion kinematics

Period $T = \dfrac{2\pi r}{v} = \dfrac{2\pi}{\omega}$

Uniform circular motion

$v_t = \text{constant} \qquad \omega = \text{constant}$

$\theta_f = \theta_i + \omega \Delta t$

Nonuniform circular motion

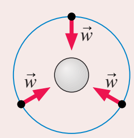
$$\theta_f = \theta_i + \omega_i \Delta t + \frac{a_t}{2r}(\Delta t)^2$$

$$\omega_f = \omega_i + \frac{a_t}{r}\Delta t$$

Orbits

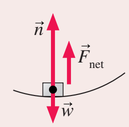
A circular orbit has radius r if

$$v = \sqrt{rg}$$



Apparent weight

Circular motion requires a net force pointing to the center. The apparent weight $w_{app} = n$ is usually not the same as the true weight w. n must be > 0 for the object to be in contact with a surface.



SUMMARY

The goal of Chapter 8 has been to learn to use Newton's third law to understand interacting systems.

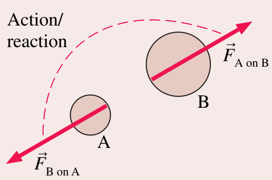
GENERAL PRINCIPLES

Newton's Third Law

Every force occurs as one member of an **action/reaction pair** of forces. The two members of an action/reaction pair:

- Act on two *different* objects.
- Are equal in magnitude but opposite in direction:

$$\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$$



Solving Interacting-System Problems

MODEL Choose the systems of interest.

VISUALIZE

Pictorial representation:
Sketch and define coordinates.
Identify acceleration constants.

Physical representation:
Draw a separate free-body diagram for each system.
Connect action/reaction pairs with dotted lines.

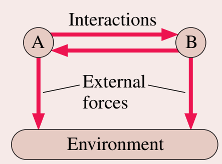
SOLVE Write Newton's second law for each system.
Include *all* forces acting *on* each system.
Use Newton's third law to equate the magnitudes of action/reaction pairs.
Include acceleration constraints and friction.

ASSESS Is the result reasonable?

IMPORTANT CONCEPTS

Interacting systems and the environment

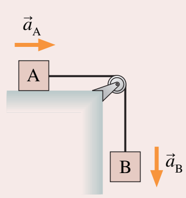
Two systems interact by exerting forces on each other.
Systems whose motion is not of interest form the environment.
The systems of interest interact with the environment, but those interactions can be considered external forces.



APPLICATIONS

Acceleration constraints

Objects that are constrained to move together must have accelerations of equal magnitude: $a_A = a_B$. This must be expressed in terms of components, such as $a_{Ax} = -a_{By}$.

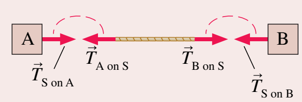


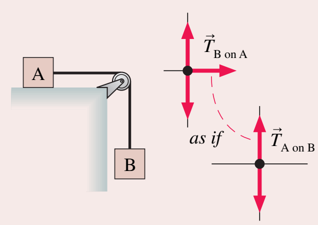
Strings and pulleys

The tension in a string or rope pulls in both directions. The tension is constant in a string if the string is:

- Massless, or
- In equilibrium

Systems connected by massless strings passing over massless, frictionless pulleys act *as if* they interact via an action/reaction pair of forces.





SUMMARY

The goal of Chapter 9 has been to introduce the ideas of impulse, momentum, and angular momentum and to learn a new problem-solving strategy based on conservation laws.

GENERAL PRINCIPLES

Law of Conservation of Momentum

The total momentum $\vec{P} = \vec{p}_1 + \vec{p}_2 + \cdots$ of an isolated system is a constant. Thus

$$\vec{P}_f = \vec{P}_i$$

Law of Conservation of Angular Momentum

The angular momentum L of a particle or system of particles in circular motion does not change unless there is a net tangential force. Thus

$$L_f = L_i$$

Solving Momentum Conservation Problems

MODEL Choose an isolated system or a system that is isolated during at least part of the problem.

VISUALIZE Draw a pictorial representation of the system before and after the interaction.

SOLVE Write the law of conservation of momentum in terms of vector components

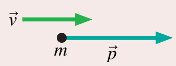
$$(p_{fx})_1 + (p_{fx})_2 + \cdots = (p_{ix})_1 + (p_{ix})_2 + \cdots$$

$$(p_{fy})_1 + (p_{fy})_2 + \cdots = (p_{iy})_1 + (p_{iy})_2 + \cdots$$

ASSESS Is the result reasonable?

IMPORTANT CONCEPTS

Momentum $\vec{p} = m\vec{v}$

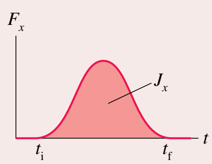


Impulse $J_x = \int_{t_i}^{t_f} F_x(t)\,dt =$ area under force curve

Impulse and momentum are related by the **impulse-momentum theorem**

$$\Delta p_x = J_x$$

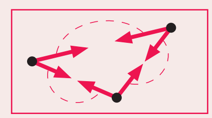
This is an alternative statement of Newton's second law.



Angular momentum $L = mrv_t$

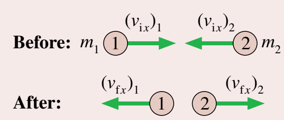
System A group of interacting particles.

Isolated system A system on which there are no external forces or the net external force is zero.



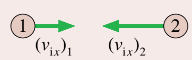
Before-and-after pictorial representation

- Define the system.
- Use two drawings to show the system *before* and *after* the interaction.
- List known information and identify what you are trying to find.

