PHYS 551 Lecture #29

Title: The Free Electron Gas in a Magnetic Field

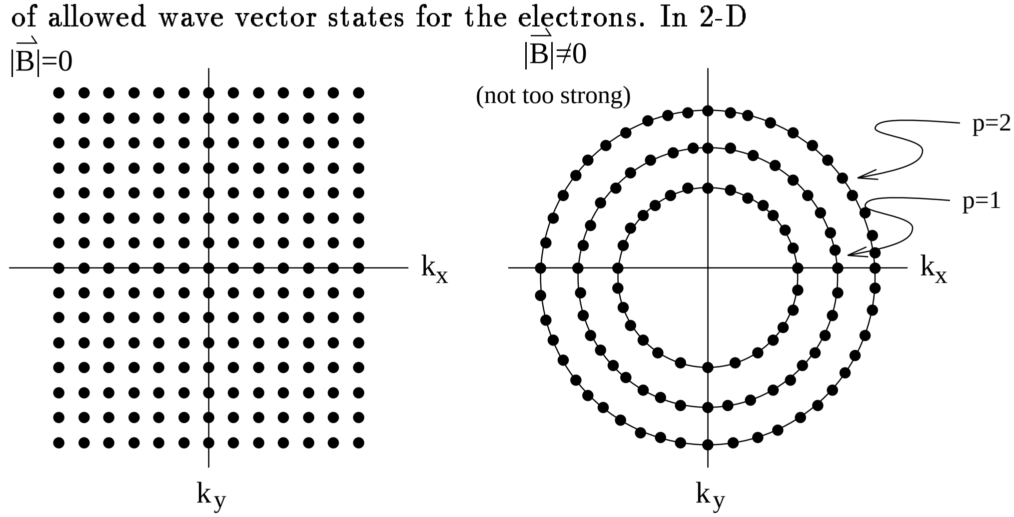
Obviously the application of a magnetic field will provide a Lorentz force perpendicular to both the electron velocity and the magnetic field. Since no work is done, the electron must follow an equipotential contour on the energy surface. This would be exactly correct if the electron were a spinless classical particle. Hence the electron must have a wave solution to the Schrödinger equation.

$$\vec{p} = \hbar\vec{k} + q\vec{A}/c \quad \text{(a Lorentz Gauge) for an electron}$$

$$\frac{1}{2m}\left(\frac{\hbar}{i}\nabla - \frac{e}{c}\vec{A}\right)^2\psi = E\psi \qquad \vec{B} = \nabla\times\vec{A} \qquad \vec{A} = -\frac{B}{2}y\,\hat{\imath} + \frac{B}{2}x\,\hat{\jmath}$$

$$\omega_c = \frac{eB}{cm} \equiv \text{cyclotron frequency}$$

If $k_z = 0$ and $\vec{B} \parallel \hat{z}$, then $E = \hbar\omega_c(p+\frac{1}{2})$ $\quad p = 0,\ 1,\ 2,\ \ldots$ which is identical to that for a harmonic oscillator. Notice that this has a profound effect on the distribution of allowed wave vector states for the electrons. In 2-D



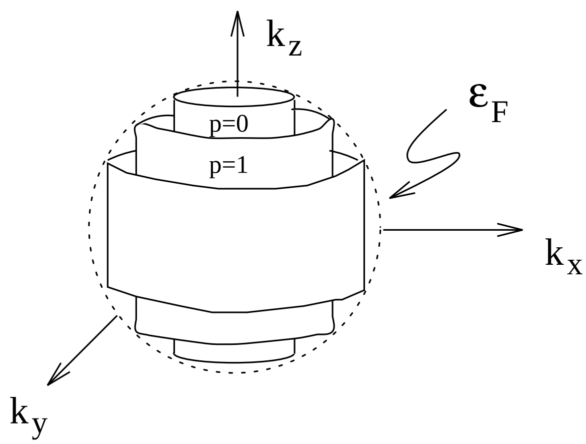
Since the volume per state is fixed, the number of states per p level (or Landau) level is proportional to the area of each ring. Ignoring spin,

