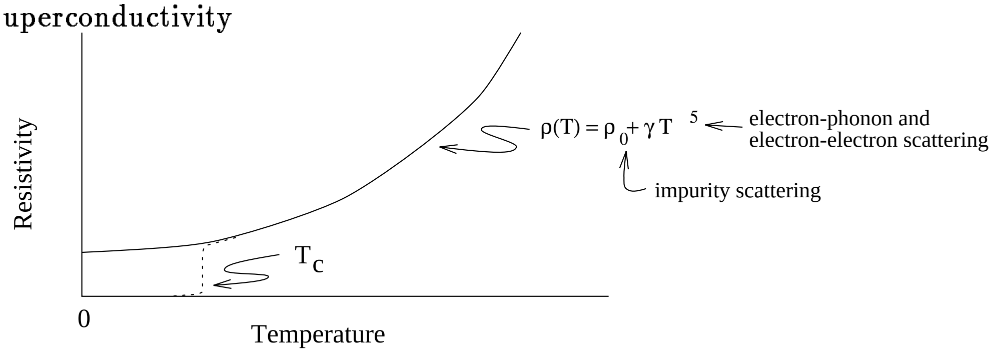
Physics 0551 Lecture #33

Title: Superconductivity



Superconductors are unusual for a variety of reasons. The first observations were made in 1911 by Kamerlingh Onnes (Denmark) on the low temperature behavior of Hg (mercury) and he observed a complete absence of ρ in the vicinity of 4°K.

Note: There is nothing in the "classical" Drude' theory of metals which can account for this behavior.

There are many systematics which accompany the occurrence of superconductivity

No monovalent Elements
No ferromagnetic elements
No Rare-earths

} i.e., spin is *not* compatible with superconductivity (conventional)

Normal BCS (Bardeen-Cooper-Schrieffer) (1957)

10^{-2}K $< T_c <$ 23°K or so Nb_3Ge 23.2°K

High T_c superconductors are at odds with conventional wisdom.

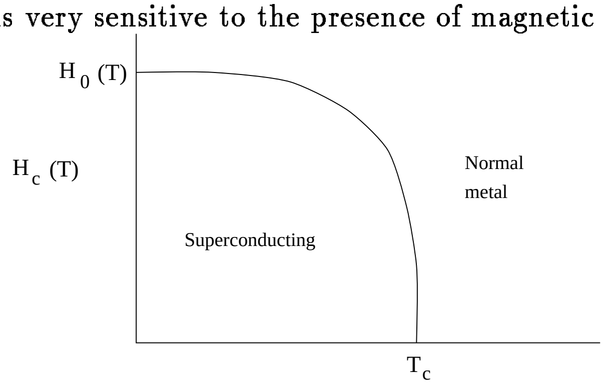
Oxides	35°K	$La_{1.8}\, Sr_{.2} CuO_4$	Bednoorz and Müller
	95°K	$(YBa_2)\, Cu_3O_7$	
	125°K	Th- .... -Cu)	

Note: Anomalous behavior in ρ is often due to structural transformation

Organic superconductor (BEDT-TTF)$_2 Cu(NCS)_2$ 10.4°K

T_c is very sensitive to applied magnetic fields.

T_c is very sensitive to the presence of magnetic impurities.



Second test for superconductivity is the Meissner/Ochsenfeld effect.

There is complete expulsion of applied magnetic fields.

B=0 inside superconducting regions of a superconductor

$\Longrightarrow$ Note that a superconductor is <u>NOT</u> a perfect conductor

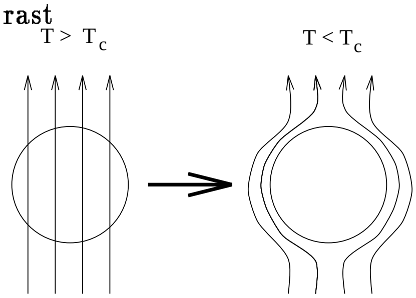
Ohm's Law $\vec{E} = \rho \vec{J}$

If $\rho = 0$, then $\vec{E} = 0$

But $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (cgs) $\quad \nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$ (SI)

If $\vec{E} = 0$ then $\frac{\partial \vec{B}}{\partial t} = 0$ $\quad \Rightarrow$ Magnetic flux cannot be expelled!

In contrast



i.e., superconductors are *perfect* diamagnets

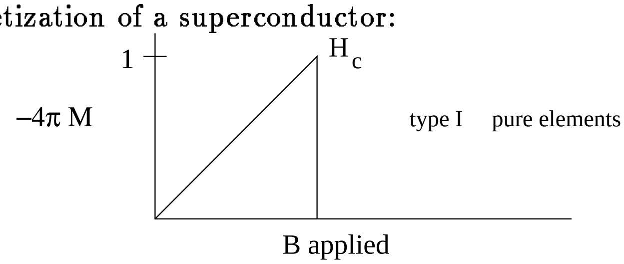
$$\vec{B} = \vec{H} + 4\pi \vec{M} \quad \text{where} \quad \chi \equiv \frac{\vec{M}}{\vec{H}} \quad \text{(cgs)}$$

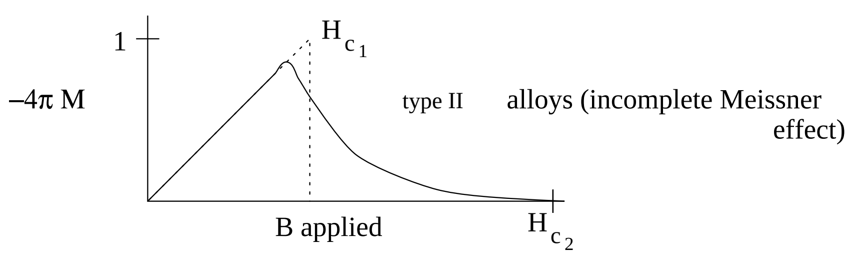
Since $\vec{B} = 0$, $\quad \chi = -\frac{1}{4\pi}$ $\quad$ (Not $-\infty$)

* Note since high T_c samples are often multiphase

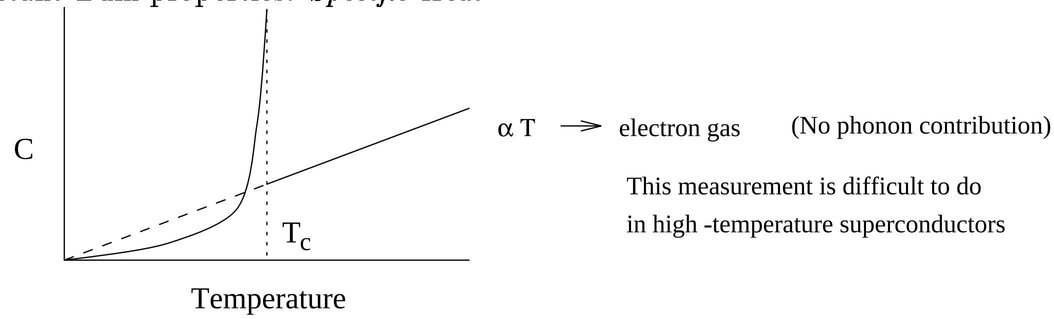
$$\chi_{\mathrm{HT}_c} < -\frac{1}{4\pi} \quad \text{or} \quad \%\ \text{superconducting} = \frac{\chi_{\mathrm{HT}_c}}{-\frac{1}{4\pi}}$$

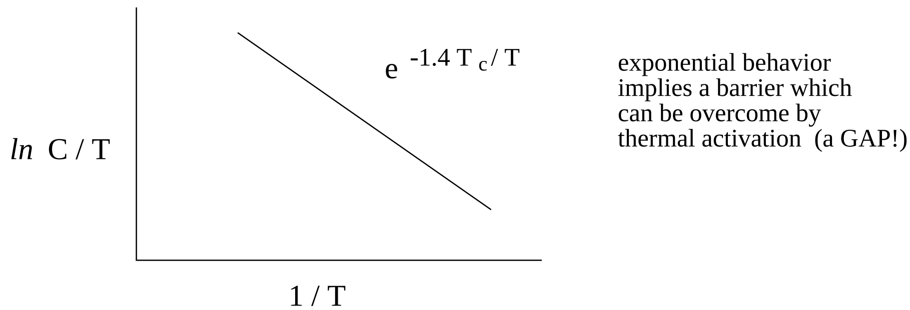
Magnetization of a superconductor:





Other important Bulk properties: *Specific Heat*





The entropy of the S.C. state is smaller than in the normal state.

Isotope effect (factor of 1/2 is approximate)

$$T_c \propto \frac{1}{M^{1/2}} \quad \Longrightarrow \text{implies a } \textit{phonon} \text{ mechanism}$$

Thermodynamics of a superconductor:

S.C. ↔ Normal Transition is a reversible process, hence a thermodynamic treatment is possible

2^{nd} Order Transition → NO LATENT HEAT

$$\begin{aligned} d(U - TS) &= -\vec{M} \cdot d\vec{H} \\ dU &= TdS - \vec{M} \cdot d\vec{H} \\ dU &= dQ - dW \end{aligned}$$

How much energy is stored?

$$\vec{M} \cdot d\vec{H}$$

$$dU = TdS - \vec{M} \cdot d\vec{B}_a \qquad B_a \Longrightarrow \text{magnetic field applied}$$

$$\text{but } M = -\frac{1}{4\pi} B_a$$

We want the change in the Free energy (F) at constant temperature

$$dF = d(U - TS) = -\vec{M} \cdot d\vec{B}_a$$

$$\text{but } \vec{M} = \chi \vec{H} = -\frac{1}{4\pi}\, \vec{B}_a$$

$$dF = \frac{1}{4\pi}\,\vec{B}_a \cdot d\vec{B}_a \qquad F_{superconducting}$$

$$0 \text{ to } B_a \text{ is } B_{applied}$$

$$F_s(B_a) - F_s(0) = B_a^2/8\pi$$

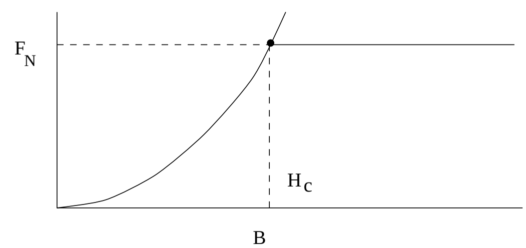
for the normal $F_N(B_a) = F_N(0)$ ignore, $\chi_N \approx 0$!