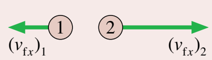


APPLICATIONS

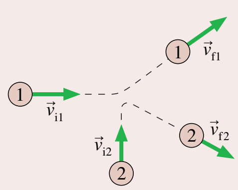
Collisions Two or more particles come together. In a perfectly inelastic collision, they stick together and move with a common final velocity.



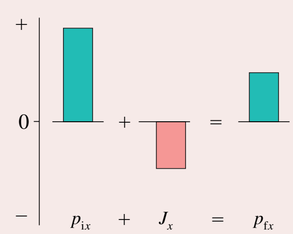
Explosions Two or more particles move away from each other.



Two dimensions No new ideas, but both the x- and y-components of P must be conserved, giving two simultaneous equations.



Momentum bar charts display the impulse-momentum theorem $p_{fx} = p_{ix} + J_x$ in graphical form.



SUMMARY

The goal of Chapter 10 has been to introduce the ideas of kinetic and potential energy and to learn a new problem-solving strategy based on conservation of energy.

GENERAL PRINCIPLES

Law of Conservation of Mechanical Energy

If there are no friction or other energy-loss processes (to be explored more thoroughly in Chapter 11), then the mechanical energy $E_{mech} = K + U$ of a system is conserved. Thus

$$K_f + U_f = K_i + U_i$$

- K is the sum of the kinetic energies of all particles.
- U is the sum of all potential energies.

Solving Energy Conservation Problems

MODEL Choose a system without friction or other losses of mechanical energy.

VISUALIZE Draw a before-and-after pictorial representation.

SOLVE Use the law of conservation of energy

$$K_f + U_f = K_i + U_i$$

ASSESS Is the result reasonable?

IMPORTANT CONCEPTS

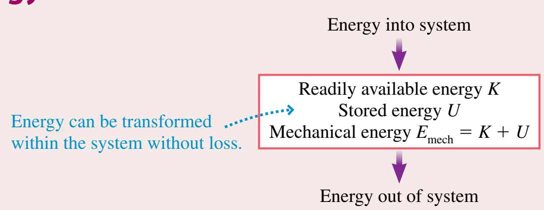
Kinetic energy is an energy of motion

$$K = \frac{1}{2}mv^2$$

Potential energy is an energy of position

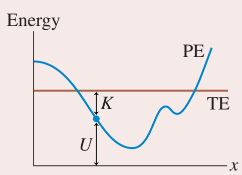
- **Gravitational:** $U_g = mgy$
- **Elastic:** $U_s = \frac{1}{2}k(\Delta s)^2$

Basic Energy Model



Energy diagrams

These diagrams show the potential energy curve PE and the total mechanical energy line TE.



- The distance from the axis to the curve is PE.
- The distance from the curve to the TE line is KE.
- A point where the TE line crosses the PE curve is a **turning point.**
- Minima in the PE curve are points of **stable equilibrium.** Maxima are points of **unstable equilibrium.**

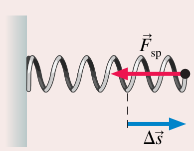
APPLICATIONS

Hooke's law

The restoring force of an ideal spring is

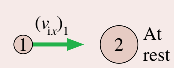
$$(F_{sp})_s = -k\Delta s$$

where k is the spring constant and $\Delta s = s - s_e$ is the displacement from equilibrium.



Perfectly elastic collisions

Both mechanical energy and momentum are conserved.



$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1 \qquad (v_{fx})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1$$

If ball 2 is moving, transform to a reference frame in which ball 2 is at rest.

SUMMARY

The goal of Chapter 11 has been to develop a more complete understanding of energy and its conservation.

GENERAL PRINCIPLES

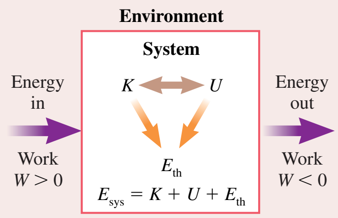
Basic Energy Model

- Energy is *transferred* to or from the system by work.
- Energy is *transformed* within the system.

Two versions of the energy equation are

$$\Delta E_{sys} = \Delta K + \Delta U + \Delta E_{th} = W_{ext}$$

$$K_f + U_f + \Delta E_{th} = K_i + U_i + W_{ext}$$



Solving Energy Problems

MODEL Identify objects in the system.

VISUALIZE Draw a before-and-after pictorial representation and an energy bar chart.

SOLVE Use the energy equation

$$K_f + U_f + \Delta E_{th} = K_i + U_i + W_{ext}$$

ASSESS Is the result reasonable?

Law of Conservation of Energy

- **Isolated system:** $W_{ext} = 0$. The total energy $E_{sys} = E_{mech} + E_{th}$ is conserved. $\Delta E_{sys} = 0$
- **Isolated, nondissipative system:** $W_{ext} = 0$ and $W_{diss} = 0$. The mechanical energy E_{mech} is conserved.

$$\Delta E_{mech} = 0 \text{ or } K_f + U_f = K_i + U_i$$

IMPORTANT CONCEPTS

The **work-kinetic energy theorem** is

$$\Delta K = W_{net} = W_c + W_{diss} + W_{ext}$$

Using $W_c = -\Delta U$ for conservative forces and $W_{diss} = -\Delta E_{th}$ for dissipative forces, this becomes the energy equation.