$$\begin{aligned} \text{Area} &= 2\pi|k_p|\Delta k_p \\ \text{Number} &= 2\pi|k_p|\Delta k_p\frac{1}{\frac{(2\pi)^2}{L^2}} \\ \text{Since} \quad E_p &= \hbar\omega_c(p+\frac{1}{2}) = \frac{\hbar^2k_p^2}{2m} \end{aligned}$$

$$\Delta E_p = \hbar\omega_c = \frac{\hbar^2}{m} k_p \Delta k_p, \qquad A = L^2, \qquad \frac{\hbar\omega_c m}{\hbar^2} = k_p \Delta k_p$$

$$\text{or} \qquad N = 2\pi \frac{\hbar\omega_c m}{\hbar^2} \frac{A}{4\pi^2} = \frac{eA}{2\pi\hbar c} B = \text{constant}$$

In 3-D $\rightarrow$ $E = \frac{\hbar^2 k_z^2}{2m} + \hbar\omega_c(p + \frac{1}{2})$ and the Landau levels are in the form of cylinders.



Notice the new forms for N and E:

$$N_{3D} = \int_0^\infty f(\epsilon) D(\epsilon) d\epsilon = \frac{V}{2\pi^2} \frac{m\omega_c}{\hbar} \sum_{p=0}^{\infty} \int_{-\infty}^{\infty} \frac{dk_z}{(e^{(\varepsilon-\mu)/k_B T} + 1)}$$

and

$$U_{3D} = \int_0^\infty \epsilon f(\epsilon) D(\epsilon) d\epsilon = \frac{V}{2\pi^2} \frac{m\omega_c}{\hbar} \sum_{p=0}^{\infty} \int_{-\infty}^{\infty} E(k_z, p) \frac{dk_z}{(e^{(\varepsilon-\mu)/k_B T} + 1)}$$

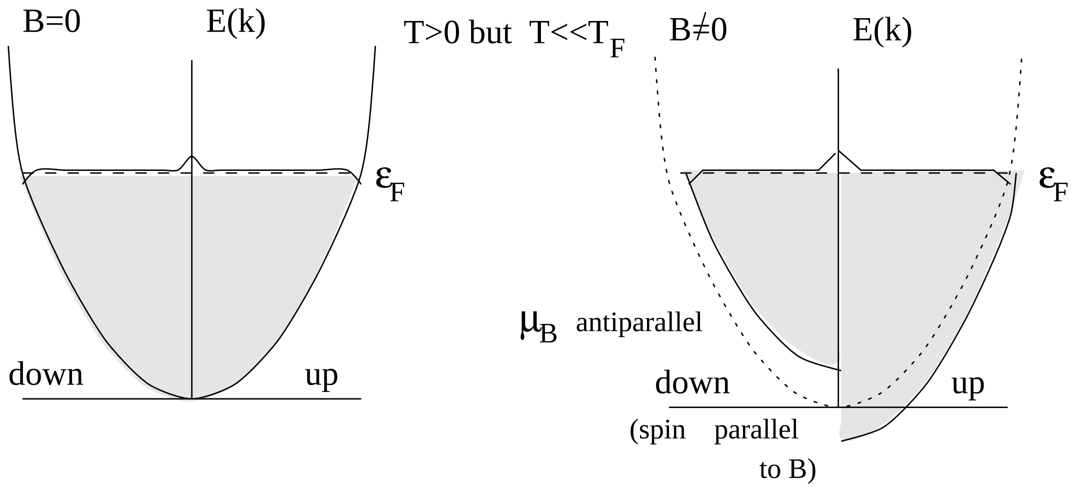
To find $\mu(T = 0, B)$, we use the top expression. Notice the peculiar progression that occurs as B is smoothly increased. Since (2-D) $\hbar^2 k_p^2/2m = \hbar\omega_c(p + \frac{1}{2})$ or $k_p \propto B^{1/2}$, as B increases the energy levels become spaced further apart. This tends to increase the Fermi energy. At the same time, the number of allowed states for a particular Landau level is also increasing. This tends to lower ϵ_F. The actual progression of ϵ_F is rather complex.