At H_c (critical field)

$$F_N(H_c) = F_S(H_c) \qquad \text{No latent heat}$$

therefore $F_N(0) = F_N(H_c) = F_S(0) + B_{H_c}^2/8\pi$

so $F_N(0) - F_s(0) = B_{H_c}^2/8\pi$



$F_s(0) < F_N(0)$ for $T < T_c$!

Since $|\vec{B}| = 0$ inside a superconductor, how does it behave microscopically? (i.e., what happens at the surface?)

London equation:

First consider a "perfect" conductor, and a momentum $\vec{E}$-field

$$\frac{d\vec{v}}{dt} = -\frac{e\vec{E}}{m} \qquad \frac{d\vec{j}}{dt} = \frac{ne^2\vec{E}}{m_e}$$

Faraday's law:

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{E} = \frac{m_e}{ne^2}\nabla \times \frac{d\vec{j}}{dt} \Longrightarrow \nabla \times \frac{d\vec{j}}{dt} = -\frac{ne^2}{mc}\frac{\partial \vec{B}}{\partial t}$$

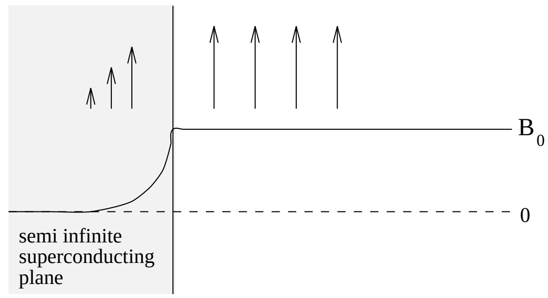
$$\frac{\partial}{\partial t}\left(\nabla \times \vec{j} + \frac{ne^2}{mc}\vec{B}\right) = 0 \tag{1}$$

$$\text{and} \qquad \nabla \times \vec{B} = \frac{4\pi}{c}\vec{j} \tag{2}$$

* These two equations relate $\vec{B}$ and $\vec{j}$ for a perfect conductor, they also allow time independent solution of $\vec{B}$ and $\vec{j}$

(1) $\nabla\times\vec{j}+\frac{ne^2}{mc}\vec{B}=const.=???\to 0$ (is what London said for a superconductor)

$\nabla\times$ both sides of (2)



$$\nabla\times(\nabla\times\vec{B})=\frac{4\pi}{c}\nabla\times\vec{j}$$

$$-\nabla^2\vec{B}=\frac{4\pi}{c}\left(-\frac{ne^2}{mc}\vec{B}\right)$$

$$\nabla^2\vec{B}-4\pi\frac{ne^2}{mc^2}\vec{B}=0 \qquad B_x(\hat{z})=B_0e^{-x/\Lambda} \qquad \text{If } B_a=B_0\hat{z}$$

$$\Lambda\equiv\left(\frac{mc^2}{4\pi ne^2}\right)^{1/2}$$ London penetration depth $\sim$ 100-1000Å

NOTES:

$$\nabla\times(\nabla\times\vec{B})\implies-\nabla^2\vec{B}+\nabla(\nabla\cdot B)$$

$$\nabla\times\vec{B}=\begin{vmatrix} i & j & k\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ B_x & B_y & B_z\end{vmatrix}=\left(\frac{\partial B_z}{\partial y}-\frac{\partial B_y}{\partial z}\right)\hat{i}+\left(\frac{\partial B_x}{\partial z}-\frac{\partial B_z}{\partial x}\right)\hat{j}+\left(\frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}\right)\hat{k}$$

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$$\begin{aligned}\left(\frac{\partial^2}{\partial x^2}-\nabla^2\right)B_x+\frac{\partial^2 B_y}{\partial x\partial y}+\frac{\partial B_z}{\partial x\partial z} &= -\nabla^2 B_y+\frac{\partial}{\partial x}\left(\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+\frac{\partial B_z}{\partial z}\right)\\ \left(\frac{\partial^2}{\partial y^2}-\nabla^2\right)B_y+\ldots\ldots &= -\nabla^2 B_y+\frac{\partial}{\partial y}(\nabla\cdot\vec{B})\\ \left(\frac{\partial^2}{\partial z^2}-\nabla^2\right)B_z+\ldots\ldots &= -\nabla^2 B_z+\frac{\partial}{\partial z}(\nabla\cdot\vec{B})\end{aligned}$$

$$-\nabla^2\vec{B}+\nabla\,(\nabla\cdot B)=\nabla\times(\nabla\times\vec{B}) \quad\text{and}\quad \nabla\cdot\vec{B}=0$$

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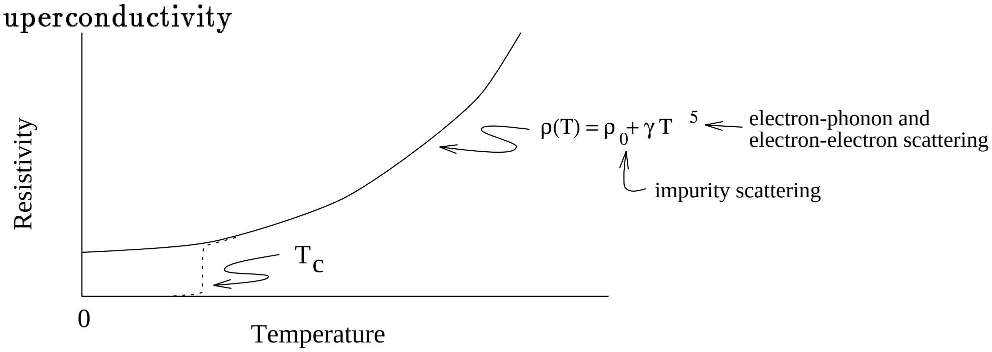

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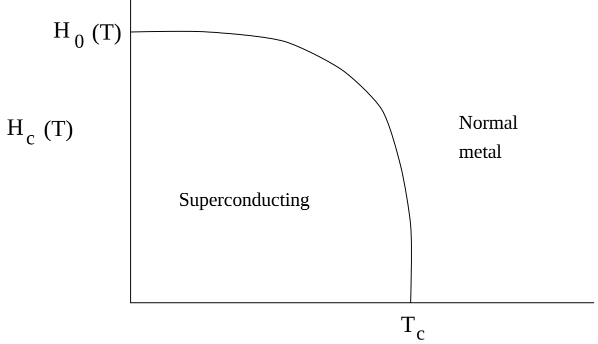

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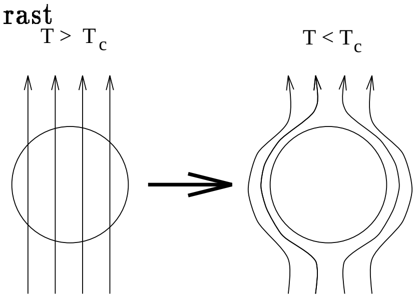

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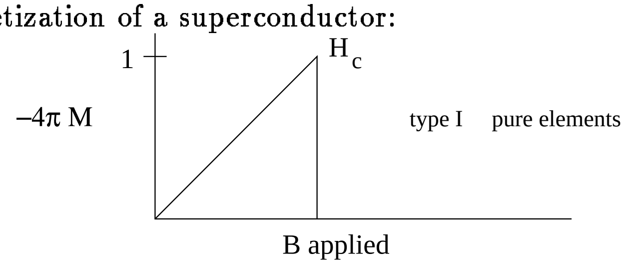

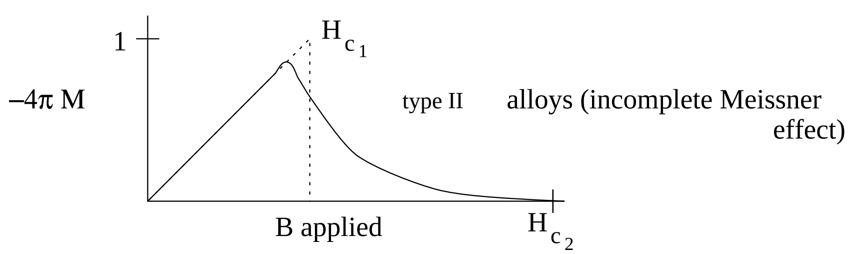

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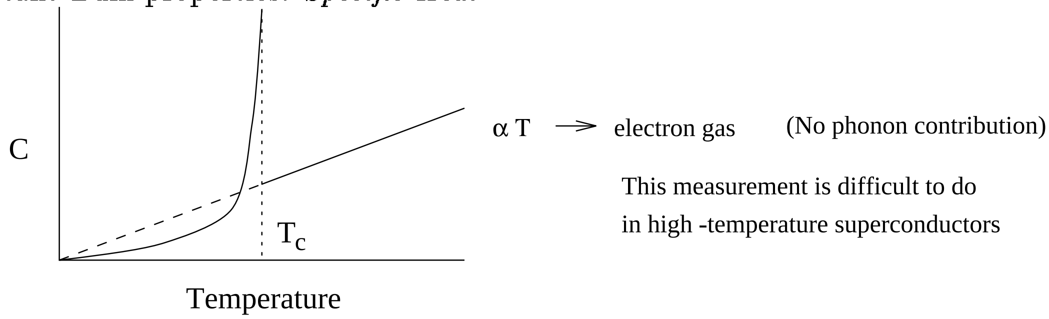

