Physics 551 Lecture #32

Title: Spin Waves (for a ferromagnetic system)

So far we have shown that the Coulomb force can give rise to an interaction in which parallel spins are the most energetically favorable. Thus the molecular field in the expression

$$H = H_{\text{applied}} + \lambda M, \quad U_{\text{exchange}} = -2(\sum_j J\ S_j) \cdot S_i$$

$$\text{becomes} \quad |\vec{\mu}| = \frac{1}{2} g\mu_B \qquad \vec{M} = \chi \vec{B} \qquad \vec{\mu} \cdot \vec{H}_{\text{ex}} = U = -2(\sum_j J S_j) \cdot S_i$$

$$H = H_{\text{applied}} - \frac{4}{g\mu_B} \sum_j J\ \vec{S}_j \qquad \text{where } \sum_j \text{ is over nearest-neighbors}$$

However notice the significance of the $\sum_j$ terms, we could write

$$U = -2J \sum_{p=1}^{N} \vec{S}_p \cdot \vec{S}_{p+1} \text{ for N spins along a line}$$

(Note: $\hbar \vec{S}_p$ is the spin angular momentum) The magnetic moment per site is $\vec{\mu}_p = -g\mu_B \vec{S}_p$ so at the p^{th} site

$$\begin{aligned} U_p &= -\vec{\mu}_p \cdot \left[(-2J/g\mu_B)(\vec{S}_{p-1} + \vec{S}_{p+1})\right] \\ U_p &= -\vec{\mu}_p \cdot \vec{B}_p \end{aligned}$$

Since the time derivative of the angular momentum $\hbar S_p$ equals the torque $\frac{d\vec{L}}{dt} = \tau = \vec{\mu}_p \times \vec{B}_p$, we get $|\vec{L}| = I/A$ and $\vec{L} = \hbar \vec{S}$. Thus

$$dS_p/dt = (-g\mu_B/\hbar)\ \vec{S}_p \times \vec{B}_p$$

This gives us equations of motion for the x, y, and z components. Let $S_z \approx S$ for small deviations of the spins $S_p^x, S_p^y << S$

$$\vec{S}_p \times \vec{B}_p \Longrightarrow \begin{pmatrix} i & j & k \\ S_p^x & S_p^y & S \\ S_{p-1}^x + S_{p+1}^x & S_{p-1}^y + S_{p+1}^y & S_{p-1}^z + S_{p+1}^z \end{pmatrix}$$

To first order in S_p

$$\begin{aligned}
\frac{dS_p^x}{dt} &= \frac{2JS}{\hbar}(2S_p^y - S_{p-1}^y - S_{p+1}^y) \\
\frac{dS_p^y}{dt} &= -\frac{2JS}{\hbar}(2S_p^x - S_{p-1}^x - S_{p+1}^x) \\
\frac{dS_p^z}{dt} &= 0
\end{aligned}$$

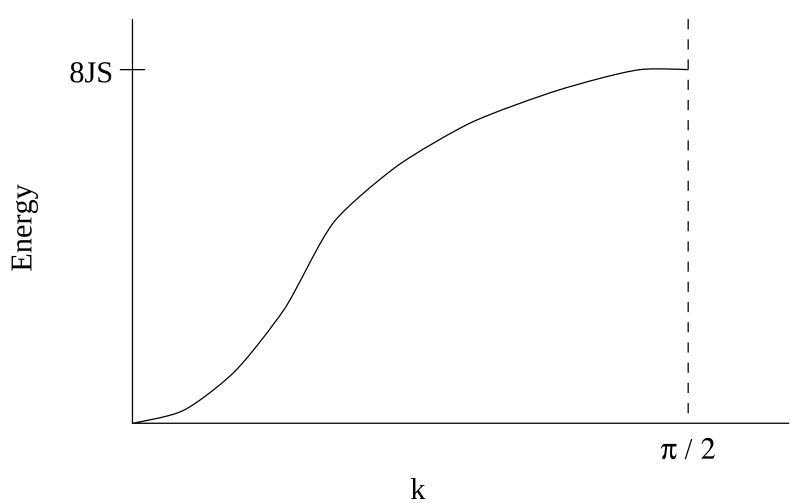
This should look familiar to the phonon problem of atoms coupled by springs except it now a 1^{st} order differential equation (as opposed to a second order differential equation for the case of phonons). Hence we solve by "guessing" solutions of the form

$$\begin{aligned}
S_p^x &= u \exp[i(pka - \omega t)] \\
S_p^y &= v \exp[i(pka - \omega t)]
\end{aligned}$$

and substitute into the top expression. Notice these solutions are spin waves. To couple the solutions the determinant must be zero. This yields:

Magnons		Phonons
$\boxed{\hbar\omega = 4J\lvert S\rvert(1-\cos\ ka)}$	vs.	$\boxed{\omega^2 = \frac{2C}{M}(1-\cos\ ka)}$

Notice that when $ka \approx 0 \quad \hbar\omega \approx 2JSk^2a^2 \propto k^2$



Physics 551 Lecture #32

Title: Spin Waves (for a ferromagnetic system)

So far we have shown that the Coulomb force can give rise to an interaction in which parallel spins are the most energetically favorable. Thus the molecular field in the expression

$$H = H_{\text{applied}} + \lambda M, \quad U_{\text{exchange}} = -2(\sum_j J\ S_j) \cdot S_i$$

$$\text{becomes} \quad |\vec{\mu}| = \frac{1}{2} g\mu_B \qquad \vec{M} = \chi \vec{B} \qquad \vec{\mu} \cdot \vec{H}_{\text{ex}} = U = -2(\sum_j J S_j) \cdot S_i$$

$$H = H_{\text{applied}} - \frac{4}{g\mu_B} \sum_j J\ \vec{S}_j \qquad \text{where } \sum_j \text{ is over nearest-neighbors}$$

However notice the significance of the $\sum_j$ terms, we could write

$$U = -2J \sum_{p=1}^{N} \vec{S}_p \cdot \vec{S}_{p+1} \text{ for N spins along a line}$$

(Note: $\hbar \vec{S}_p$ is the spin angular momentum) The magnetic moment per site is $\vec{\mu}_p = -g\mu_B \vec{S}_p$ so at the p^{th} site

$$\begin{aligned} U_p &= -\vec{\mu}_p \cdot \left[(-2J/g\mu_B)(\vec{S}_{p-1} + \vec{S}_{p+1}) \right] \\ U_p &= -\vec{\mu}_p \cdot \vec{B}_p \end{aligned}$$

Since the time derivative of the angular momentum $\hbar S_p$ equals the torque $\frac{d\vec{L}}{dt} = \tau = \vec{\mu}_p \times \vec{B}_p$, we get $|\vec{L}| = I/A$ and $\vec{L} = \hbar \vec{S}$. Thus

$$dS_p/dt = (-g\mu_B/\hbar)\ \vec{S}_p \times \vec{B}_p$$

This gives us equations of motion for the x, y, and z components. Let $S_z \approx S$ for small deviations of the spins $S_p^x, S_p^y << S$

$$\vec{S}_p \times \vec{B}_p \Longrightarrow \begin{pmatrix} i & j & k \\ S_p^x & S_p^y & S \\ S_{p-1}^x + S_{p+1}^x & S_{p-1}^y + S_{p+1}^y & S_{p-1}^z + S_{p+1}^z \end{pmatrix}$$

