The Movement of Electrons in a Magnetic Field and the Ratio of an Electron's Charge to its Mass

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Purpose:

1. To observe the behaviour of a beam of electrons in a uniform magnetic field ($\vec{B}$).
2. To determine the charge to mass (e/m) ratio of an electron.

Introduction:

This experiment was performed to observe the motion of electrons in a uniform B and to calculate the e/m ratio. By varying the voltage the electrons are accelerated through, and therefore their speed, and the current through the coils, and therefore the magnetic field, the e/m ratio can be found. This can be done through the manipulation of three equations: ① $eV = \frac{mv^2}{2}$ ② $F = evB = \frac{mv^2}{r}$ ③ $B = \left(\frac{4}{5}\right)^{\frac{3}{2}} \frac{\mu_0 N I}{R}$

rearranging for v in equation 1, $v = \sqrt{\frac{2eV}{m}}$

rearranging equation 2 for e/m, $\frac{e}{m} = \frac{v}{Br}$

substituting for v and B, $\frac{e}{m} = \frac{\sqrt{\frac{2eV}{m}}}{\left(\frac{4}{5}\right)^{\frac{3}{2}} \frac{\mu_0 N I}{R} \cdot r}$

squaring both sides, $\frac{e^{\cancel{2}}}{m^{\cancel{2}}} = \left(\frac{2\cancel{e}V}{\cancel{m}}\right)\left(\frac{5^3}{4^3} \cdot \frac{R^2}{\mu_0^2 N^2 I^2 r^2}\right)$

substituting N=130, R=0.15m
μ =4π*10^-7(T *m /A or N*s^2/C^2)

$$\frac{e}{m} = \frac{(2)(5^3)(R^2)}{(4^3)(N^2)(\mu_0^2)} \cdot \frac{V}{I^2 r^2}$$

$$= \frac{(2)(125)(.15)^2}{(64)(130)^2(4\pi\times10^{-7})^2} \cdot \frac{V}{I^2 r^2} \rightarrow 3.29\times10^6 \frac{V}{I^2 r^2}$$

The values for N and R are used because they apply to the equipment used to create the B in this experiment. R is the radius of the coils and N is the number of loops in the coils. V is the potential difference used to accelerate the electrons to their exiting speed, v. The radius of the path the electrons follow is denoted by r. μ_0 is the constant for the permeability of free space. The units for the constant created by this derivation are C^4/kg^2, and were found by simplifying all the values involved.

$$\frac{R^2}{\mu_0^2} = \frac{m^2}{\left(\frac{N\cdot s^2}{C^2}\right)^2} = \frac{\cancel{m^2}}{\frac{kg^2\cdot \cancel{s^4}\cdot \cancel{m^2}}{\frac{\cancel{s^4}}{C^4}}} = \frac{\frac{1}{kg^2}}{\frac{1}{C^4}} = \frac{C^4}{kg^2}$$

By graphing the relationship of V to I^2 for a constant radius, the slope of this line can be substituted for the variables V and I ^2. r can be measured using a scale projector, and then substituted in to the derived equation along with the slope of the graph to yield a value for e/m.

References:

Young and Freedman, 10th ed., pp. 873-879, pp. 913-915.

Equipment:

Fine beam tube
Helmholtz coils
Box enclosure with two metal terminal boxes.
Scale projector
Regulated power supply and leads
12V DC power supply for coil
1 digital ammeter/multimeter
1 Hewlett-Packard 427A Voltmeter

Experimental Procedure:

Preliminary:

Increase the voltage on the filament until the electron beam can be seen and note the characteristics of the beam. Do not exceed 250 V, the fine beam tube's safety limit. Increase the current through the coils and note the changes in the electron beam. Slowly and carefully rotate the fine beam tube through 90° and note the changes in the path of the electron beam.