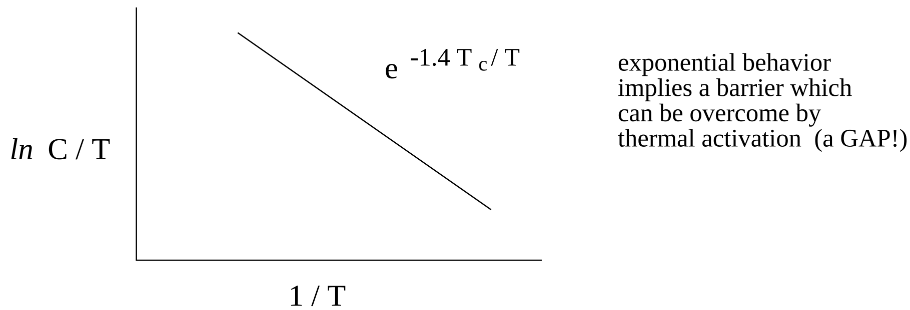

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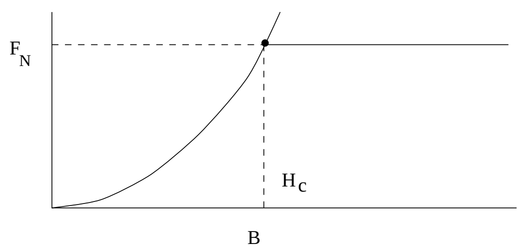

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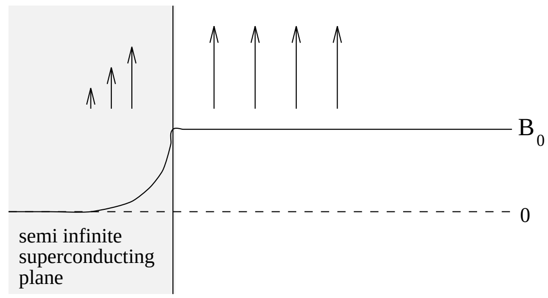

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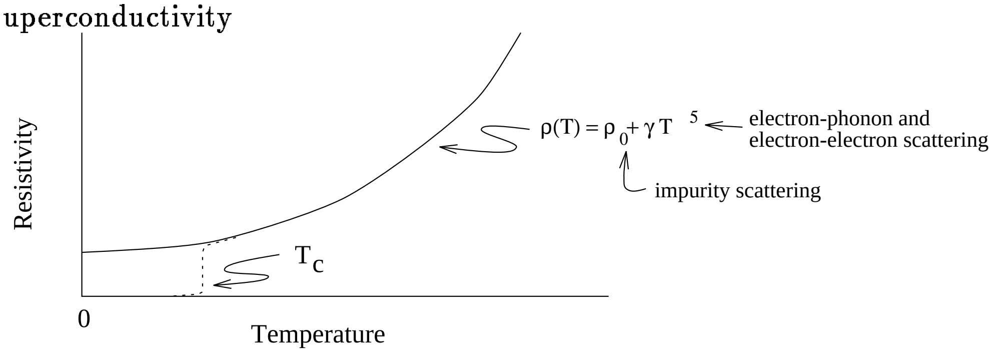

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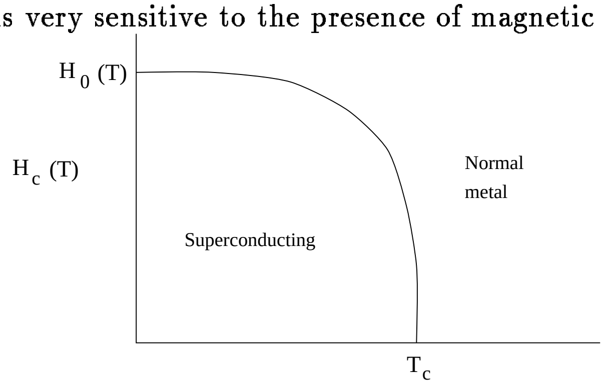

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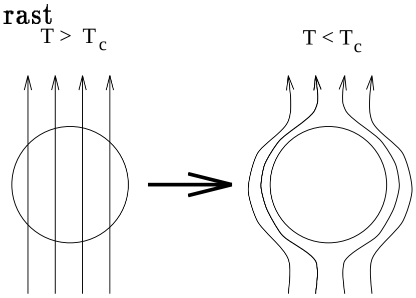

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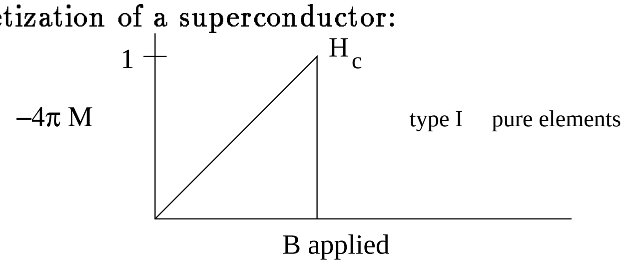

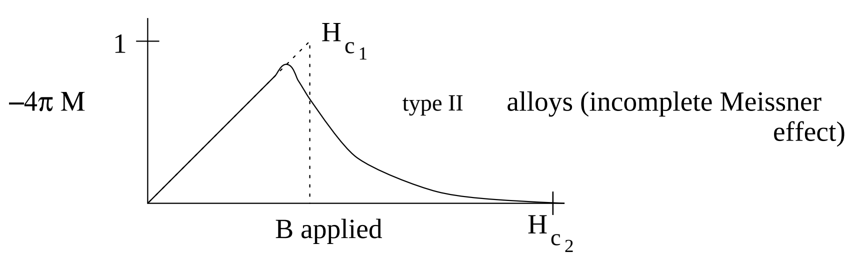

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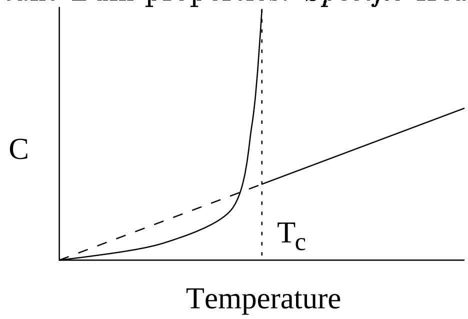

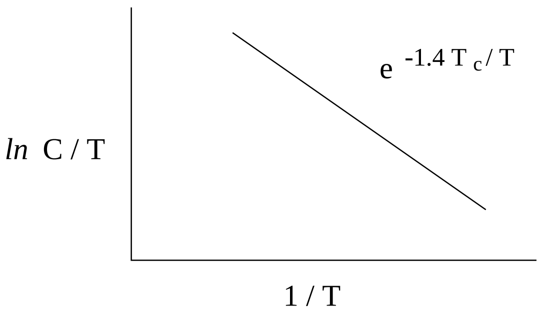

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$$F_s(B_a) - F_s(0) = B_a^2/8\pi$$

for the normal $F_N(B_a) = F_N(0)$ ignore, $\chi_N \approx 0$!

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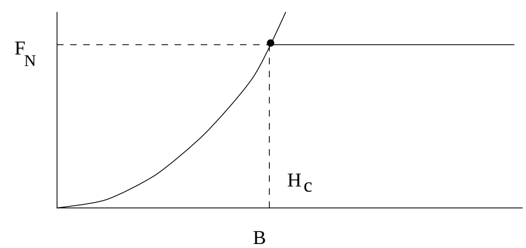

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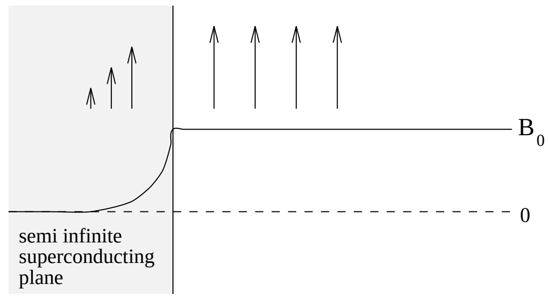

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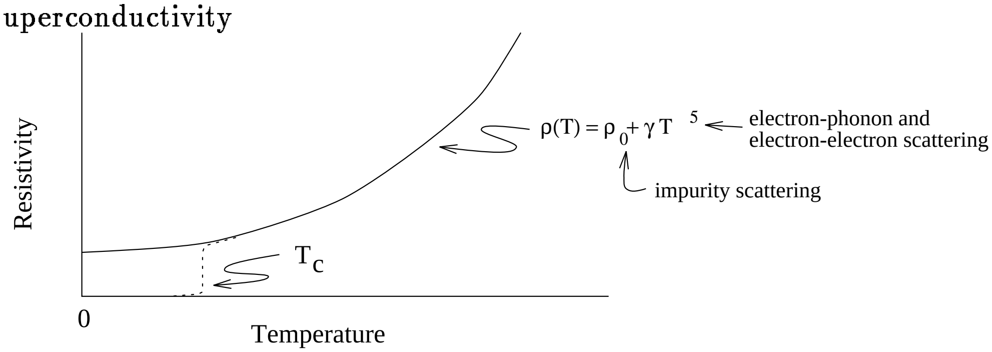
Physics 0551 Lecture #33

Title: Superconductivity



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} i.e., spin is *not* compatible with superconductivity (conventional)

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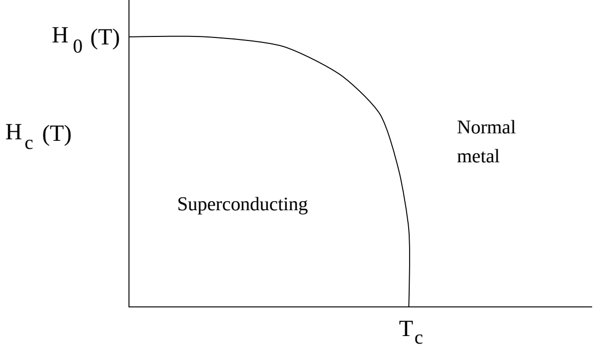

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There is complete expulsion of applied magnetic fields.