To first order in S_p

$$\begin{aligned}
\frac{dS_p^x}{dt} &= \frac{2JS}{\hbar}(2S_p^y - S_{p-1}^y - S_{p+1}^y) \\
\frac{dS_p^y}{dt} &= -\frac{2JS}{\hbar}(2S_p^x - S_{p-1}^x - S_{p+1}^x) \\
\frac{dS_p^z}{dt} &= 0
\end{aligned}$$

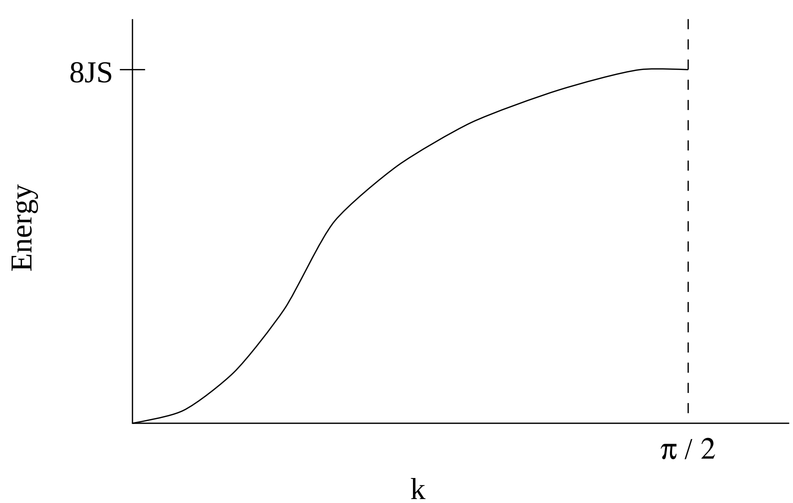
This should look familiar to the phonon problem of atoms coupled by springs except it now a 1^{st} order differential equation (as opposed to a second order differential equation for the case of phonons). Hence we solve by "guessing" solutions of the form

$$\begin{aligned}
S_p^x &= u \exp[i(pka - \omega t)] \\
S_p^y &= v \exp[i(pka - \omega t)]
\end{aligned}$$

and substitute into the top expression. Notice these solutions are spin waves. To couple the solutions the determinant must be zero. This yields:

Magnons		Phonons
$\hbar\omega = 4J\lvert S\rvert(1 - \cos\ ka)$	vs.	$\omega^2 = \frac{2C}{M}(1 - \cos\ ka)$

Notice that when $ka \approx 0 \quad \hbar\omega \approx 2JSk^2a^2 \propto k^2$



Physics 551   Lecture #32

Title: Spin Waves (for a ferromagnetic system)

So far we have shown that the Coulomb force can give rise to an interaction in which parallel spins are the most energetically favorable. Thus the molecular field in the expression

$$H = H_{\text{applied}} + \lambda M, \quad U_{\text{exchange}} = -2(\sum_j J\ S_j)\cdot S_i$$

$$\text{becomes} \quad |\vec{\mu}| = \frac{1}{2} g\mu_B \qquad \vec{M} = \chi\vec{B} \qquad \vec{\mu}\cdot\vec{H}_{\text{ex}} = U = -2(\sum_j JS_j)\cdot S_i$$

$$H = H_{\text{applied}} - \frac{4}{g\mu_B}\sum_j J\ \vec{S}_j \qquad \text{where } \sum_j \text{ is over nearest-neighbors}$$

However notice the significance of the $\sum_j$ terms, we could write

$$U = -2J\sum_{p=1}^{N} \vec{S}_p\cdot\vec{S}_{p+1} \text{ for N spins along a line}$$

(Note: $\hbar\vec{S}_p$ is the spin angular momentum) The magnetic moment per site is $\vec{\mu}_p = -g\mu_B\vec{S}_p$ so at the p^{th} site

$$\begin{aligned} U_p &= -\vec{\mu}_p\cdot\left[(-2J/g\mu_B)(\vec{S}_{p-1}+\vec{S}_{p+1})\right] \\ U_p &= -\vec{\mu}_p\cdot\vec{B}_p \end{aligned}$$

Since the time derivative of the angular momentum $\hbar S_p$ equals the torque $\frac{d\vec{L}}{dt} = \tau = \vec{\mu}_p\times\vec{B}_p$, we get $|\vec{L}| = I/A$ and $\vec{L} = \hbar\vec{S}$. Thus

$$dS_p/dt = (-g\mu_B/\hbar)\ \vec{S}_p\times\vec{B}_p$$

This gives us equations of motion for the x, y, and z components. Let $S_z \approx S$ for small deviations of the spins $S_p^x, S_p^y << S$

$$\vec{S}_p\times\vec{B}_p \Longrightarrow \begin{pmatrix} i & j & k \\ S_p^x & S_p^y & S \\ S_{p-1}^x + S_{p+1}^x & S_{p-1}^y + S_{p+1}^y & S_{p-1}^z + S_{p+1}^z \end{pmatrix}$$

To first order in S_p

$$\begin{aligned} \frac{dS_p^x}{dt} &= \frac{2JS}{\hbar}(2S_p^y - S_{p-1}^y - S_{p+1}^y) \\ \frac{dS_p^y}{dt} &= -\frac{2JS}{\hbar}(2S_p^x - S_{p-1}^x - S_{p+1}^x) \\ \frac{dS_p^z}{dt} &= 0 \end{aligned}$$

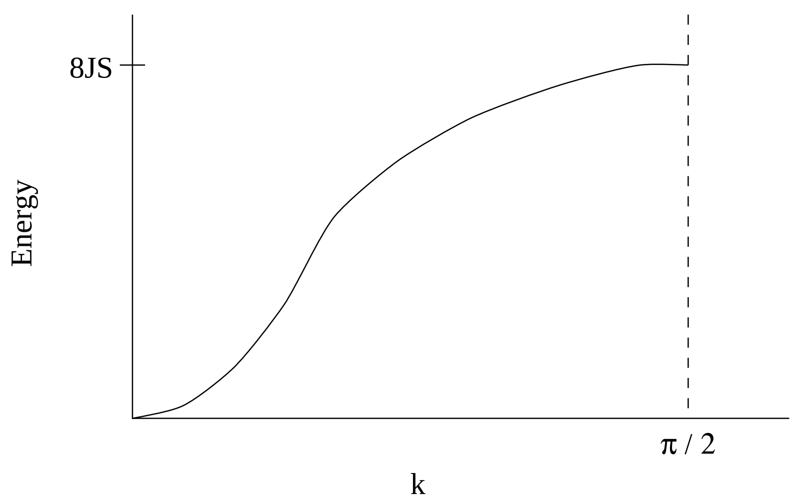
This should look familiar to the phonon problem of atoms coupled by springs except it now a 1^{st} order differential equation (as opposed to a second order differential equation for the case of phonons). Hence we solve by "guessing" solutions of the form

$$\begin{aligned} S_p^x &= u \exp[i(pka - \omega t)] \\ S_p^y &= v \exp[i(pka - \omega t)] \end{aligned}$$

and substitute into the top expression. Notice these solutions are spin waves. To couple the solutions the determinant must be zero. This yields:

Magnons		Phonons
$\hbar\omega = 4J\vert S\vert(1 - \cos\ ka)$	vs.	$\omega^2 = \frac{2C}{M}(1 - \cos\ ka)$

Notice that when $ka \approx 0 \quad \hbar\omega \approx 2JSk^2a^2 \propto k^2$



Physics 551 Lecture #32

Title: Spin Waves (for a ferromagnetic system)

So far we have shown that the Coulomb force can give rise to an interaction in which parallel spins are the most energetically favorable. Thus the molecular field in the expression

$$H = H_{\text{applied}} + \lambda M, \quad U_{\text{exchange}} = -2(\sum_j J\ S_j) \cdot S_i$$

$$\text{becomes} \quad |\vec{\mu}| = \frac{1}{2} g\mu_B \qquad \vec{M} = \chi \vec{B} \qquad \vec{\mu} \cdot \vec{H}_{\text{ex}} = U = -2(\sum_j J S_j) \cdot S_i$$

$$H = H_{\text{applied}} - \frac{4}{g\mu_B} \sum_j J\ \vec{S}_j \qquad \text{where } \sum_j \text{ is over nearest-neighbors}$$

However notice the significance of the $\sum_j$ terms, we could write

$$U = -2J \sum_{p=1}^{N} \vec{S}_p \cdot \vec{S}_{p+1} \text{ for N spins along a line}$$

(Note: $\hbar \vec{S}_p$ is the spin angular momentum) The magnetic moment per site is $\vec{\mu}_p = -g\mu_B \vec{S}_p$ so at the p^{th} site

$$\begin{aligned} U_p &= -\vec{\mu}_p \cdot \left[(-2J/g\mu_B)(\vec{S}_{p-1} + \vec{S}_{p+1})\right] \\ U_p &= -\vec{\mu}_p \cdot \vec{B}_p \end{aligned}$$

Since the time derivative of the angular momentum $\hbar S_p$ equals the torque $\frac{d\vec{L}}{dt} = \tau = \vec{\mu}_p \times \vec{B}_p$, we get $|\vec{L}| = I/A$ and $\vec{L} = \hbar \vec{S}$. Thus

$$dS_p/dt = (-g\mu_B/\hbar)\ \vec{S}_p \times \vec{B}_p$$

This gives us equations of motion for the x, y, and z components. Let $S_z \approx S$ for small deviations of the spins $S_p^x, S_p^y << S$

$$\vec{S}_p \times \vec{B}_p \Longrightarrow \begin{pmatrix} i & j & k \\ S_p^x & S_p^y & S \\ S_{p-1}^x + S_{p+1}^x & S_{p-1}^y + S_{p+1}^y & S_{p-1}^z + S_{p+1}^z \end{pmatrix}$$

To first order in S_p

$$\begin{aligned}\frac{dS_p^x}{dt} &= \frac{2JS}{\hbar}(2S_p^y - S_{p-1}^y - S_{p+1}^y)\\ \frac{dS_p^y}{dt} &= -\frac{2JS}{\hbar}(2S_p^x - S_{p-1}^x - S_{p+1}^x)\\ \frac{dS_p^z}{dt} &= 0\end{aligned}$$

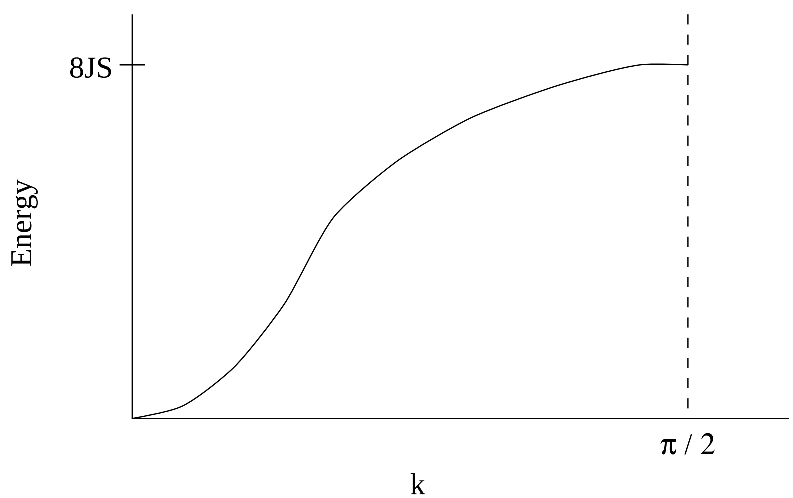
This should look familiar to the phonon problem of atoms coupled by springs except it now a 1^{st} order differential equation (as opposed to a second order differential equation for the case of phonons). Hence we solve by "guessing" solutions of the form

$$\begin{aligned}S_p^x &= u\exp[i(pka - \omega t)]\\ S_p^y &= v\exp[i(pka - \omega t)]\end{aligned}$$

and substitute into the top expression. Notice these solutions are spin waves. To couple the solutions the determinant must be zero. This yields:

Magnons		Phonons
$\hbar\omega = 4J\|S\|(1 - \cos\ ka)$	vs.	$\omega^2 = \frac{2C}{M}(1 - \cos\ ka)$

Notice that when $ka \approx 0 \quad \hbar\omega \approx 2JSk^2a^2 \propto k^2$



Physics 551 Lecture #32

Title: Spin Waves (for a ferromagnetic system)

So far we have shown that the Coulomb force can give rise to an interaction in which parallel spins are the most energetically favorable. Thus the molecular field in the expression

$$H = H_{\text{applied}} + \lambda M, \quad U_{\text{exchange}} = -2(\sum_j J\ S_j) \cdot S_i$$

$$\text{becomes} \quad |\vec{\mu}| = \frac{1}{2} g\mu_B \qquad \vec{M} = \chi \vec{B} \qquad \vec{\mu} \cdot \vec{H}_{\text{ex}} = U = -2(\sum_j J S_j) \cdot S_i$$

$$H = H_{\text{applied}} - \frac{4}{g\mu_B} \sum_j J\ \vec{S}_j \qquad \text{where } \sum_j \text{ is over nearest-neighbors}$$

However notice the significance of the $\sum_j$ terms, we could write

$$U = -2J \sum_{p=1}^{N} \vec{S}_p \cdot \vec{S}_{p+1} \text{ for N spins along a line}$$

(Note: $\hbar \vec{S}_p$ is the spin angular momentum) The magnetic moment per site is $\vec{\mu}_p = -g\mu_B \vec{S}_p$ so at the p^{th} site

$$\begin{aligned} U_p &= -\vec{\mu}_p \cdot \left[(-2J/g\mu_B)(\vec{S}_{p-1} + \vec{S}_{p+1}) \right] \\ U_p &= -\vec{\mu}_p \cdot \vec{B}_p \end{aligned}$$