The **work** done by a force on a particle as it moves from s_i to s_f is

$$W = \int_{s_i}^{s_f} F_s\, ds = \text{area under the force curve}$$

$$= \vec{F} \cdot \Delta\vec{r} \text{ if } \vec{F} \text{ is a constant force}$$

Conservative forces are forces for which the work is independent of the path followed. The work done by a conservative force can be represented as a **potential energy**

$$\Delta U = U_f - U_i = -W_c(\text{i} \rightarrow \text{f})$$

A conservative force is found from the potential energy by

$$F = -dU/ds = \text{negative of the slope of the PE curve}$$

Dissipative forces transform **macroscopic energy** into thermal energy, which is the **microscopic energy** of the atoms and molecules.

$$\Delta E_{th} = -W_{diss}$$

APPLICATIONS

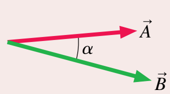
Power is the rate at which energy is transferred or transformed:

$$P = \frac{dE_{sys}}{dt}$$

For a particle moving with velocity $\vec{v}$, the power delivered to the particle by force $\vec{F}$ is $P = \vec{F} \cdot \vec{v} = Fv\cos\theta$.

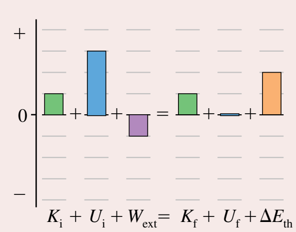
Dot product

$$\vec{A} \cdot \vec{B} = AB\cos\alpha = A_xB_x + A_yB_y$$



Energy bar charts display the energy equation

$K_f + U_f + \Delta E_{th} = K_i + U_i + W_{ext}$ in graphical form.



SUMMARY

The goal of Chapter 12 has been to use Newton's theory of gravity to understand the motion of satellites and planets.

GENERAL PRINCIPLES

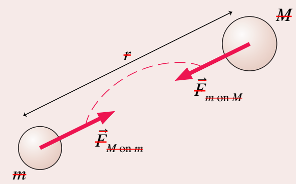
Newton's Theory of Gravity

1. Two objects with masses M and m a distance r apart exert attractive **gravitational forces** on each other of magnitude

$$F_{M \text{ on } m} = F_{m \text{ on } M} = \frac{GMm}{r^2}$$

where the **gravitational constant** is $G = 6.67 \times 10^{-11}\ \text{N m}^2/\text{kg}^2$.

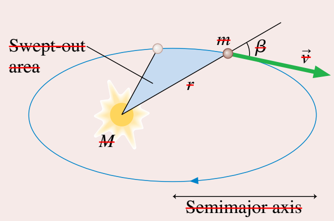
2. Gravitational mass and inertial mass are equivalent.
3. Newton's three laws of motion apply to satellites, planets, and stars.



IMPORTANT CONCEPTS

Orbital motion of a planet (or satellite) is described by **Kepler's laws:**

1. Orbits are ellipses with the sun (or planet) at one focus.
2. A line between the sun and the planet sweeps out equal areas during equal intervals of time.
3. The square of the planet's period T is proportional to the cube of the orbit's semimajor axis.



Circular orbits are a special case of an ellipse. For a circular orbit around a mass M,

$$v = \sqrt{\frac{GM}{r}} \quad \text{and} \quad T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

Conservation of angular momentum

The angular momentum $L = mrv\sin\beta$ remains constant throughout the orbit. Kepler's second law is a consequence of this law.

Orbital energetics

A satellite's mechanical energy $E_{mech} = K + U_g$ is conserved, where the gravitational potential energy is

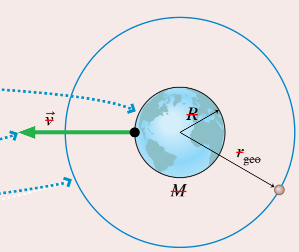
$$U_g = -\frac{GMm}{r}$$

For circular orbits, $K = -\frac{1}{2}U_g$ and $E_{mech} = \frac{1}{2}U_g$. Negative total energy is characteristic of a **bound system.**

APPLICATIONS

For a planet of mass M and radius R,

- The acceleration due to gravity on the surface is $g_{surface} = \dfrac{GM}{R^2}$
- The **escape speed** is $v_{escape} = \sqrt{\dfrac{2GM}{R}}$
- The radius of a **geosynchronous orbit** is $r_{geo} = \left(\dfrac{GM}{4\pi^2}T^2\right)^{1/3}$



SUMMARY

The goal of Chapter 13 has been to understand the physics of rotating objects.

GENERAL PRINCIPLES

Rotational Dynamics

Every point on a **rigid body** rotating about a fixed axis has the same angular velocity ω and angular acceleration α.

Newton's second law for rotational motion is

$$\alpha = \frac{\tau_{net}}{I}$$

Use rotational kinematics to find angles and angular velocities.

Conservation Laws

Energy is conserved for an isolated system.

- Pure rotation $E = K_{rot} + U_g = \frac{1}{2}I\omega^2 + Mgy_{cm}$
- Rolling $E = K_{rot} + K_{cm} + U_g = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv_{cm}{}^2 + Mgy_{cm}$

Angular momentum is conserved if $\vec{\tau}_{net} = \vec{0}$.

- Particle $\vec{L} = m\vec{r} \times \vec{p}$
- Rigid body rotating about axis of symmetry $\vec{L} = I\vec{\omega}$

IMPORTANT CONCEPTS

Angular velocity

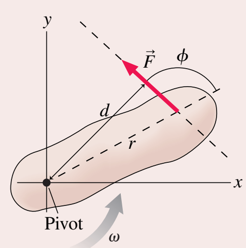
$$\omega = \frac{d\theta}{dt}$$

Angular acceleration is the rotational equivalent of acceleration

$$\alpha = \frac{d\omega}{dt}$$

Torque is the rotational equivalent of force

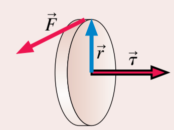
$$\tau = rF\sin\phi = rF_t = dF$$



Vector description of rotation

Torque $\vec{\tau} = \vec{r} \times \vec{F}$

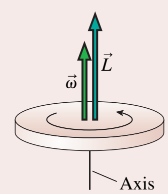
Angular velocity $\vec{\omega}$ points along the rotation axis in the direction of the right-hand rule.