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If $\rho = 0$, then $\vec{E} = 0$

But $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (cgs) $\quad \nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$ (SI)

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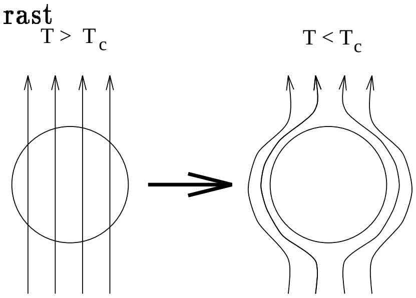

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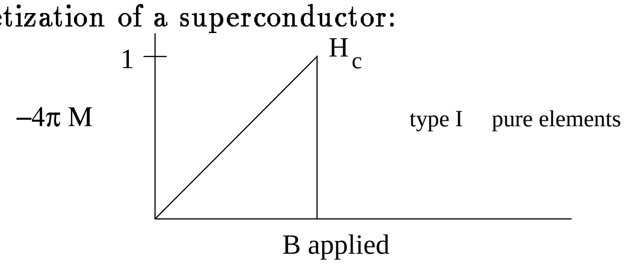

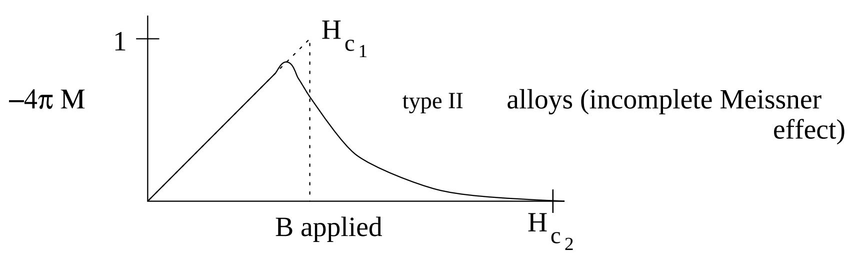

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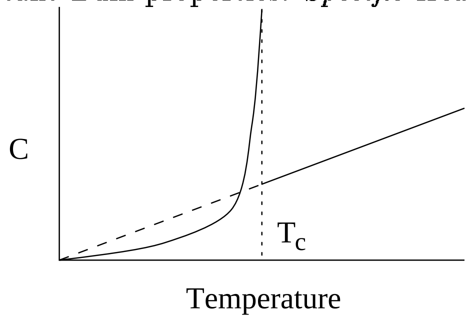
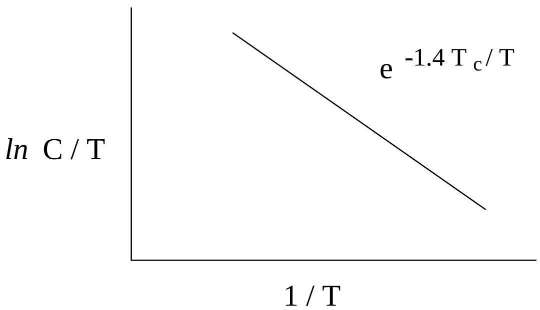


α T $\rightarrow$ electron gas (No phonon contribution)

This measurement is difficult to do in high -temperature superconductors



exponential behavior implies a barrier which can be overcome by thermal activation (a GAP!)

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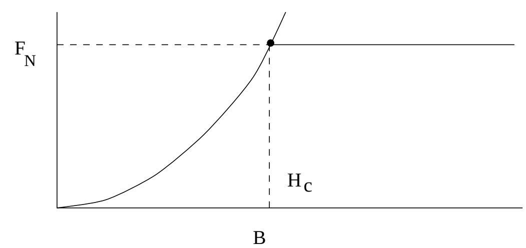

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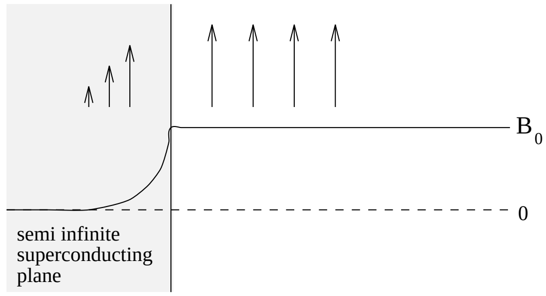

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Physics 0551 Lecture #33

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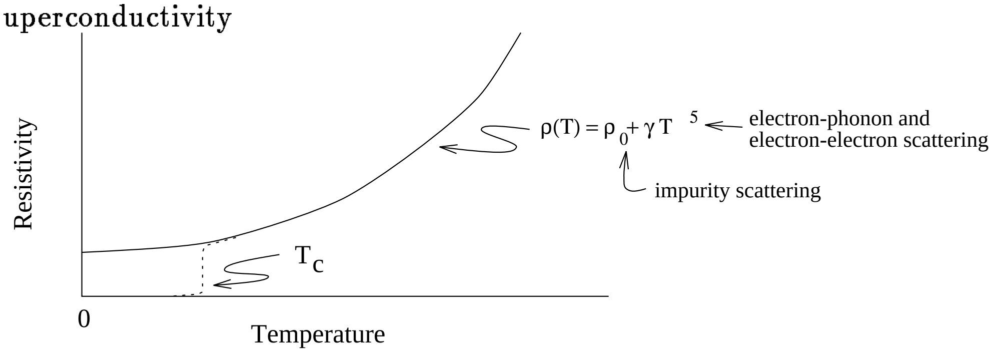

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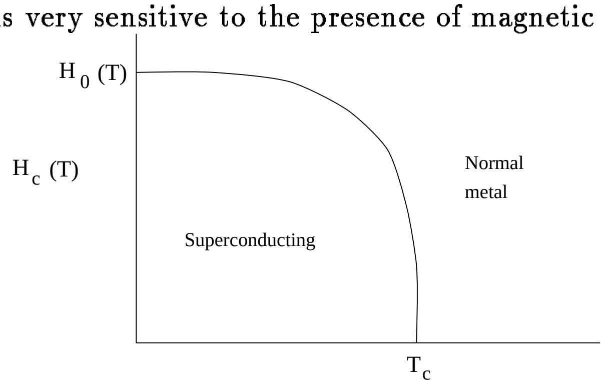

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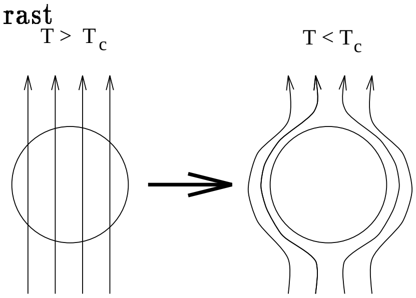

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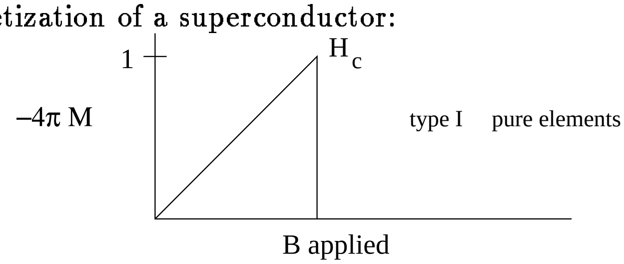

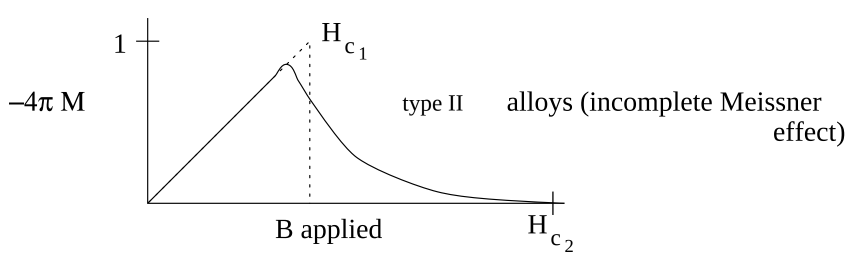

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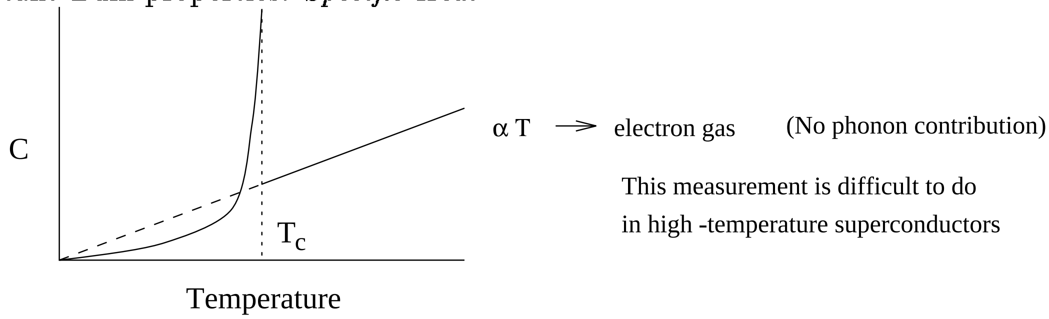