Since the time derivative of the angular momentum $\hbar S_p$ equals the torque $\frac{d\vec{L}}{dt} = \tau = \vec{\mu}_p \times \vec{B}_p$, we get $|\vec{L}| = I/A$ and $\vec{L} = \hbar \vec{S}$. Thus

$$dS_p/dt = (-g\mu_B/\hbar)\ \vec{S}_p \times \vec{B}_p$$

This gives us equations of motion for the x, y, and z components. Let $S_z \approx S$ for small deviations of the spins $S_p^x, S_p^y << S$

$$\vec{S}_p \times \vec{B}_p \Longrightarrow \begin{pmatrix} i & j & k \\ S_p^x & S_p^y & S \\ S_{p-1}^x + S_{p+1}^x & S_{p-1}^y + S_{p+1}^y & S_{p-1}^z + S_{p+1}^z \end{pmatrix}$$

To first order in S_p

$$\frac{dS_p^x}{dt} = \frac{2JS}{\hbar}(2S_p^y - S_{p-1}^y - S_{p+1}^y)$$
$$\frac{dS_p^y}{dt} = -\frac{2JS}{\hbar}(2S_p^x - S_{p-1}^x - S_{p+1}^x)$$
$$\frac{dS_p^z}{dt} = 0$$

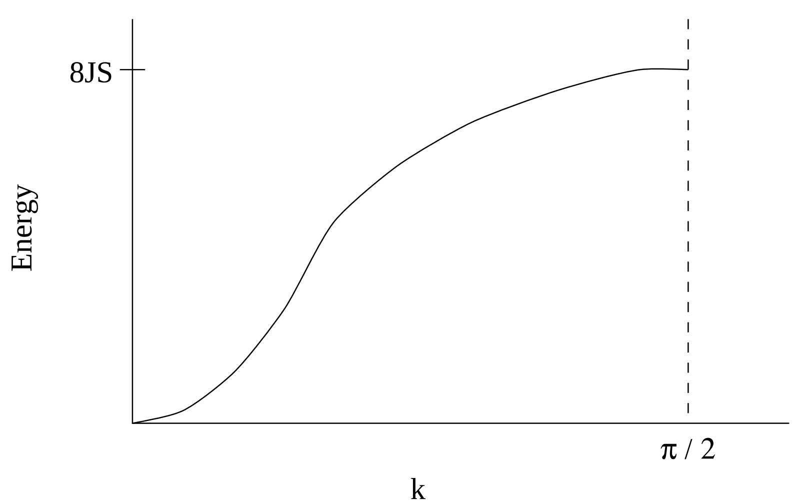
This should look familiar to the phonon problem of atoms coupled by springs except it now a 1^{st} order differential equation (as opposed to a second order differential equation for the case of phonons). Hence we solve by "guessing" solutions of the form

$$S_p^x = u\exp[i(pka - \omega t)]$$
$$S_p^y = v\exp[i(pka - \omega t)]$$

and substitute into the top expression. Notice these solutions are spin waves. To couple the solutions the determinant must be zero. This yields:

Magnons		Phonons
$\hbar\omega = 4J\vert S\vert(1 - \cos\ ka)$	vs.	$\omega^2 = \frac{2C}{M}(1 - \cos\ ka)$

Notice that when $ka \approx 0 \quad \hbar\omega \approx 2JSk^2a^2 \propto k^2$



Physics 551   Lecture #32

Title: Spin Waves (for a ferromagnetic system)

So far we have shown that the Coulomb force can give rise to an interaction in which parallel spins are the most energetically favorable. Thus the molecular field in the expression

$$H = H_{\text{applied}} + \lambda M, \quad U_{\text{exchange}} = -2(\sum_j J\ S_j) \cdot S_i$$

$$\text{becomes} \quad |\vec{\mu}| = \frac{1}{2} g\mu_B \qquad \vec{M} = \chi \vec{B} \qquad \vec{\mu} \cdot \vec{H}_{\text{ex}} = U = -2(\sum_j J S_j) \cdot S_i$$

$$H = H_{\text{applied}} - \frac{4}{g\mu_B} \sum_j J\ \vec{S}_j \qquad \text{where } \sum_j \text{ is over nearest-neighbors}$$

However notice the significance of the $\sum_j$ terms, we could write

$$U = -2J \sum_{p=1}^{N} \vec{S}_p \cdot \vec{S}_{p+1} \text{ for N spins along a line}$$

(Note: $\hbar \vec{S}_p$ is the spin angular momentum) The magnetic moment per site is $\vec{\mu}_p = -g\mu_B \vec{S}_p$ so at the p^{th} site

$$\begin{aligned} U_p &= -\vec{\mu}_p \cdot \left[(-2J/g\mu_B)(\vec{S}_{p-1} + \vec{S}_{p+1})\right] \\ U_p &= -\vec{\mu}_p \cdot \vec{B}_p \end{aligned}$$

Since the time derivative of the angular momentum $\hbar S_p$ equals the torque $\frac{d\vec{L}}{dt} = \tau = \vec{\mu}_p \times \vec{B}_p$, we get $|\vec{L}| = I/A$ and $\vec{L} = \hbar \vec{S}$. Thus

$$dS_p/dt = (-g\mu_B/\hbar)\ \vec{S}_p \times \vec{B}_p$$

This gives us equations of motion for the x, y, and z components. Let $S_z \approx S$ for small deviations of the spins $S_p^x, S_p^y << S$

$$\vec{S}_p \times \vec{B}_p \Longrightarrow \begin{pmatrix} i & j & k \\ S_p^x & S_p^y & S \\ S_{p-1}^x + S_{p+1}^x & S_{p-1}^y + S_{p+1}^y & S_{p-1}^z + S_{p+1}^z \end{pmatrix}$$

To first order in S_p

$$\begin{aligned}
\frac{dS_p^x}{dt} &= \frac{2JS}{\hbar}(2S_p^y - S_{p-1}^y - S_{p+1}^y) \\
\frac{dS_p^y}{dt} &= -\frac{2JS}{\hbar}(2S_p^x - S_{p-1}^x - S_{p+1}^x) \\
\frac{dS_p^z}{dt} &= 0
\end{aligned}$$

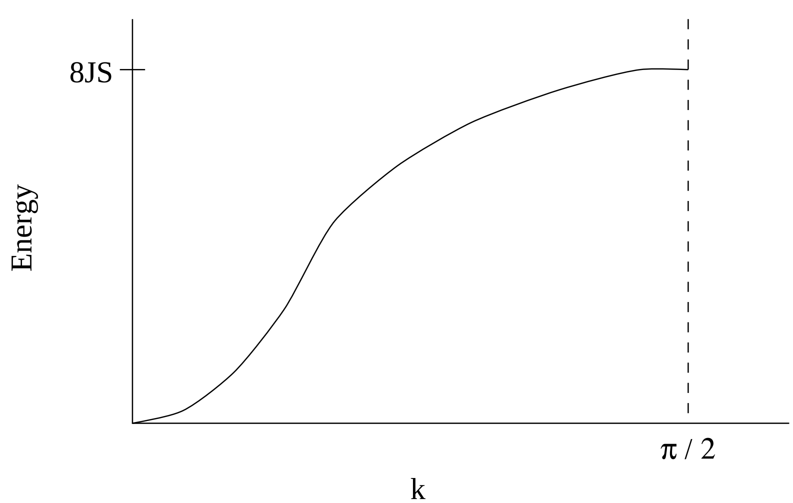
This should look familiar to the phonon problem of atoms coupled by springs except it now a 1^{st} order differential equation (as opposed to a second order differential equation for the case of phonons). Hence we solve by "guessing" solutions of the form

$$\begin{aligned}
S_p^x &= u\exp[i(pka - \omega t)] \\
S_p^y &= v\exp[i(pka - \omega t)]
\end{aligned}$$

and substitute into the top expression. Notice these solutions are spin waves. To couple the solutions the determinant must be zero. This yields:

Magnons		Phonons
$\boxed{\hbar\omega = 4J\vert S\vert(1 - \cos\ ka)}$	vs.	$\boxed{\omega^2 = \frac{2C}{M}(1 - \cos\ ka)}$

Notice that when $ka \approx 0 \quad \hbar\omega \approx 2JSk^2a^2 \propto k^2$



Physics 551 Lecture #32

Title: Spin Waves (for a ferromagnetic system)

So far we have shown that the Coulomb force can give rise to an interaction in which parallel spins are the most energetically favorable. Thus the molecular field in the expression

$$H = H_{\text{applied}} + \lambda M, \quad U_{\text{exchange}} = -2(\sum_j J\ S_j) \cdot S_i$$

$$\text{becomes} \quad |\vec{\mu}| = \frac{1}{2} g\mu_B \qquad \vec{M} = \chi\vec{B} \qquad \vec{\mu}\cdot\vec{H}_{\text{ex}} = U = -2(\sum_j JS_j)\cdot S_i$$

$$H = H_{\text{applied}} - \frac{4}{g\mu_B}\sum_j J\ \vec{S}_j \qquad \text{where } \sum_j \text{ is over nearest-neighbors}$$

However notice the significance of the $\sum_j$ terms, we could write

$$U = -2J\sum_{p=1}^{N} \vec{S}_p \cdot \vec{S}_{p+1} \text{ for N spins along a line}$$

(Note: $\hbar\vec{S}_p$ is the spin angular momentum) The magnetic moment per site is $\vec{\mu}_p = -g\mu_B\vec{S}_p$ so at the p^{th} site

$$\begin{aligned} U_p &= -\vec{\mu}_p \cdot \Big[(-2J/g\mu_B)(\vec{S}_{p-1} + \vec{S}_{p+1})\Big] \\ U_p &= -\vec{\mu}_p \cdot \vec{B}_p \end{aligned}$$

Since the time derivative of the angular momentum $\hbar S_p$ equals the torque $\frac{d\vec{L}}{dt} = \tau = \vec{\mu}_p \times \vec{B}_p$, we get $|\vec{L}| = I/A$ and $\vec{L} = \hbar\vec{S}$. Thus

$$dS_p/dt = (-g\mu_B/\hbar)\ \vec{S}_p \times \vec{B}_p$$

This gives us equations of motion for the x, y, and z components. Let $S_z \approx S$ for small deviations of the spins $S_p^x, S_p^y << S$

$$\vec{S}_p \times \vec{B}_p \Longrightarrow \begin{pmatrix} i & j & k \\ S_p^x & S_p^y & S \\ S_{p-1}^x + S_{p+1}^x & S_{p-1}^y + S_{p+1}^y & S_{p-1}^z + S_{p+1}^z \end{pmatrix}$$

To first order in S_p

$$\begin{aligned}
\frac{dS_p^x}{dt} &= \frac{2JS}{\hbar}(2S_p^y - S_{p-1}^y - S_{p+1}^y) \\
\frac{dS_p^y}{dt} &= -\frac{2JS}{\hbar}(2S_p^x - S_{p-1}^x - S_{p+1}^x) \\
\frac{dS_p^z}{dt} &= 0
\end{aligned}$$

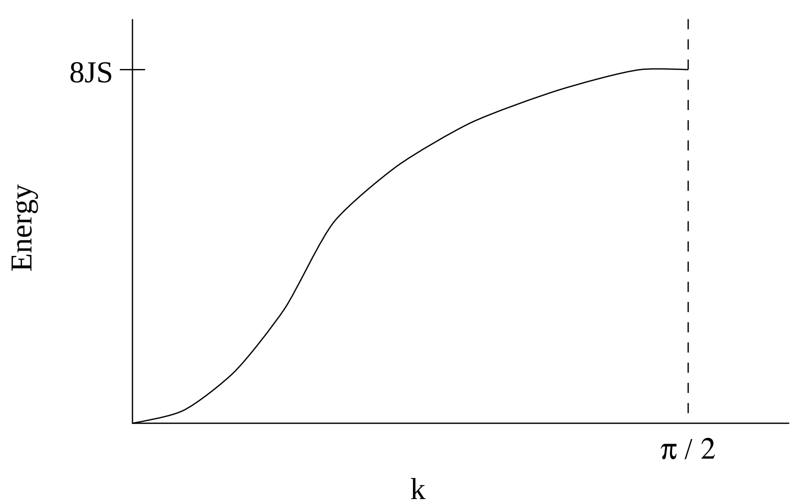
This should look familiar to the phonon problem of atoms coupled by springs except it now a 1^{st} order differential equation (as opposed to a second order differential equation for the case of phonons). Hence we solve by "guessing" solutions of the form

$$\begin{aligned}
S_p^x &= u \exp[i(pka - \omega t)] \\
S_p^y &= v \exp[i(pka - \omega t)]
\end{aligned}$$

and substitute into the top expression. Notice these solutions are spin waves. To couple the solutions the determinant must be zero. This yields:

Magnons		Phonons
$\hbar\omega = 4J\vert S\vert(1 - \cos\ ka)$	vs.	$\omega^2 = \frac{2C}{M}(1 - \cos\ ka)$

Notice that when $ka \approx 0$ $\quad \hbar\omega \approx 2JSk^2a^2 \propto k^2$



Physics 551 Lecture #32

Title: Spin Waves (for a ferromagnetic system)

So far we have shown that the Coulomb force can give rise to an interaction in which parallel spins are the most energetically favorable. Thus the molecular field in the expression

$$H = H_{\text{applied}} + \lambda M, \quad U_{\text{exchange}} = -2(\sum_j J\ S_j)\cdot S_i$$

$$\text{becomes} \quad |\vec{\mu}| = \frac{1}{2} g\mu_B \qquad \vec{M} = \chi\vec{B} \qquad \vec{\mu}\cdot\vec{H}_{\text{ex}} = U = -2(\sum_j JS_j)\cdot S_i$$

$$H = H_{\text{applied}} - \frac{4}{g\mu_B}\sum_j J\ \vec{S}_j \qquad \text{where } \sum_j \text{ is over nearest-neighbors}$$

However notice the significance of the $\sum_j$ terms, we could write

$$U = -2J\sum_{p=1}^{N} \vec{S}_p \cdot \vec{S}_{p+1} \text{ for N spins along a line}$$

(Note: $\hbar\vec{S}_p$ is the spin angular momentum) The magnetic moment per site is $\vec{\mu}_p = -g\mu_B\vec{S}_p$ so at the p^{th} site

$$\begin{aligned} U_p &= -\vec{\mu}_p \cdot \left[(-2J/g\mu_B)(\vec{S}_{p-1} + \vec{S}_{p+1})\right] \\ U_p &= -\vec{\mu}_p \cdot \vec{B}_p \end{aligned}$$