Exercise 1:

Set the current on the Helmholtz coils to approximately 1.3 A and note the changes in the path of the electron beam for increases and decreases in the voltage applied to the filament. Set the voltage of the filament to approximately 180 V and note the changes in the path of the electron beam for increases and decreases of the current through the coils.

Exercise 2:

Create a circular beam path with a radius of about 4.0 cm by setting the current to 1.7 A and adjusting the voltage. Do not exceed the maximum voltage for the filament while creating the circle, instead adjust the current to compensate. Record the radius of the circle and the values of the voltage and current for about ten readings decreasing from ~1.7 A to 1.3 A. The values of V and I should be recorded in table format for the constant radius created.

Observations:

Preliminary:

The electron beam appeared at 164.6 V on the external voltmeter. The beam is pointed directly up and diverged slightly. As the current through the coils was increased to 1.5249 A, the radius of the circle the beam forms decreased to ~2.5cm. As the tube was rotated through 90°, the circle became a helix. The more the tube is rotated, the more the helix stretches out.

Preliminary Questions:

a) If all of the electron's $\vec{v}$ was perpendicular to the $\vec{B}$, then all the $\vec{v}$ contributed to the force ($\vec{F}$) that is produced by the interaction of $\vec{v}$ and $\vec{B}$ and the path becomes circular.

b)i) The part of the $\vec{v}$ that is parallel to the $\vec{B}$ is unaffected by $\vec{B}$, so $\vec{v}$ remains constant for that component.

ii) The part of the electron's $\vec{v}$ that was perpendicular to the $\vec{B}$ was affected by $\vec{B}$ and this created a force that caused the path of the beam to curl.

The combination of these two paths created a helix. As the tube was rotated, more of the $\vec{v}$ is in the direction of $\vec{B}$, so the helix became more stretched.

Exercise 1: The effect of the voltage of electrons on the radius of their path

I = 1.3059 A

As the voltage of the filament, and therefore the exiting speed of the electrons was increased, the radius of the electrons path increased. When the voltage was decreased, the radius decreased as well.

V= 180.1 V

As the current through the coils, and therefore the $\vec{B}$, was increased, the radius of the electron beam's path decreased. When the current through the coils was decreased, the radius of the circle increased.