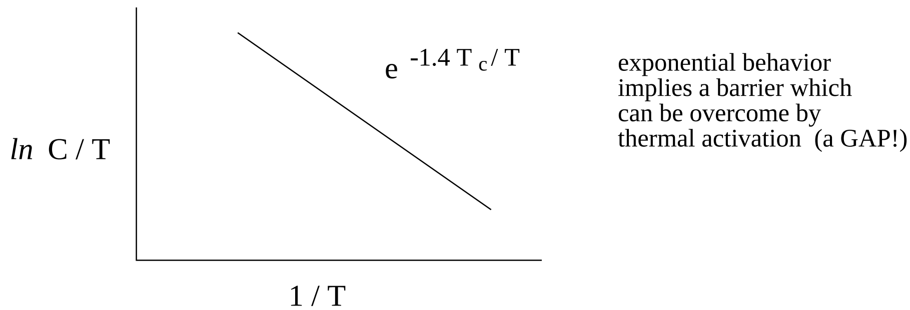

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$$\begin{aligned} d(U - TS) &= -\vec{M} \cdot d\vec{H} \\ dU &= TdS - \vec{M} \cdot d\vec{H} \\ dU &= dQ - dW \end{aligned}$$

How much energy is stored?

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for the normal $F_N(B_a) = F_N(0)$ ignore, $\chi_N \approx 0$!

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$$F_N(H_c) = F_S(H_c) \qquad \text{No latent heat}$$

therefore $F_N(0) = F_N(H_c) = F_S(0) + B_{H_c}^2/8\pi$

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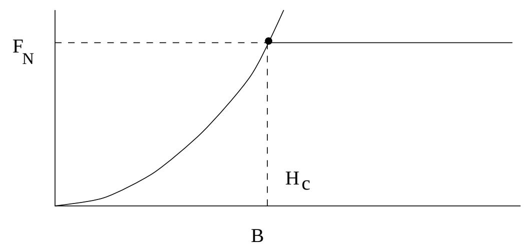

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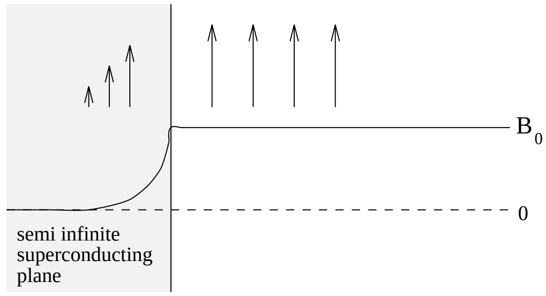
$\nabla\times$ both sides of (2)



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$$\nabla\times(\nabla\times\vec{B}) \implies -\nabla^2\vec{B}+\nabla(\nabla\cdot B)$$

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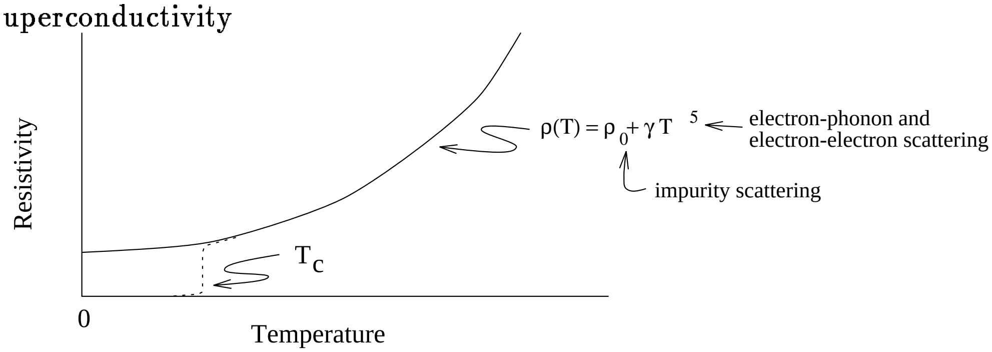
Physics 0551 Lecture #33

Title: Superconductivity



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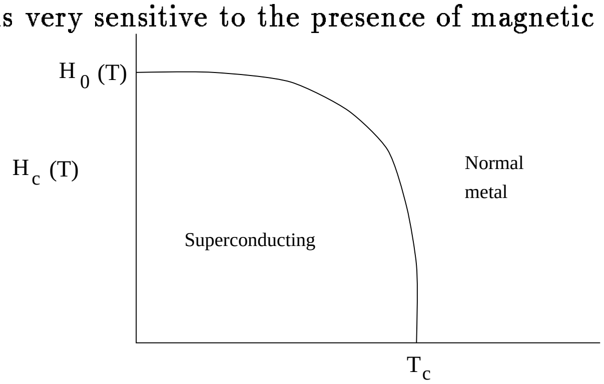

Second test for superconductivity is the Meissner/Ochsenfeld effect.

There is complete expulsion of applied magnetic fields.

B=0 inside superconducting regions of a superconductor

$\Longrightarrow$ Note that a superconductor is <u>NOT</u> a perfect conductor

Ohm's Law $\vec{E} = \rho \vec{J}$

If $\rho = 0$, then $\vec{E} = 0$

But $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (cgs) $\quad \nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$ (SI)

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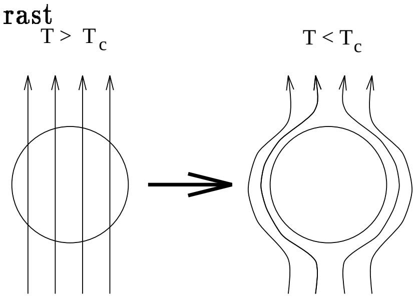
In contrast



i.e., superconductors are *perfect* diamagnets

$$\vec{B} = \vec{H} + 4\pi \vec{M} \quad \text{where} \quad \chi \equiv \frac{\vec{M}}{\vec{H}} \quad \text{(cgs)}$$

Since $\vec{B} = 0$, $\quad \chi = -\frac{1}{4\pi}$ $\quad$ (Not $-\infty$)

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$$\chi_{\mathrm{HT}_c} < -\frac{1}{4\pi} \quad \text{or} \quad \% \text{ superconducting} = \frac{\chi_{\mathrm{HT}_c}}{-\frac{1}{4\pi}}$$

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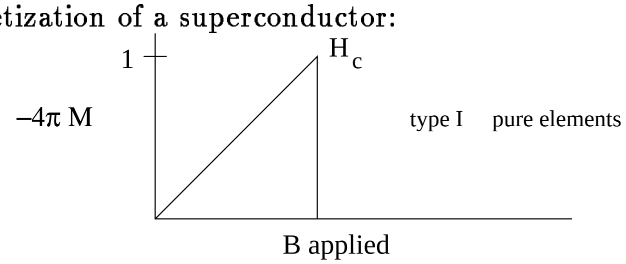
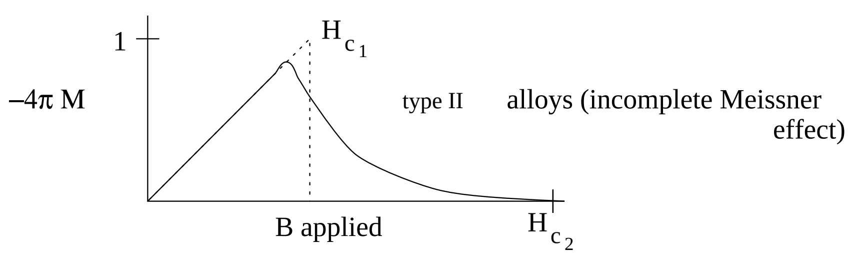
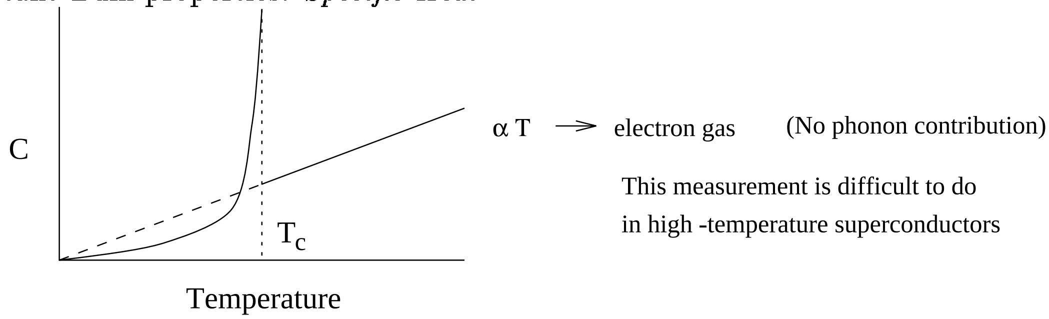
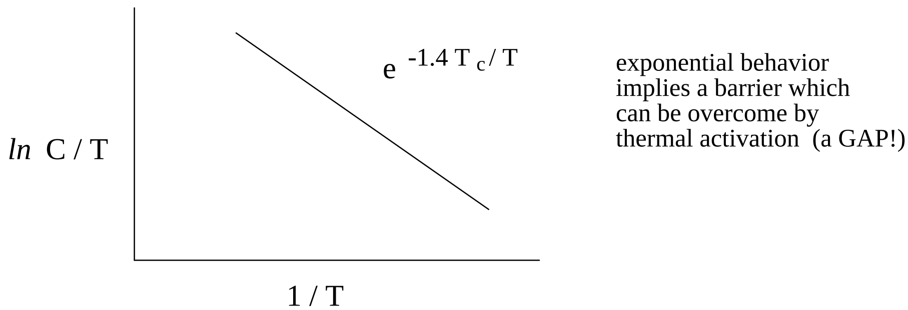