Since the time derivative of the angular momentum $\hbar S_p$ equals the torque $\frac{d\vec{L}}{dt} = \tau = \vec{\mu}_p \times \vec{B}_p$, we get $|\vec{L}| = I/A$ and $\vec{L} = \hbar\vec{S}$. Thus

$$dS_p/dt = (-g\mu_B/\hbar)\ \vec{S}_p \times \vec{B}_p$$

This gives us equations of motion for the x, y, and z components. Let $S_z \approx S$ for small deviations of the spins $S_p^x, S_p^y << S$

$$\vec{S}_p \times \vec{B}_p \Longrightarrow \begin{pmatrix} i & j & k \\ S_p^x & S_p^y & S \\ S_{p-1}^x + S_{p+1}^x & S_{p-1}^y + S_{p+1}^y & S_{p-1}^z + S_{p+1}^z \end{pmatrix}$$

To first order in S_p

$$\begin{aligned}
\frac{dS_p^x}{dt} &= \frac{2JS}{\hbar}(2S_p^y - S_{p-1}^y - S_{p+1}^y) \\
\frac{dS_p^y}{dt} &= -\frac{2JS}{\hbar}(2S_p^x - S_{p-1}^x - S_{p+1}^x) \\
\frac{dS_p^z}{dt} &= 0
\end{aligned}$$

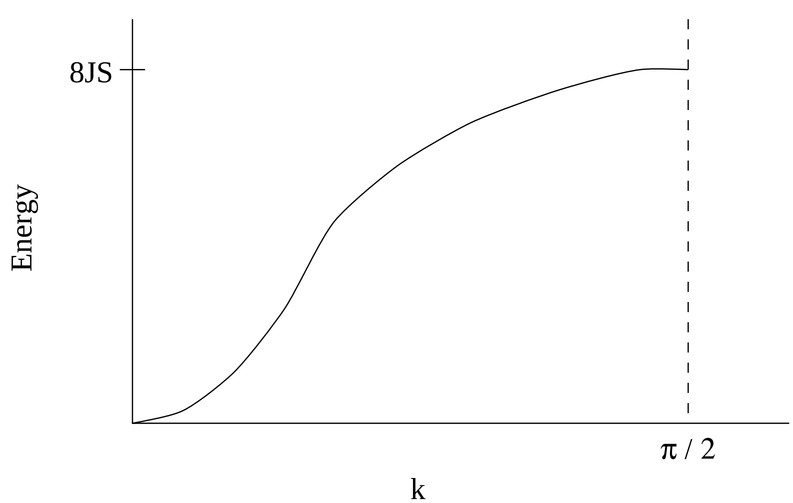
This should look familiar to the phonon problem of atoms coupled by springs except it now a 1^{st} order differential equation (as opposed to a second order differential equation for the case of phonons). Hence we solve by "guessing" solutions of the form

$$\begin{aligned}
S_p^x &= u \exp[i(pka - \omega t)] \\
S_p^y &= v \exp[i(pka - \omega t)]
\end{aligned}$$

and substitute into the top expression. Notice these solutions are spin waves. To couple the solutions the determinant must be zero. This yields:

Magnons		Phonons
$\boxed{\hbar\omega = 4J\lvert S\rvert(1 - \cos\ ka)}$	vs.	$\boxed{\omega^2 = \frac{2C}{M}(1 - \cos\ ka)}$

Notice that when $ka \approx 0 \quad \hbar\omega \approx 2JSk^2a^2 \propto k^2$



Physics 551 Lecture #32

Title: Spin Waves (for a ferromagnetic system)

So far we have shown that the Coulomb force can give rise to an interaction in which parallel spins are the most energetically favorable. Thus the molecular field in the expression

$$H = H_{\text{applied}} + \lambda M, \quad U_{\text{exchange}} = -2(\sum_j J\ S_j) \cdot S_i$$

$$\text{becomes} \quad |\vec{\mu}| = \frac{1}{2} g\mu_B \qquad \vec{M} = \chi \vec{B} \qquad \vec{\mu} \cdot \vec{H}_{\text{ex}} = U = -2(\sum_j J S_j) \cdot S_i$$

$$H = H_{\text{applied}} - \frac{4}{g\mu_B} \sum_j J\ \vec{S}_j \qquad \text{where } \sum_j \text{ is over nearest-neighbors}$$

However notice the significance of the $\sum_j$ terms, we could write

$$U = -2J \sum_{p=1}^{N} \vec{S}_p \cdot \vec{S}_{p+1} \text{ for N spins along a line}$$

(Note: $\hbar \vec{S}_p$ is the spin angular momentum) The magnetic moment per site is $\vec{\mu}_p = -g\mu_B \vec{S}_p$ so at the p^{th} site

$$\begin{aligned} U_p &= -\vec{\mu}_p \cdot \left[(-2J/g\mu_B)(\vec{S}_{p-1} + \vec{S}_{p+1})\right] \\ U_p &= -\vec{\mu}_p \cdot \vec{B}_p \end{aligned}$$

Since the time derivative of the angular momentum $\hbar S_p$ equals the torque $\frac{d\vec{L}}{dt} = \tau = \vec{\mu}_p \times \vec{B}_p$, we get $|\vec{L}| = I/A$ and $\vec{L} = \hbar \vec{S}$. Thus

$$dS_p/dt = (-g\mu_B/\hbar)\ \vec{S}_p \times \vec{B}_p$$

This gives us equations of motion for the x, y, and z components. Let $S_z \approx S$ for small deviations of the spins $S_p^x, S_p^y << S$

$$\vec{S}_p \times \vec{B}_p \Longrightarrow \begin{pmatrix} i & j & k \\ S_p^x & S_p^y & S \\ S_{p-1}^x + S_{p+1}^x & S_{p-1}^y + S_{p+1}^y & S_{p-1}^z + S_{p+1}^z \end{pmatrix}$$

To first order in S_p

$$\frac{dS_p^x}{dt} = \frac{2JS}{\hbar}(2S_p^y - S_{p-1}^y - S_{p+1}^y)$$
$$\frac{dS_p^y}{dt} = -\frac{2JS}{\hbar}(2S_p^x - S_{p-1}^x - S_{p+1}^x)$$
$$\frac{dS_p^z}{dt} = 0$$

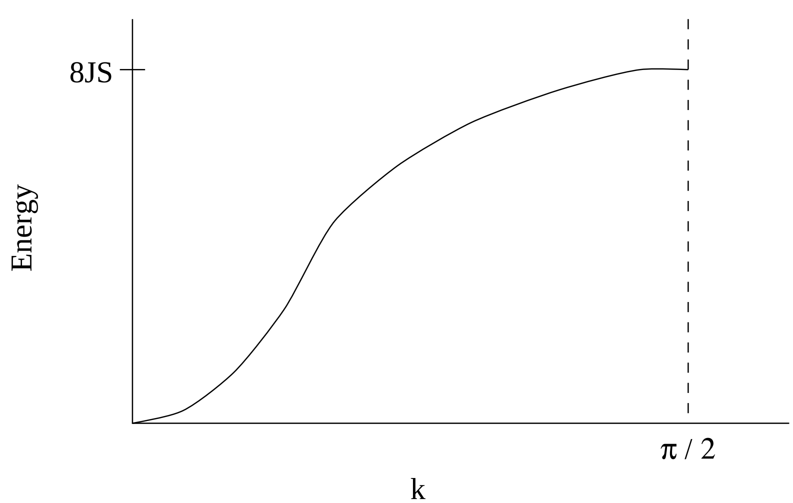
This should look familiar to the phonon problem of atoms coupled by springs except it now a 1^{st} order differential equation (as opposed to a second order differential equation for the case of phonons). Hence we solve by "guessing" solutions of the form

$$S_p^x = u\exp[i(pka - \omega t)]$$
$$S_p^y = v\exp[i(pka - \omega t)]$$

and substitute into the top expression. Notice these solutions are spin waves. To couple the solutions the determinant must be zero. This yields:

Magnons		Phonons
$\hbar\omega = 4J\lvert S\rvert(1 - \cos\ ka)$	vs.	$\omega^2 = \frac{2C}{M}(1 - \cos\ ka)$

Notice that when $ka \approx 0 \quad \hbar\omega \approx 2JSk^2a^2 \propto k^2$



Physics 551 Lecture #32

Title: Spin Waves (for a ferromagnetic system)

So far we have shown that the Coulomb force can give rise to an interaction in which parallel spins are the most energetically favorable. Thus the molecular field in the expression

$$H = H_{\text{applied}} + \lambda M, \quad U_{\text{exchange}} = -2(\sum_j J\ S_j) \cdot S_i$$

$$\text{becomes} \quad |\vec{\mu}| = \frac{1}{2} g\mu_B \qquad \vec{M} = \chi \vec{B} \qquad \vec{\mu} \cdot \vec{H}_{\text{ex}} = U = -2(\sum_j J S_j) \cdot S_i$$

$$H = H_{\text{applied}} - \frac{4}{g\mu_B} \sum_j J\ \vec{S}_j \qquad \text{where } \sum_j \text{ is over nearest-neighbors}$$

However notice the significance of the $\sum_j$ terms, we could write

$$U = -2J \sum_{p=1}^{N} \vec{S}_p \cdot \vec{S}_{p+1} \text{ for N spins along a line}$$

(Note: $\hbar \vec{S}_p$ is the spin angular momentum) The magnetic moment per site is $\vec{\mu}_p = -g\mu_B \vec{S}_p$ so at the p^{th} site

$$\begin{aligned} U_p &= -\vec{\mu}_p \cdot \left[(-2J/g\mu_B)(\vec{S}_{p-1} + \vec{S}_{p+1})\right] \\ U_p &= -\vec{\mu}_p \cdot \vec{B}_p \end{aligned}$$

Since the time derivative of the angular momentum $\hbar S_p$ equals the torque $\frac{d\vec{L}}{dt} = \tau = \vec{\mu}_p \times \vec{B}_p$, we get $|\vec{L}| = I/A$ and $\vec{L} = \hbar \vec{S}$. Thus

$$dS_p/dt = (-g\mu_B/\hbar)\ \vec{S}_p \times \vec{B}_p$$

This gives us equations of motion for the x, y, and z components. Let $S_z \approx S$ for small deviations of the spins $S_p^x, S_p^y << S$

$$\vec{S}_p \times \vec{B}_p \Longrightarrow \begin{pmatrix} i & j & k \\ S_p^x & S_p^y & S \\ S_{p-1}^x + S_{p+1}^x & S_{p-1}^y + S_{p+1}^y & S_{p-1}^z + S_{p+1}^z \end{pmatrix}$$

To first order in S_p

$$\begin{aligned}\frac{dS_p^x}{dt} &= \frac{2JS}{\hbar}(2S_p^y - S_{p-1}^y - S_{p+1}^y) \\ \frac{dS_p^y}{dt} &= -\frac{2JS}{\hbar}(2S_p^x - S_{p-1}^x - S_{p+1}^x) \\ \frac{dS_p^z}{dt} &= 0\end{aligned}$$

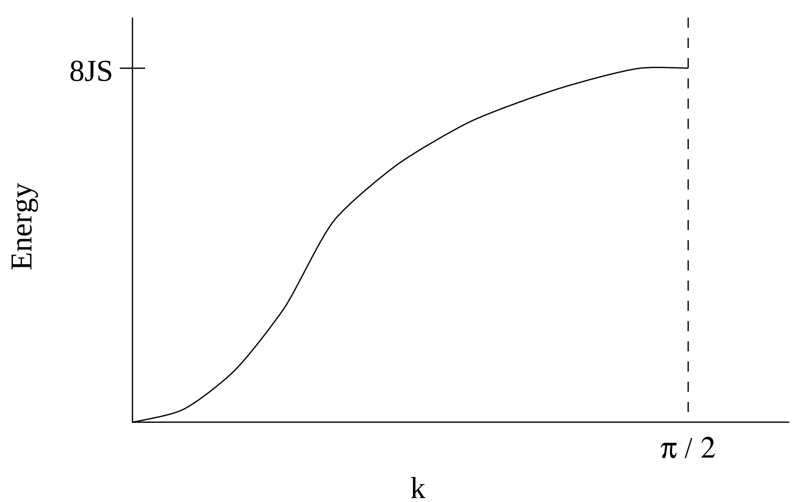
This should look familiar to the phonon problem of atoms coupled by springs except it now a 1^{st} order differential equation (as opposed to a second order differential equation for the case of phonons). Hence we solve by "guessing" solutions of the form

$$\begin{aligned}S_p^x &= u\exp[i(pka - \omega t)] \\ S_p^y &= v\exp[i(pka - \omega t)]\end{aligned}$$

and substitute into the top expression. Notice these solutions are spin waves. To couple the solutions the determinant must be zero. This yields:

Magnons		Phonons
$\hbar\omega = 4J\|S\|(1 - \cos\ ka)$	vs.	$\omega^2 = \frac{2C}{M}(1 - \cos\ ka)$

Notice that when $ka \approx 0 \quad \hbar\omega \approx 2JSk^2a^2 \propto k^2$



Physics 551 Lecture #32

Title: Spin Waves (for a ferromagnetic system)

So far we have shown that the Coulomb force can give rise to an interaction in which parallel spins are the most energetically favorable. Thus the molecular field in the expression

$$H = H_{\text{applied}} + \lambda M, \quad U_{\text{exchange}} = -2(\sum_j J\ S_j) \cdot S_i$$

$$\text{becomes} \quad |\vec{\mu}| = \frac{1}{2} g\mu_B \qquad \vec{M} = \chi \vec{B} \qquad \vec{\mu} \cdot \vec{H}_{\text{ex}} = U = -2(\sum_j J S_j) \cdot S_i$$

$$H = H_{\text{applied}} - \frac{4}{g\mu_B} \sum_j J\ \vec{S}_j \qquad \text{where } \sum_j \text{ is over nearest-neighbors}$$

However notice the significance of the $\sum_j$ terms, we could write

$$U = -2J \sum_{p=1}^{N} \vec{S}_p \cdot \vec{S}_{p+1} \text{ for N spins along a line}$$