For a rigid body rotating about an axis of symmetry, the angular momentum is $\vec{L} = I\vec{\omega}$.

Newton's second law is

$$\frac{d\vec{L}}{dt} = \vec{\tau}_{net}$$



A system of particles on which there is no net force undergoes unconstrained rotation about the center of mass

$$x_{cm} = \frac{1}{M}\int x\,dm \qquad y_{cm} = \frac{1}{M}\int y\,dm$$

The gravitational torque on a body can be found by treating the body as a particle with all the mass M concentrated at the center of mass.

The moment of inertia

$$I = \int r^2\,dm$$

is the rotational equivalent of mass. The moment of inertia depends on how the mass is distributed around the axis. If I_{cm} is known, the I about a parallel axis distance d away is given by the **parallel-axis theorem** $I = I_{cm} + Md^2$.

APPLICATIONS

Rotational kinematics

$$\omega_f = \omega_i + \alpha\Delta t$$

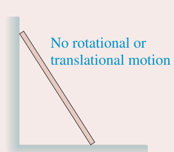
$$\theta_f = \theta_i + \omega_i\Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\,\Delta\theta$$

$$v_t = r\omega \qquad a_t = r\alpha$$

Rigid-body equilibrium

An object is in total equilibrium only if both $\vec{F}_{net} = \vec{0}$ and $\vec{\tau}_{net} = \vec{0}$.

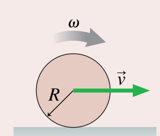


Rolling motion

For an object that rolls without slipping

$$v_{cm} = R\omega$$

$$K = K_{rot} + K_{cm}$$



SUMMARY

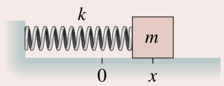
The goal of Chapter 14 has been to understand systems that oscillate with simple harmonic motion.

GENERAL PRINCIPLES

Dynamics

SHM occurs when a linear restoring force acts to return a system to an equilibrium position.

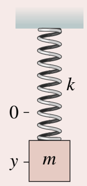
Horizontal spring



$(F_{net})_x = -kx$

Vertical spring

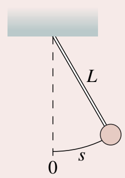
The origin is at the equilibrium position $\Delta L = mg/k$.



$(F_{net})_y = -ky$

$\omega = \sqrt{\dfrac{k}{m}} \qquad T = 2\pi\sqrt{\dfrac{m}{k}}$

Pendulum

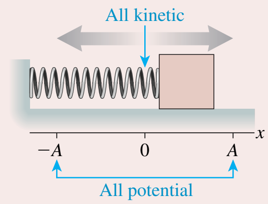


$(F_{net})_t = -\left(\dfrac{mg}{L}\right)s$

$\omega = \sqrt{\dfrac{g}{L}} \qquad T = 2\pi\sqrt{\dfrac{L}{g}}$

Energy

If there is **no friction** or dissipation, kinetic and potential energy are alternately transformed into each other, but the total mechanical energy $E = K + U$ is conserved.

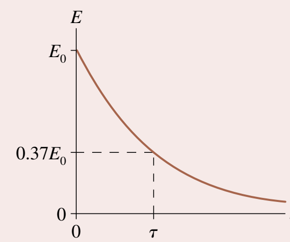


$$E = \frac{1}{2}mv_x^{\,2} + \frac{1}{2}kx^2$$
$$= \frac{1}{2}m(v_{max})^2$$
$$= \frac{1}{2}kA^2$$

In a **damped system,** the energy decays exponentially

$$E = E_0 e^{-t/\tau}$$

where τ is the time constant.



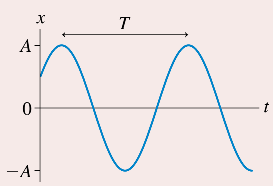
IMPORTANT CONCEPTS

Simple harmonic motion (SHM) is a sinusoidal oscillation with period T and amplitude A.

Frequency $f = \dfrac{1}{T}$

Angular frequency

$$\omega = 2\pi f = \frac{2\pi}{T}$$



Position $x(t) = A\cos(\omega t + \phi_0)$

$$= A\cos\left(\frac{2\pi t}{T} + \phi_0\right)$$

Velocity $v_x(t) = -v_{max}\sin(\omega t + \phi_0)$ with maximum speed $v_{max} = \omega A$

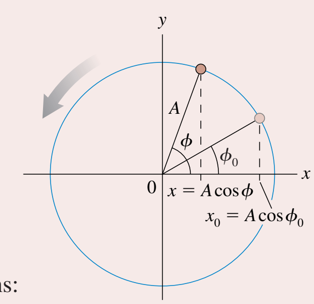
Acceleration $a_x = -\omega^2 x$

SHM is the projection onto the x-axis of **uniform circular motion.**

$\phi = \omega t + \phi_0$ is the phase

The position at time t is

$$x(t) = A\cos\phi$$
$$= A\cos(\omega t + \phi_0)$$



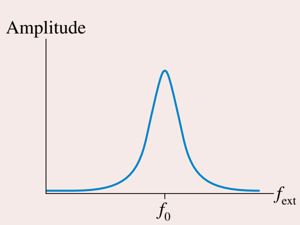
The phase constant ϕ_0 determines the initial conditions:

$x_0 = A\cos\phi_0 \qquad v_{0x} = -\omega A\sin\phi_0$

APPLICATIONS

Resonance

When a system is driven by a periodic external force, it responds with a large-amplitude oscillation if $f_{ext} \approx f_0$ where f_0 is the system's natural oscillation frequency, or **resonant frequency.**

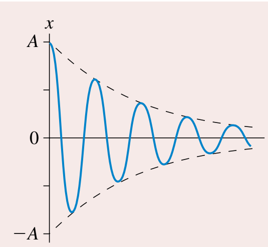


Damping

If there is a drag force $\vec{D} = -b\vec{v}$, where b is the damping constant, then (for lightly damped systems)

$$x(t) = Ae^{-bt/2m}\cos(\omega t + \phi_0)$$

The time constant for energy loss is $\tau = m/b$.