Other important Bulk properties: *Specific Heat*





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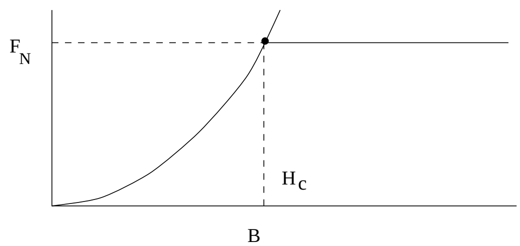

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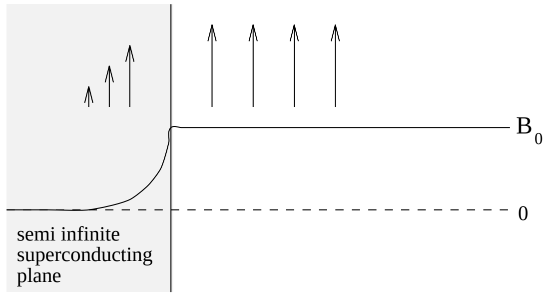

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Physics 0551 Lecture #33

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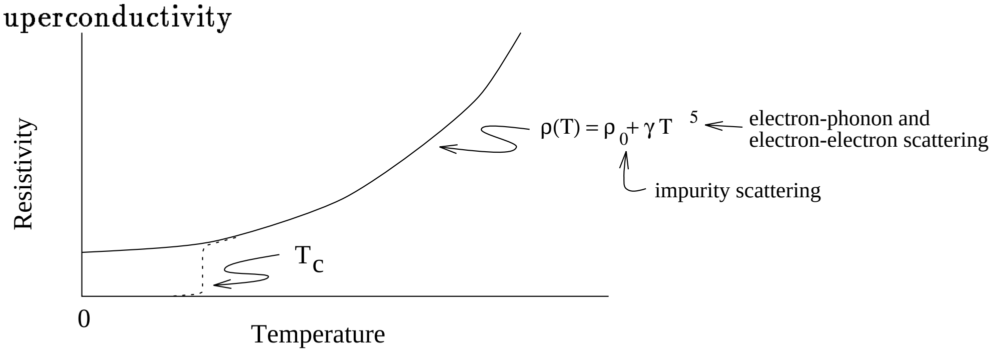

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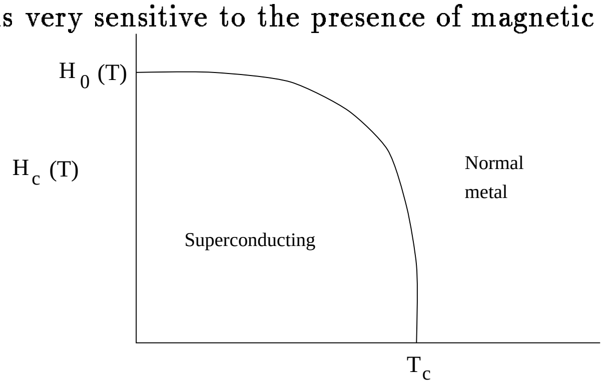

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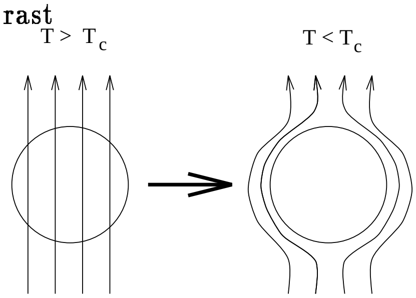

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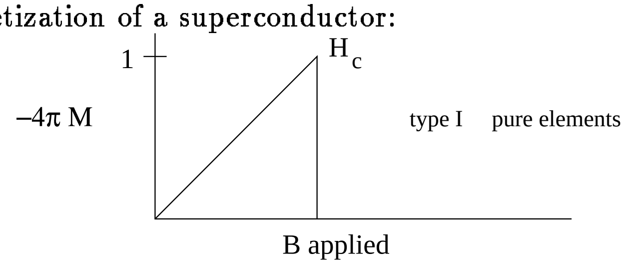

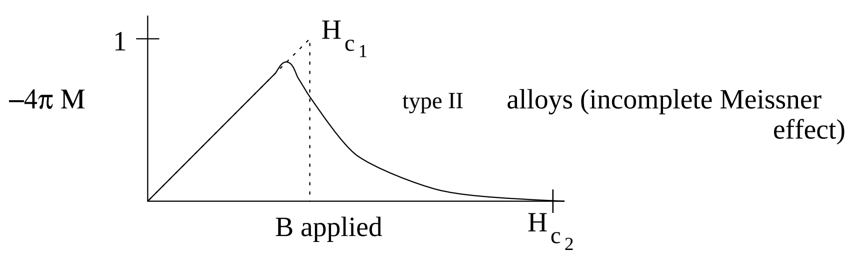

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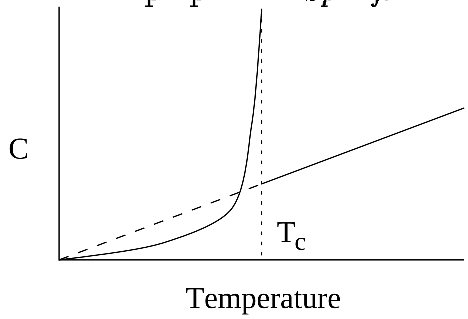
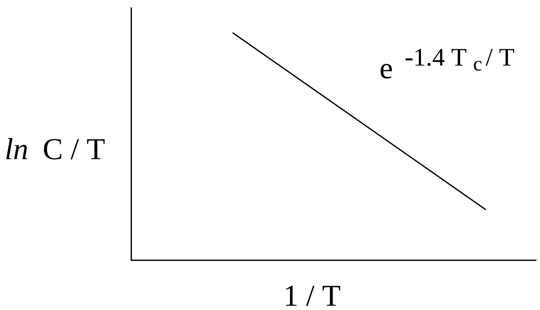


α T → electron gas (No phonon contribution)

This measurement is difficult to do in high -temperature superconductors



exponential behavior implies a barrier which can be overcome by thermal activation (a GAP!)

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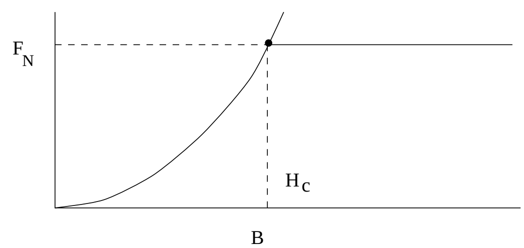

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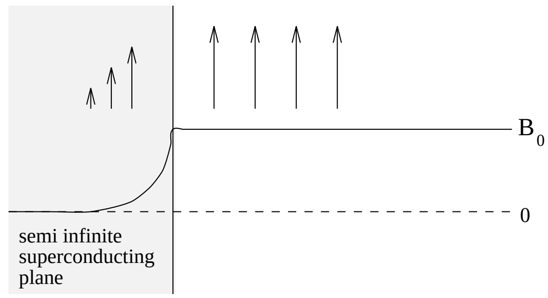

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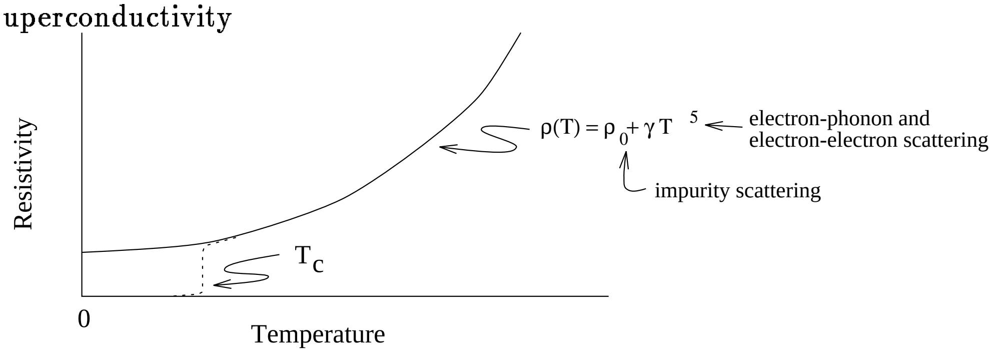
Physics 0551 Lecture #33

Title: Superconductivity



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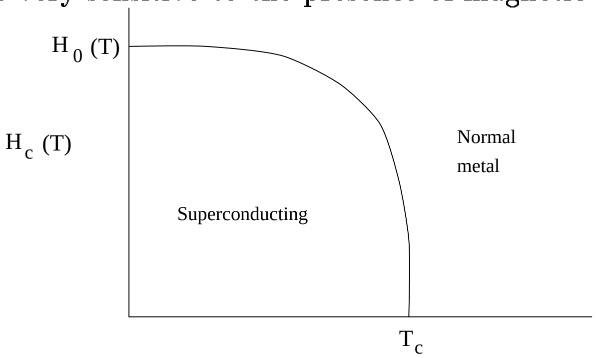