(Note: $\hbar \vec{S}_p$ is the spin angular momentum) The magnetic moment per site is $\vec{\mu}_p = -g\mu_B \vec{S}_p$ so at the p^{th} site

$$\begin{aligned} U_p &= -\vec{\mu}_p \cdot \left[(-2J/g\mu_B)(\vec{S}_{p-1} + \vec{S}_{p+1})\right] \\ U_p &= -\vec{\mu}_p \cdot \vec{B}_p \end{aligned}$$

Since the time derivative of the angular momentum $\hbar S_p$ equals the torque $\frac{d\vec{L}}{dt} = \tau = \vec{\mu}_p \times \vec{B}_p$, we get $|\vec{L}| = I/A$ and $\vec{L} = \hbar \vec{S}$. Thus

$$dS_p/dt = (-g\mu_B/\hbar)\ \vec{S}_p \times \vec{B}_p$$

This gives us equations of motion for the x, y, and z components. Let $S_z \approx S$ for small deviations of the spins $S_p^x, S_p^y << S$

$$\vec{S}_p \times \vec{B}_p \Longrightarrow \begin{pmatrix} i & j & k \\ S_p^x & S_p^y & S \\ S_{p-1}^x + S_{p+1}^x & S_{p-1}^y + S_{p+1}^y & S_{p-1}^z + S_{p+1}^z \end{pmatrix}$$

To first order in S_p

$$\frac{dS_p^x}{dt} = \frac{2JS}{\hbar}(2S_p^y - S_{p-1}^y - S_{p+1}^y)$$
$$\frac{dS_p^y}{dt} = -\frac{2JS}{\hbar}(2S_p^x - S_{p-1}^x - S_{p+1}^x)$$
$$\frac{dS_p^z}{dt} = 0$$

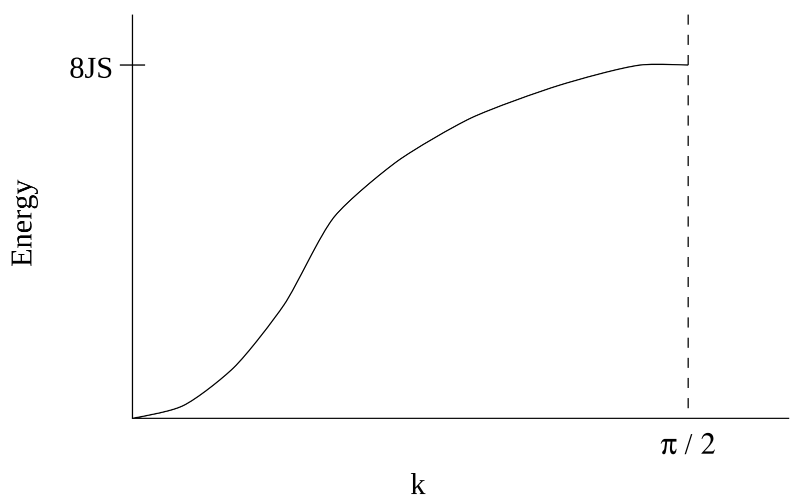
This should look familiar to the phonon problem of atoms coupled by springs except it now a 1^{st} order differential equation (as opposed to a second order differential equation for the case of phonons). Hence we solve by "guessing" solutions of the form

$$S_p^x = u\exp[i(pka - \omega t)]$$
$$S_p^y = v\exp[i(pka - \omega t)]$$

and substitute into the top expression. Notice these solutions are spin waves. To couple the solutions the determinant must be zero. This yields:

Magnons		Phonons
$\hbar\omega = 4J\|S\|(1 - \cos\ ka)$	vs.	$\omega^2 = \frac{2C}{M}(1 - \cos\ ka)$

Notice that when $ka \approx 0 \quad \hbar\omega \approx 2JSk^2a^2 \propto k^2$



Physics 551 Lecture #32

Title: Spin Waves (for a ferromagnetic system)

So far we have shown that the Coulomb force can give rise to an interaction in which parallel spins are the most energetically favorable. Thus the molecular field in the expression

$$H = H_{\text{applied}} + \lambda M, \quad U_{\text{exchange}} = -2(\sum_j J\ S_j) \cdot S_i$$

$$\text{becomes} \quad |\vec{\mu}| = \frac{1}{2} g\mu_B \qquad \vec{M} = \chi \vec{B} \qquad \vec{\mu} \cdot \vec{H}_{\text{ex}} = U = -2(\sum_j J S_j) \cdot S_i$$

$$H = H_{\text{applied}} - \frac{4}{g\mu_B} \sum_j J\ \vec{S}_j \qquad \text{where } \sum_j \text{ is over nearest-neighbors}$$

However notice the significance of the $\sum_j$ terms, we could write

$$U = -2J \sum_{p=1}^{N} \vec{S}_p \cdot \vec{S}_{p+1} \text{ for N spins along a line}$$

(Note: $\hbar \vec{S}_p$ is the spin angular momentum) The magnetic moment per site is $\vec{\mu}_p = -g\mu_B \vec{S}_p$ so at the p^{th} site

$$\begin{aligned} U_p &= -\vec{\mu}_p \cdot \left[(-2J/g\mu_B)(\vec{S}_{p-1} + \vec{S}_{p+1})\right] \\ U_p &= -\vec{\mu}_p \cdot \vec{B}_p \end{aligned}$$

Since the time derivative of the angular momentum $\hbar S_p$ equals the torque $\frac{d\vec{L}}{dt} = \tau = \vec{\mu}_p \times \vec{B}_p$, we get $|\vec{L}| = I/A$ and $\vec{L} = \hbar \vec{S}$. Thus

$$dS_p/dt = (-g\mu_B/\hbar)\ \vec{S}_p \times \vec{B}_p$$

This gives us equations of motion for the x, y, and z components. Let $S_z \approx S$ for small deviations of the spins $S_p^x, S_p^y << S$

$$\vec{S}_p \times \vec{B}_p \Longrightarrow \begin{pmatrix} i & j & k \\ S_p^x & S_p^y & S \\ S_{p-1}^x + S_{p+1}^x & S_{p-1}^y + S_{p+1}^y & S_{p-1}^z + S_{p+1}^z \end{pmatrix}$$

To first order in S_p

$$\frac{dS_p^x}{dt} = \frac{2JS}{\hbar}(2S_p^y - S_{p-1}^y - S_{p+1}^y)$$
$$\frac{dS_p^y}{dt} = -\frac{2JS}{\hbar}(2S_p^x - S_{p-1}^x - S_{p+1}^x)$$
$$\frac{dS_p^z}{dt} = 0$$

This should look familiar to the phonon problem of atoms coupled by springs except it now a 1^{st} order differential equation (as opposed to a second order differential equation for the case of phonons). Hence we solve by "guessing" solutions of the form

$$S_p^x = u \exp[i(pka - \omega t)]$$
$$S_p^y = v \exp[i(pka - \omega t)]$$

and substitute into the top expression. Notice these solutions are spin waves. To couple the solutions the determinant must be zero. This yields:

Magnons		Phonons
$\hbar\omega = 4J\lvert S\rvert(1 - \cos\ ka)$	vs.	$\omega^2 = \frac{2C}{M}(1 - \cos\ ka)$

Notice that when $ka \approx 0$ $\quad \hbar\omega \approx 2JSk^2a^2 \propto k^2$