SUMMARY

The goal of Chapter 15 has been to understand macroscopic systems that flow or deform.

GENERAL PRINCIPLES

Fluid Statics

Gases

- Freely moving particles
- Compressible
- Pressure primarily thermal
- Pressure constant in a laboratory-size container

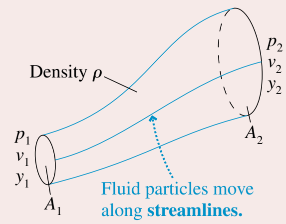
Liquids

- Loosely bound particles
- Incompressible
- Pressure primarily gravitational
- Hydrostatic pressure at depth d is $p = p_0 + \rho g d$

Fluid Dynamics

Ideal-fluid model

- Incompressible
- Smooth, laminar flow
- Nonviscous
- Irrotational



Equation of continuity

$$v_1 A_1 = v_2 A_2$$

Bernoulli's equation

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

Bernoulli's equation is a statement of energy conservation.

IMPORTANT CONCEPTS

Density $\rho = m/V$, where m is mass and V is volume.

Pressure $p = F/A$, where F is the magnitude of the fluid force and A is the area on which the force acts.

- Exists at all points in a fluid
- Pushes equally in all directions
- Constant along a horizontal line
- Gauge pressure $p_g = p - 1$ atm

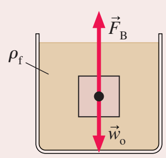
APPLICATIONS

Buoyancy is the upward force of a fluid on an object.

Archimedes' principle

The magnitude of the buoyant force equals the weight of the fluid displaced by the object.

Sink	$\rho_{avg} > \rho_f$	$F_B < w_o$
Rise to surface	$\rho_{avg} < \rho_f$	$F_B > w_o$
Neutrally buoyant	$\rho_{avg} = \rho_f$	$F_B = w_o$

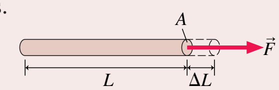


Elasticity describes the deformation of solids and liquids under stress.

Linear stretch and compression:

$$(F/A) = Y(\Delta L/L)$$

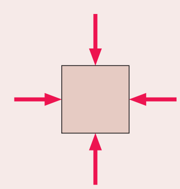
Tensile stress; Young's modulus; Strain



Volume compression:

$$p = -B(\Delta V/V)$$

Bulk modulus; Volume strain



SUMMARY

The goal of Chapter 16 has been to learn the characteristics of macroscopic systems.

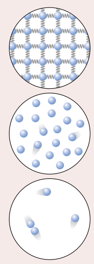
GENERAL PRINCIPLES

Three Phases of Matter

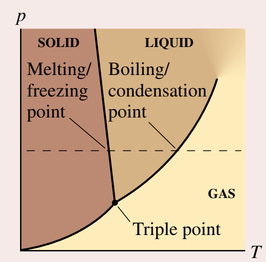
Solid Rigid, definite shape. Nearly incompressible.

Liquid Molecules loosely held together by molecular bonds, but able to move around. Nearly incompressible.

Gas Molecules move freely through space. Compressible.



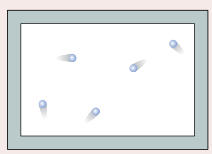
The different phases exist for different conditions of temperature T and pressure p. The boundaries separating the regions of a phase diagram are lines of phase equilibrium. Any amounts of the two phases can coexist in equilibrium. The **triple point** is the one value of temperature and pressure at which all three phases can coexist in equilibrium.



IMPORTANT CONCEPTS

Ideal-Gas Model

- Atoms and molecules are small, hard spheres that travel freely through space except for occasional collisions with each other or the walls.
- The molecules have a distribution of speeds.
- The model is valid when the density is low and the temperature well above the condensation point.



Ideal-Gas Law

The **state variables** of an ideal gas are related by the ideal-gas law

$$pV = nRT \quad \text{or} \quad pV = Nk_BT$$

where $R = 8.31$ J/mol K is the universal gas constant and $k_B = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant.

p, V, and T *must* be in SI units of Pa, m^3, and K. For a gas in a sealed container, with constant n:

$$\frac{p_2V_2}{T_2} = \frac{p_1V_1}{T_1}$$

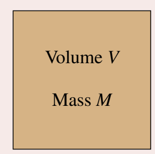
Counting atoms and moles

A macroscopic sample of matter consists of N atoms (or molecules), each of mass m (the **atomic** or **molecular mass**):

$$N = \frac{M}{m}$$

Alternatively, we can state that the sample consists of n **moles**

$$n = \frac{N}{N_A} \quad \text{or} \quad \frac{M(\text{in grams})}{M_{mol}}$$



$N_A = 6.02 \times 10^{23}\ \text{mol}^{-1}$ is **Avogadro's number.**

The numerical value of the molar mass M_{mol}, in g/mol, equals the numerical value of the atomic or molecular mass m in u. The atomic or molecular mass m, in atomic mass units u, is well approximated by the **atomic mass number** A.

$$1\ \text{u} = 1.661 \times 10^{-27}\ \text{kg}$$

The **number density** of the sample is $\frac{N}{V}$.