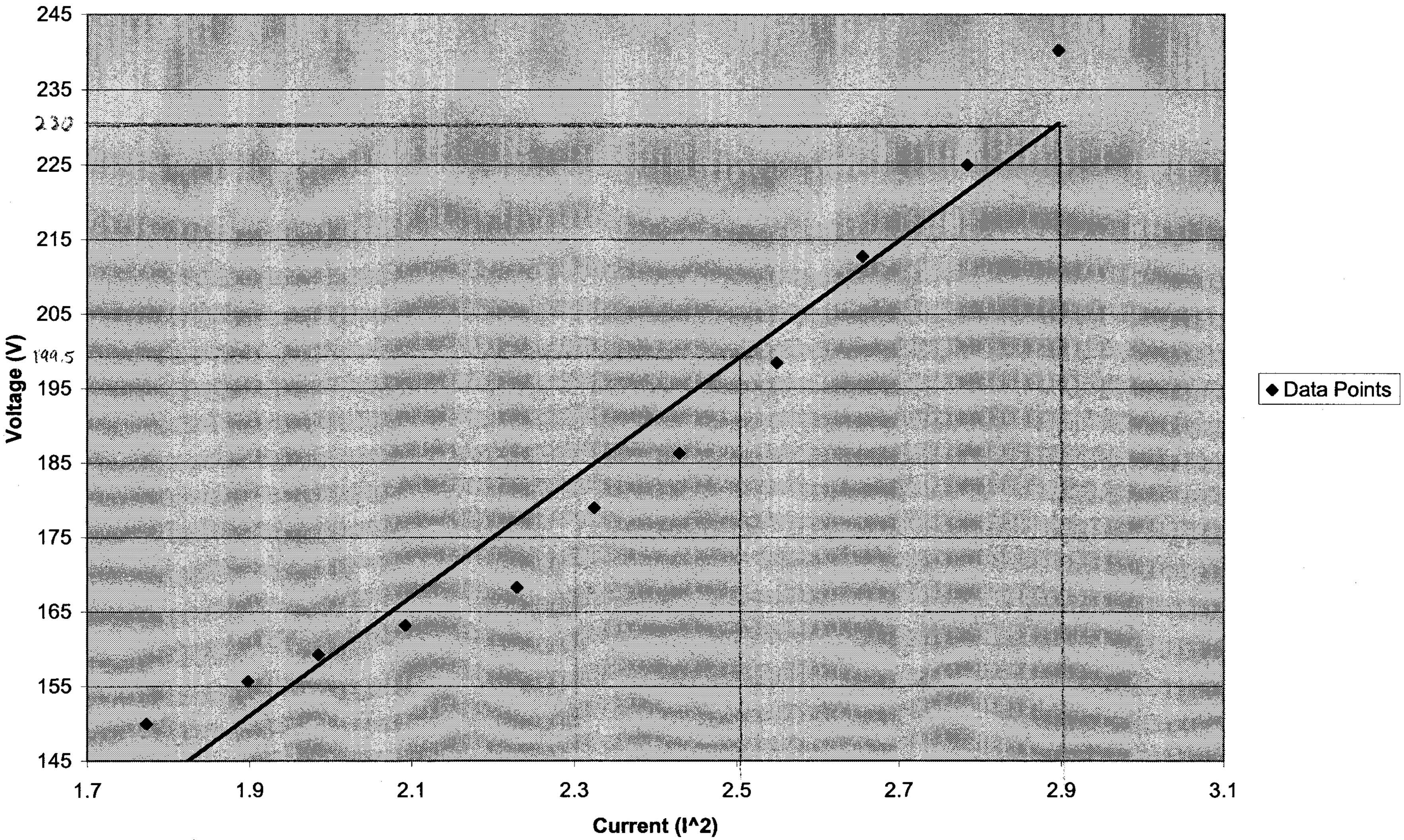
Exercise 2:

Table 1: The Effect of Current (I) on Voltage (V) with Respect to a Constant Radius (r)

Radius ~3.7cm.

Trial	Current (A)	Voltage (V)
1	1.7017	240.2
2	1.6685	225.0
3	1.6293	212.8
4	1.5967	198.5
5	1.5586	186.3
6	1.5247	179.0
7	1.4931	168.3
8	1.4465	163.2
9	1.4091	159.3
10	1.3777	155.7
11	1.3314	150.0

The Effect of Current(I^2) on the Voltage of Electrons(V) with Respect to a Constant Radius(r)



$$\text{Slope }(S) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{230 - 199.5}{2.9 - 2.5} = 76.25 \left(\frac{V}{A^2}\right) \rightarrow 76 \frac{V}{A^2}$$

Discussion:

From the preliminary observations and the equation $\vec{F} = q\vec{v} \times \vec{B}$, and the right hand rule for positive moving charges it yields, it can be deduced that the direction of $\vec{B}$ is into the apparatus, away from the viewer. The only correct perpendicular that would have created a $\vec{F}$ right when the $\vec{v}$ was up would be out for a positive charge and in for a negative charge. Since electrons were used, the direction of $\vec{B}$ must have been into the apparatus. The electrons formed a circular path because $\vec{F}$ was always perpendicular to $\vec{v}$ because of its definition according to the equation, and $\vec{F}$ was constantly changing direction as $\vec{v}$ changed direction.