Pauli susceptibility:

There are complications which are also apparent. Electrons have spin (up or down). An electron has a net magnetic moment of $|\mu_B| \equiv$ Bohr magnetron $= |e\hbar/2m| = 9.3 \times 10^{-24}$ joule/tesla (SI units). The application of a magnetic field will produce a net energy difference between the two spin types.

$$U = -\vec{\mu}_B \cdot \vec{B} \begin{pmatrix} +, \ \vec{\mu}_B \text{ anti-parallel to } \vec{B} \text{ (spin parallel)} \\ -, \ \vec{\mu}_B \text{ parallel to } \vec{B} \text{ (spin anti-parallel)} \end{pmatrix}$$

This energy difference will bias the statistics.



Now we can see that if B $\neq$ 0, there will be an excess of one spin type.
This excess will give a net magnetic moment, $\vec{M}$

$$\vec{\mu}_B = -g\mu_B\vec{J} \qquad g = 2$$

$$\vec{M} = \vec{\mu}_B(N_{\text{parallel}} - N_{\text{anti-parallel}}) \qquad \epsilon > \epsilon_F \quad \epsilon < \epsilon_F$$

Assume $f(\epsilon) \approx 0, \quad f(\epsilon) \approx 1$

$$N_{\text{parallel}} = \frac{1}{2}\int_0^{\epsilon_F+\mu_B B} \mathcal{D}(\epsilon)d\epsilon \qquad \frac{E}{N} = \frac{2}{3}\epsilon_F$$

$$N_{\text{anti-parallel}} = \frac{1}{2}\int_0^{\epsilon_F-\mu_B B} \mathcal{D}(\epsilon)d\epsilon$$

(the factor of $\frac{1}{2}$ comes from single spin types)

$$\mathcal{D}(\epsilon) = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\epsilon^{1/2}$$

$$= C\epsilon^{1/2} = \frac{3N}{2\epsilon_F}\epsilon^{1/2}$$

$$N_{||} = \frac{1}{2}\,\frac{2}{3}\,C\,\epsilon^{3/2}\Big|_0^{\epsilon_F+\mu_B B} = \frac{C}{3}(\epsilon_F + \mu_B B)^{3/2}$$

$$N_{AP} = \frac{1}{2}\,\frac{2}{3}\,C\,\epsilon^{3/2}\Big|_0^{\epsilon_F-\mu_B B} = \frac{C}{3}(\epsilon_F - \mu_B B)^{3/2}$$

$$\text{if } \mu_B B << \epsilon_F \qquad (1+\chi)^{3/2} \approx 1 + \frac{3}{2}\chi$$

So

$$N_{||} = \frac{C\epsilon_F}{3}(1 + \frac{3}{2}\frac{\mu_B B}{\epsilon_F}) \qquad N_{AP} = \frac{C\epsilon_F}{3}(1 - \frac{3}{2}\frac{\mu_B B}{\epsilon_F})$$

$$N_{||} - N_{AP} = \frac{C\epsilon_F}{3} 3 \frac{\mu_B B}{\epsilon_F} = C\mu_B B$$

$$|M| = \mu_B \frac{3N\mu_B B}{2\epsilon_F}$$

If we define a quantity χ such that $\vec{M} = \chi\vec{H}$, where $\vec{H}$ is the magnetic intensity. If χ is small, $\vec{B} = \vec{H} + 4\pi\vec{M} \approx \vec{H}$

Then

$$\chi = \frac{3N\mu_B^2}{2\epsilon_F} \equiv \text{ Pauli susceptibility}$$

Because all of the electrons contribute to the response and also the effect of the electrons near ϵ_F is small, χ_{pauli} is nearly temperature independent.