APPLICATIONS

Temperature scales

$$T_F = \frac{9}{5}T_C + 32° \qquad T_K = T_C + 273$$

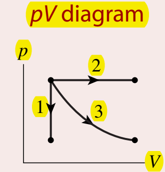
The Kelvin temperature scale is based on:

- Absolute zero at $T_0 = 0$ K
- The triple point of water at $T_3 = 273.16$ K

Three basic gas processes

1. **Isochoric,** or constant volume
2. **Isobaric,** or constant pressure
3. **Isothermal,** or constant temperature

pV diagram



SUMMARY

The goal of Chapter 17 has been to expand our understanding of energy and to develop the first law of thermodynamics as a general statement of energy conservation.

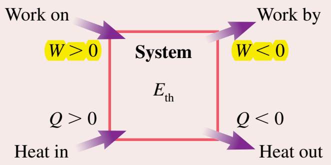
GENERAL PRINCIPLES

First Law of Thermodynamics

$\Delta E_{th} = W + Q$

The first law is a general statement of energy conservation.

Work W and heat Q depend on the process by which the system is changed.



The change in the system depends only on the total energy exchanged $W + Q$, not on the process.

Energy

Thermal energy E_{th} Microscopic energy of moving molecules and stretched molecular bonds. ΔE_{th} depends on the initial/final states but is independent of the process.

Work W Energy transferred to the system by forces in a mechanical interaction.

Heat Q Energy transferred to the system via atomic-level collisions when there is a temperature difference. A thermal interaction.

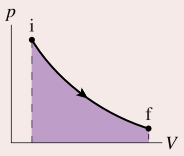
IMPORTANT CONCEPTS

The work done on a gas is

$$W = -\int_{V_i}^{V_f} p\,dV$$

540

$$= -(\text{area under the } pV \text{ curve})$$



The heat of transformation L is the energy needed to cause 1 kg of substance to undergo a phase change

$$Q = \pm ML$$

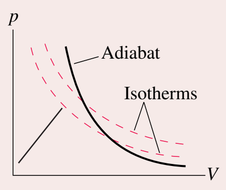
The specific heat c of a substance is the energy needed to raise the temperature of 1 kg by 1 K.

$$Q = Mc\Delta T$$

The molar specific heat C is the energy needed to raise the temperature of 1 mol by 1 K.

$$Q = nC\Delta T$$

An adiabatic process is one for which $Q = 0$. Gases move along an **adiabat** for which $pV^\gamma = \text{constant}$, where $\gamma = C_P/C_V$ is the **specific heat ratio.** An adiabatic process changes the temperature of the gas without heating it.



The molar specific heat of gases depends on the *process* by which the temperature is changed:

C_V = molar specific heat at **constant volume.**

C_P = molar specific heat at **constant pressure.**

$C_P = C_V + R$, where R is the universal gas constant.

Calorimetry When two or more systems interact thermally, they come to a common final temperature determined by

$$Q_{net} = Q_1 + Q_2 + \ldots = 0$$

SUMMARY OF BASIC GAS PROCESSES

Process	Definition	Stays constant	Work	Heat
Isochoric	$\Delta V = 0$	V and p/T	$W = 0$	$Q = nC_V\Delta T$
Isobaric	$\Delta p = 0$	p and V/T	$W = -p\Delta V$	$Q = nC_P\Delta T$
Isothermal	$\Delta T = 0$	T and pV	$W = -nRT\ln(V_f/V_i)$	$\Delta E_{th} = 0$
Adiabatic	$Q = 0$	pV^γ	$W = \Delta E_{th}$	$Q = 0$
All gas processes		Ideal-gas law	$pV = nRT$	
		First law	$\Delta E_{th} = W + Q = nC_V\Delta T$	

SUMMARY

The goal of Chapter 18 has been to understand the properties of a macroscopic system in terms of the microscopic behavior of its molecules.

GENERAL PRINCIPLES

Kinetic theory, the **micro/macro connection,** relates the macroscopic properties of a system to the motion and collisions of its atoms and molecules.

The Equipartition Theorem

Tells us how collisions distribute the energy in the system. The energy stored in each mode of the system (each **degree of freedom**) is $\frac{1}{2}Nk_BT$ or, in terms of moles, $\frac{1}{2}nRT$.

The Second Law of Thermodynamics

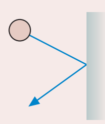
Tells us how collisions move a system toward equilibrium. The entropy of an isolated system can only increase or, in equilibrium, stay the same.

- Order turns into disorder and randomness.
- Systems run down.
- Heat energy is transferred spontaneously from the hotter to the colder system, never from colder to hotter.

IMPORTANT CONCEPTS

Pressure is due to the force of the molecules colliding with the walls.

$$p = \frac{1}{3}\frac{N}{V}mv_{rms}^2 = \frac{2}{3}\frac{N}{V}\epsilon_{avg}$$

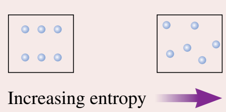


The average translational kinetic energy of a molecule is

$\epsilon_{avg} = \frac{3}{2}k_BT$. The temperature of the gas $T = \frac{2}{3k_B}\epsilon_{avg}$

measures the average translational kinetic energy.

Entropy measures the probability that a macroscopic state will occur or, equivalently, the amount of disorder in a system.

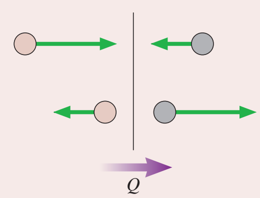


The thermal energy of a system is

E_{th} = translational kinetic energy + rotational kinetic energy + vibrational energy

- **Monatomic gas** $E_{th} = \frac{3}{2}Nk_BT = \frac{3}{2}nRT$
- **Diatomic gas** $E_{th} = \frac{5}{2}Nk_BT = \frac{5}{2}nRT$
- **Elemental solid** $E_{th} = 3Nk_BT = 3nRT$

Heat is energy transferred via collisions from more-energetic molecules on one side to less-energetic molecules on the other. Equilibrium is reached when $(\epsilon_1)_{avg} = (\epsilon_2)_{avg}$, which implies $T_{1f} = T_{2f}$.