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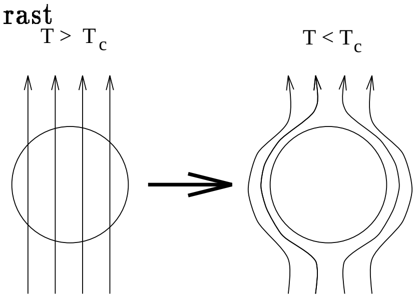

i.e., superconductors are *perfect* diamagnets

$$\vec{B} = \vec{H} + 4\pi\vec{M} \qquad \text{where} \qquad \chi \equiv \frac{\vec{M}}{\vec{H}} \qquad \text{(cgs)}$$

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$$\chi_{\mathrm{HT}_c} < -\frac{1}{4\pi} \qquad \text{or} \quad \%\ \text{superconducting} = \frac{\chi_{\mathrm{HT}_c}}{-\frac{1}{4\pi}}$$

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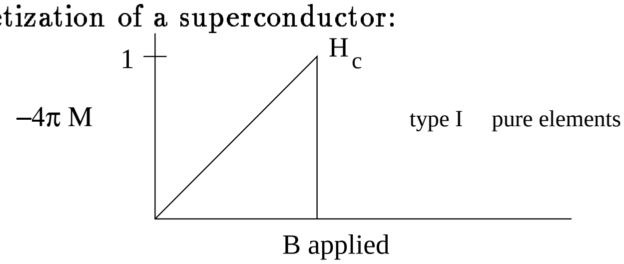

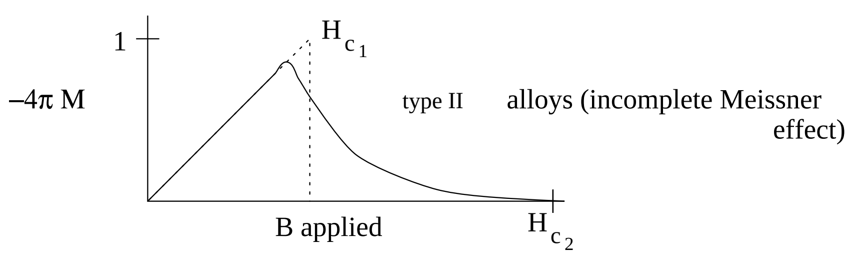

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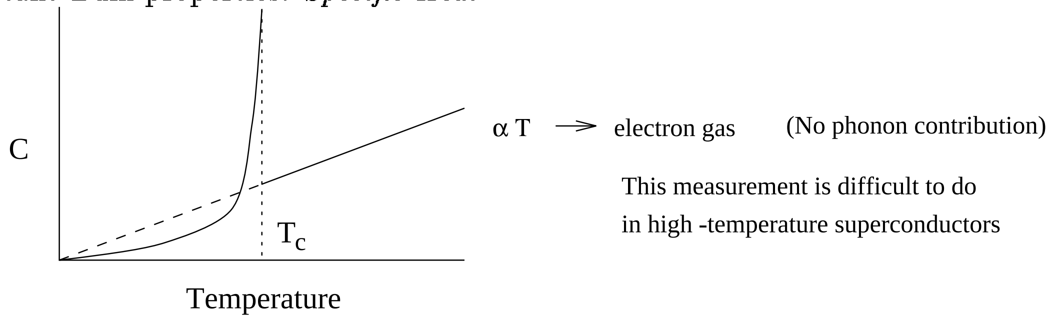

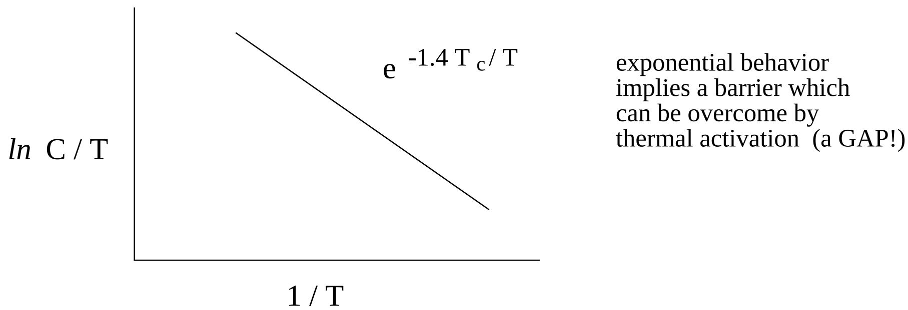

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$$F_s(B_a) - F_s(0) = B_a^2/8\pi$$

for the normal $F_N(B_a) = F_N(0)$ ignore, $\chi_N \approx 0$!

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$$F_N(H_c) = F_S(H_c) \qquad \text{No latent heat}$$

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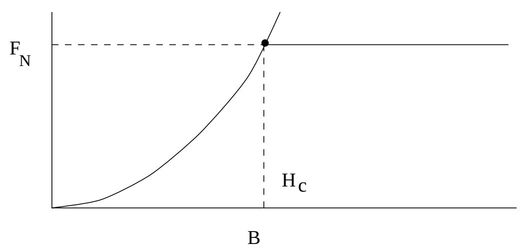

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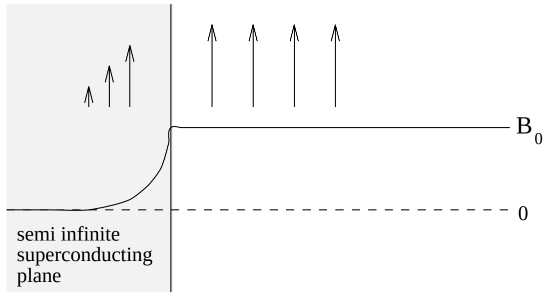

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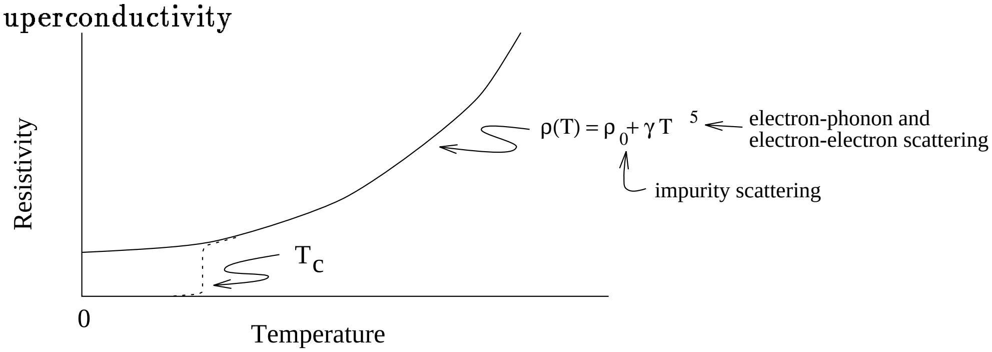

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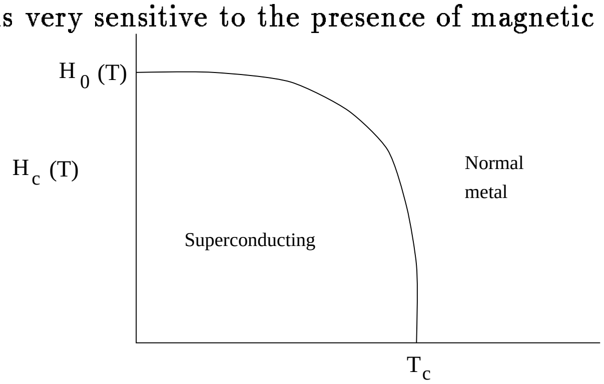

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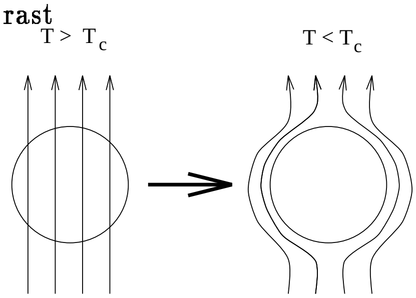

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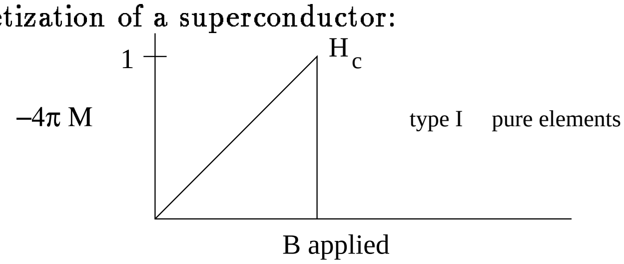

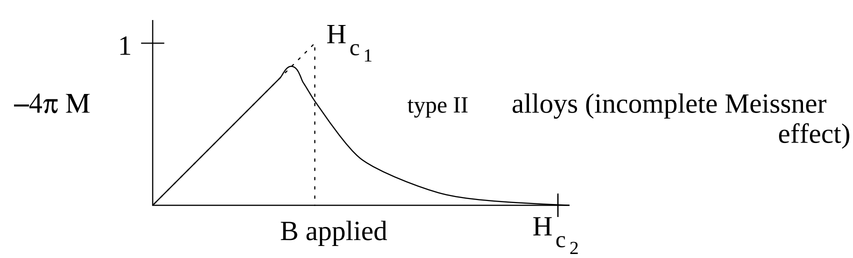

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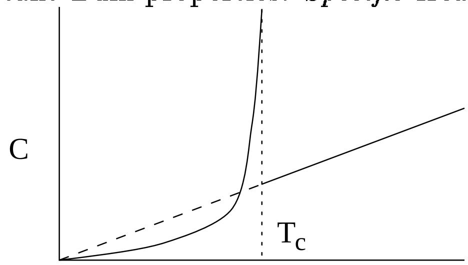