As the tube was rotated, the circular path of the electrons became a helix. This helix had fewer turns as the tube was turned. This was because as the tube rotated, the $\vec{v}$ of the electrons was changing direction. As the $\vec{v}$ changed direction, the components that make up the $\vec{v}$ were altered. The component perpendicular to $\vec{B}$ decreased and the component parallel to $\vec{B}$ increased. The component that stayed perpendicular to $\vec{B}$ contributed the bent path of the electrons, and the component that was parallel to $\vec{B}$ and unaffected by it, contributed to the constant velocity of the beam. The combination of these two factors created the helix path, where the electron beam's path still circled, it just continued at a constant speed while it circled.

From exercise 1, the relationship between the radius and the current through the coils, and the relationship between the radius and the voltage on the filament was determined. As the voltage increased, the speed increased as well. Through a rearrangement of equation 2, it can be seen that radius was proportional to r and this relationship was upheld by the observations made. As the current through the coils was increased, the radius decreased. This relationship was noted through equation 2 and the $\vec{F} = q\vec{v} \times \vec{B}$ equation. As the current increased, the field increased (equation 2), the force increased and therefore the radius decreased because of the stronger force (F = mv^2/r).

From exercise 2, the values listed in the observations, and the graph attached was plotted from that table of values. The slope was calculated from the graph to be 76 V/A^2. Using this value and the radius in m, the value of the e/m ratio was found to be 1.83 * 10^11C/kg. The actual value of e/m is 1.76*10^11C/kg. The percent difference for this rendition of the experiment was 3.98%. The calculations were as follows:

$$\frac{e}{m} = (3.29\times10^{6})(76)\left(\frac{1}{.037}\right)^2$$

$$= 1.83\times10^{11}\ \frac{C}{kg}$$

$$\text{units} = \frac{C^4}{kg^2} \cdot \frac{\frac{J}{C}}{\frac{C^2}{s^2} \cdot m^2}$$

$$= \frac{C^4}{kg^2} \cdot \frac{kg\, m^2}{C\, s^2} \cdot \frac{s^2}{C^2 \cdot m^2}$$

$$= \frac{C}{kg}$$

$$\text{percent difference} = \frac{\text{actual} - \text{theoretical}}{\text{theoretical}} \times 100\%$$

$$= \frac{1.83\times10^{11} - 1.76\times10^{11}}{1.76\times10^{11}} \times 100\%$$

$$= 0.0397727 \times 100\%$$

$$= 3.98\%$$

Conclusions:

The force created by the movement of electrons follows the left hand rule created by the $\vec{F} = q\vec{v} \times \vec{B}$ formula. This was how the circular path was created. When some of the electron's speed was not perpendicular to the $\vec{B}$, then it did not contribute to the $\vec{F}$ and the electrons moves at a constant $\vec{v}$ with respect to that direction. A combination of both kinds of components creates a helix.

The radius of an electron's path was proportional to its exiting speed from the filament and therefore the voltage used to accelerate it. It was also inversely proportional to the current, and therefore the $\vec{B}$ that is created.

The ratio of e/m for this experiment was 1.83 * 10^11 C/kg. The percent difference between this value and the accepted value for e/m was 3.96%. This error could have been caused by; the width of the beam causing inaccurate measurement of the radius, the values on the voltmeter and ammeter not being correct to the right amount of digits, and round off error in the calculation of the constant and the slope.

Please include this sheet with your report for marking.

Physics-1010 Formal Lab Report: "The Motion of Electrons in a Magnetic Field"

Name: Valerie Plaus ID: 0185498

Lab Time: Tues. 8:30 Mark: 26 out of 30

Required Element	Out of:	Mark Assigned:
Neatness, presentation, clarity, grammer	5 marks	3
Unit Maintenance	2 marks	2
Data Accuracy	2 marks	2
Sources of Error	3 marks	3
Graphing	5 marks	4
Purpose/Intro/Equipment References/Procedures	5 marks	5
Observations	3 marks	3
Calculations	2 marks	2
Conclusions	2 marks	2

Comments: Include title page. Make sure written neat and well aligned (no arrows). Don't switch between pen and pencil on graph.