APPLICATIONS

The root-mean-square speed v_{rms} is the square root of the average of the squares of the molecular speeds:

$$v_{rms} = \sqrt{(v^2)_{avg}}$$

For molecules of mass m at temperature T,

$$v_{rms} = \sqrt{\frac{3k_BT}{m}}$$

Molar specific heats can be predicted from the thermal energy because $\Delta E_{th} = nC\Delta T$.

- **Monatomic gas** $C_V = \frac{3}{2}R$
- **Diatomic gas** $C_V = \frac{5}{2}R$
- **Elemental solid** $C = 3R$

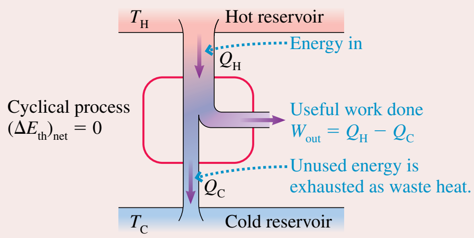
SUMMARY

The goal of Chapter 19 has been to investigate the physical principles that govern the operation of heat engines and refrigerators.

GENERAL PRINCIPLES

Heat Engines

Devices which transform heat into work. They require two energy reservoirs at different temperatures.



Thermal efficiency

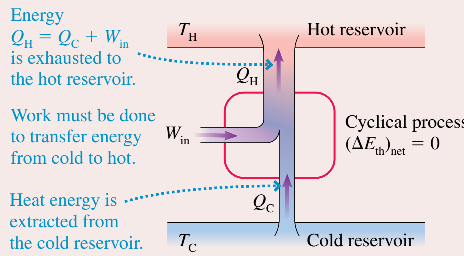
$$\eta = \frac{W_{out}}{Q_H} = \frac{\text{what you get}}{\text{what you pay}}$$

Second law limit:

$$\eta \le 1 - \frac{T_C}{T_H}$$

Refrigerators

Devices which use work to transfer heat from a colder object to a hotter object.



Coefficient of performance

$$K = \frac{Q_C}{W_{in}} = \frac{\text{what you get}}{\text{what you pay}}$$

Second law limit:

$$K \le \frac{T_C}{T_H - T_C}$$

IMPORTANT CONCEPTS

A perfectly reversible engine (a **Carnot engine**) can be operated as either a heat engine or a refrigerator between the same two energy reservoirs by reversing the cycle and with no other changes.

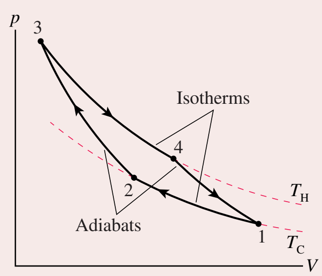
- A **Carnot heat engine** has the maximum possible thermal efficiency of any heat engine operating between T_H and T_C.

$$\eta_{Carnot} = 1 - \frac{T_C}{T_H}$$

- A **Carnot refrigerator** has the maximum possible coefficient of performance of any refrigerator operating between T_H and T_C.

$$K_{Carnot} = \frac{T_C}{T_H - T_C}$$

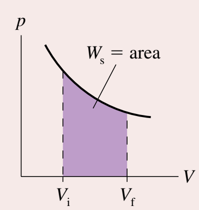
The Carnot cycle for a gas engine consists of two isothermal processes and two adiabatic processes.



An energy reservoir is a part of the environment so large in comparison to the system that its temperature doesn't change as the system extracts heat energy from or exhausts heat energy to the reservoir. All heat engines and refrigerators operate between two energy reservoirs at different temperatures T_H and T_C.

The **work** W_s done *by* the system has the opposite sign to the work done *on* the system.

W_s = area under pV curve



APPLICATIONS

To analyze a heat engine or refrigerator:

MODEL Identify each process in the cycle.

VISUALIZE Draw the pV diagram of the cycle.

SOLVE There are several steps:

- Determine p, V, and T at the beginning and end of each process.
- Calculate ΔE_{th}, W_s, and Q for each process.
- Determine W_{in} or W_{out}, Q_H, and Q_C.
- Calculate $\eta = W_{out}/Q_H$ or $K = Q_C/W_{in}$.

ASSESS Verify $(\Delta E_{th})_{net} = 0$. Check signs.

SUMMARY

The goal of Chapter 20 has been to learn the basic properties of traveling waves.

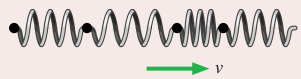
GENERAL PRINCIPLES

The Wave Model

This model is based on the idea of a traveling wave, which is an organized disturbance traveling at a well-defined **wave speed** v.

- In transverse waves the particles of the medium move perpendicular to the direction in which the wave travels.

- In longitudinal waves the particles of the medium move parallel to the direction in which the wave travels.



A wave transfers **energy,** but no material or substance is transferred outward from the source.

Three basic types of waves:

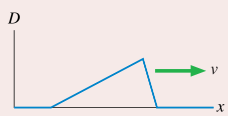
- Mechanical waves travel through a material medium such as water or air.
- Electromagnetic waves require no material medium and can travel through a vacuum.
- Matter waves describe the wavelike characteristics of atomic-level particles.

For mechanical waves, the speed of the wave is a property of the medium. Speed does not depend on the size or shape of the wave.

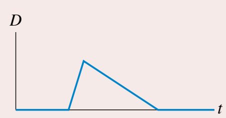
IMPORTANT CONCEPTS

The **displacement** D of a wave is a function of both position (where) and time (when).

- A snapshot graph shows the wave's displacement as a function of position at a single instant of time.



- A history graph shows the wave's displacement as a function of time at a single point in space.



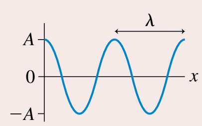
A wave traveling in the positive x-direction with speed v must be a function of the form $D(x - vt)$.

A wave traveling in the negative x-direction with speed v must be a function of the form $D(x + vt)$.

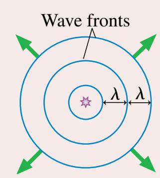
Sinusoidal waves are periodic in both time (period T) and space (wavelength λ).

$$D(x, t) = A\sin[2\pi(x/\lambda - t/T) + \phi_0]$$
$$= A\sin(kx - \omega t + \phi_0)$$

where A is the **amplitude,** $k = 2\pi/\lambda$ is the **wave number,** $\omega = 2\pi f = 2\pi/T$ is the **angular frequency,** and ϕ_0 is the **phase constant** that describes initial conditions.



One-dimensional waves



Two- and three-dimensional waves

APPLICATIONS

Wave speeds for some specific waves:

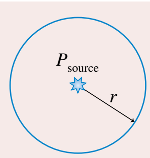
- **String** (transverse): $v = \sqrt{T_s/\mu}$
- **Sound** (longitudinal): $v = 343$ m/s in 20°C air
- **Light** (transverse): $v = c/n$, where $c = 3.00 \times 10^8$ m/s is the speed of light in a vacuum and n is the material's **index of refraction.**

The wave intensity is the power-to-area ratio

$$I = P/A$$

For a circular or spherical wave

$$I = P_{source}/4\pi r^2$$



The Doppler effect occurs when a wave source and detector are moving with respect to each other: the frequency detected differs from the frequency f_0 emitted.

Approaching source

$$f_+ = \frac{f_0}{1 - v_s/v}$$

Observer approaching a source

$$f_+ = (1 + v_o/v)f_0$$

Receding source

$$f_- = \frac{f_0}{1 + v_s/v}$$

Observer receding from a source

$$f_- = (1 - v_o/v)f_0$$

The Doppler effect for light uses a result derived from the theory of relativity.

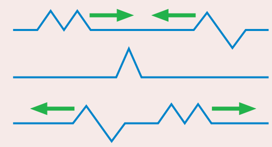
SUMMARY

The goal of Chapter 21 has been to understand and use the idea of superposition.

GENERAL PRINCIPLES

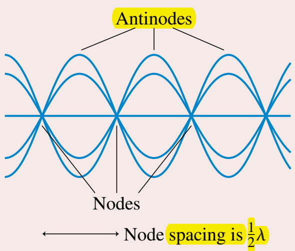
Principle of Superposition

The displacement of a medium when more than one wave is present is the sum of the displacements due to each individual wave.



IMPORTANT CONCEPTS

Standing waves are due to the superposition of two traveling waves moving in opposite directions.



The amplitude at position x is

$$A(x) = 2a \sin kx$$

where a is the amplitude of each wave.

The boundary conditions determine which standing wave frequencies and wavelengths are allowed.

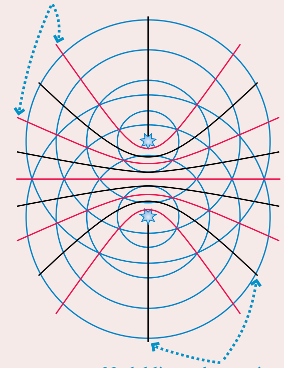
Interference

In general, the superposition of two or more waves into a single wave is called interference.

Maximum constructive interference occurs where crests are aligned with crests and troughs with troughs. These waves are in phase. The maximum displacement is $A = 2a$.

Perfect destructive interference occurs where crests are aligned with troughs. These waves are out of phase. The amplitude is $A = 0$.

Interference depends on the phase difference $\Delta\phi$ between the two waves.



$$\text{Constructive: } \Delta\phi = 2\pi\frac{\Delta r}{\lambda} + \Delta\phi_0 = 2m\pi$$

$$\text{Destructive: } \Delta\phi = 2\pi\frac{\Delta r}{\lambda} + \Delta\phi_0 = 2(m + \tfrac{1}{2})\pi$$

Δr is the path-length difference of the two waves and $\Delta\phi_0$ is any phase difference between the sources. For identical sources (in phase, $\Delta\phi_0 = 0$):

Interference is constructive if the path-length difference $\Delta r = m\lambda$.

Interference is destructive if the path-length difference $\Delta r = (m + \frac{1}{2})\lambda$.

The amplitude at a point where the phase difference is $\Delta\phi$ is $A = \left|2a\cos\left(\frac{\Delta\phi}{2}\right)\right|$

APPLICATIONS

Boundary conditions

Strings, electromagnetic waves, and sound waves in closed-closed tubes must have nodes at both ends.

$$\lambda_m = \frac{2L}{m} \qquad f_m = m\frac{v}{2L} = mf_1$$

where $m = 1, 2, 3, \ldots$

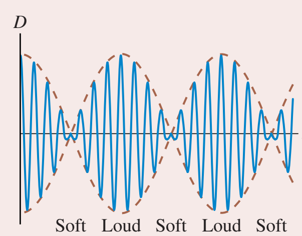
The frequencies and wavelengths are the same for a sound wave in an open-open tube, which has antinodes at both ends.

A sound wave in an open-closed tube must have a node at the closed end but an antinode at the open end. This leads to

$$\lambda_m = \frac{4L}{m} \qquad f_m = m\frac{v}{4L} = mf_1$$

where $m = 1, 3, 5, 7, \ldots$

Beats (loud-soft-loud-soft modulations of intensity) occur when two waves of slightly different frequency are superimposed.



The beat frequency between waves of frequencies f_1 and f_2 is

$$f_{beat} = f_1 - f_2$$