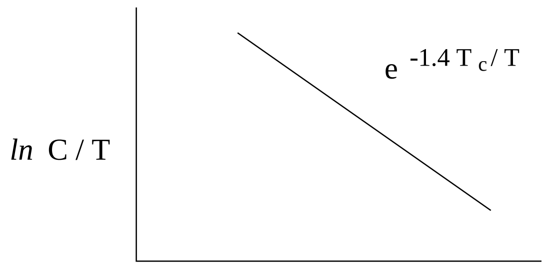

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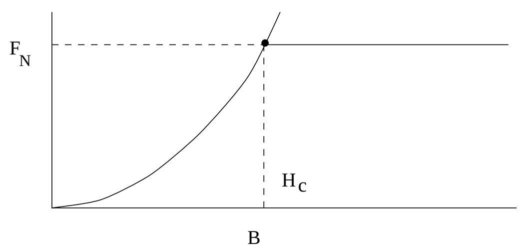

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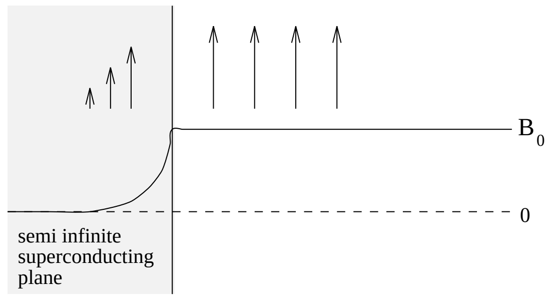

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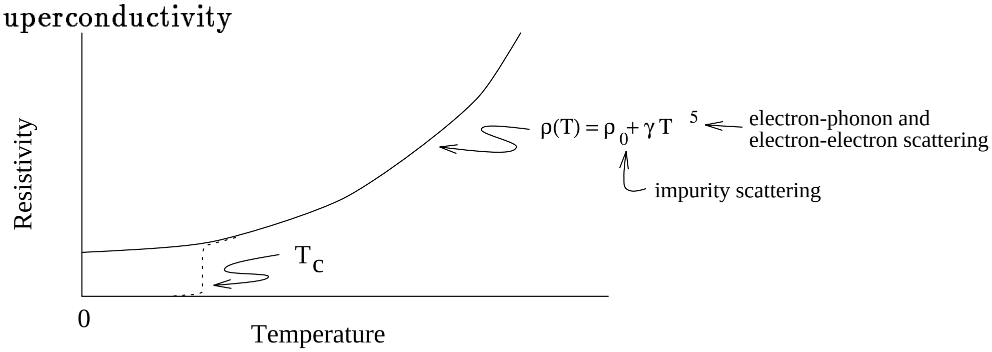
Physics 0551 Lecture #33

Title: Superconductivity



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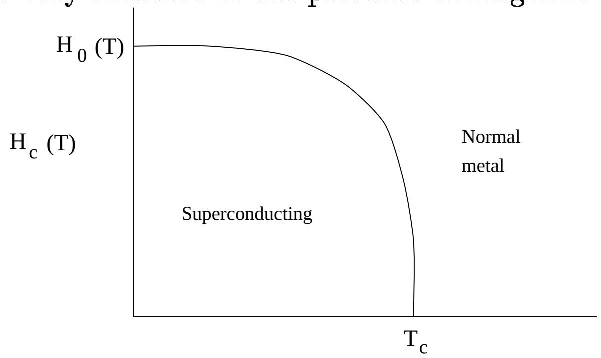

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There is complete expulsion of applied magnetic fields.

B=0 inside superconducting regions of a superconductor

$\Longrightarrow$ Note that a superconductor is <u>NOT</u> a perfect conductor

Ohm's Law $\vec{E} = \rho \vec{J}$

If $\rho = 0$, then $\vec{E} = 0$

But $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (cgs) $\quad \nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$ (SI)

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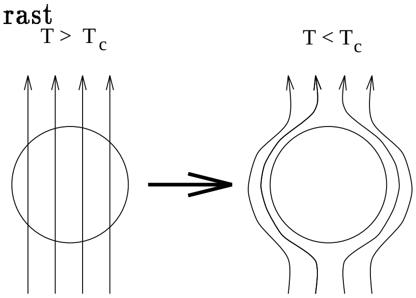

i.e., superconductors are *perfect* diamagnets

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$$\chi_{\mathrm{HT}_c} < -\frac{1}{4\pi} \quad \text{or} \quad \%\ \text{superconducting} = \frac{\chi_{\mathrm{HT}_c}}{-\frac{1}{4\pi}}$$

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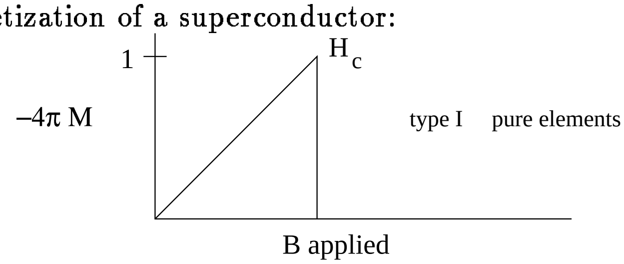

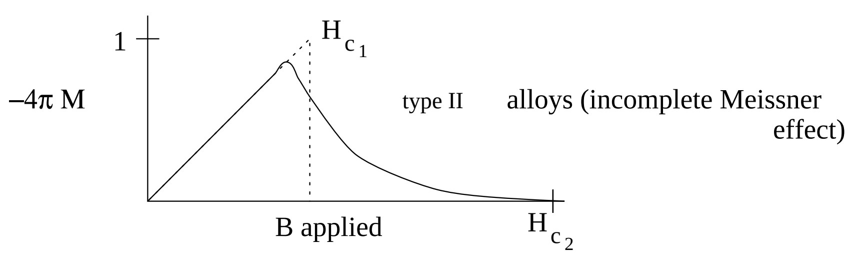

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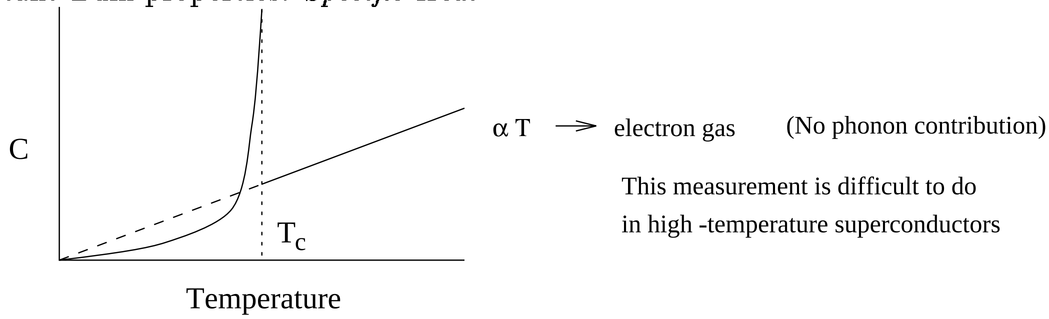

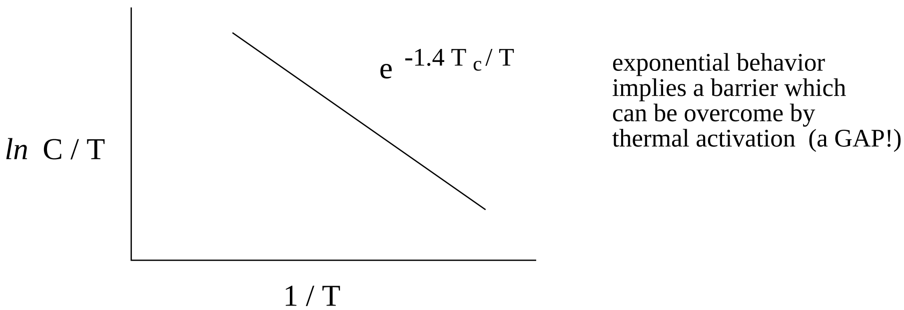

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We want the change in the Free energy (F) at constant temperature

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$$0 \text{ to } B_a \text{ is } B_{applied}$$

$$F_s(B_a) - F_s(0) = B_a^2/8\pi$$

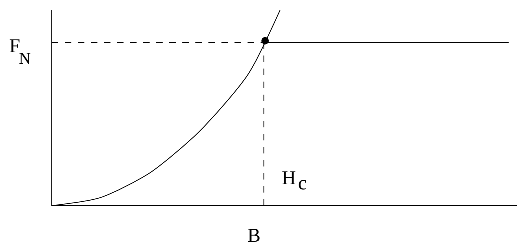
for the normal $F_N(B_a) = F_N(0)$ ignore, $\chi_N \approx 0!$

At H_c (critical field)

$$F_N(H_c) = F_S(H_c) \qquad \text{No latent heat}$$

therefore $F_N(0) = F_N(H_c) = F_S(0) + B_{H_c}^2/8\pi$

so $F_N(0) - F_s(0) = B_{H_c}^2/8\pi$



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Since $|\vec{B}| = 0$ inside a superconductor, how does it behave microscopically? (i.e., what happens at the surface?)

London equation:

First consider a "perfect" conductor, and a momentum $\vec{E}$-field

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$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

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$$\frac{\partial}{\partial t}\left(\nabla \times \vec{j} + \frac{ne^2}{mc}\vec{B}\right) = 0 \qquad (1)$$

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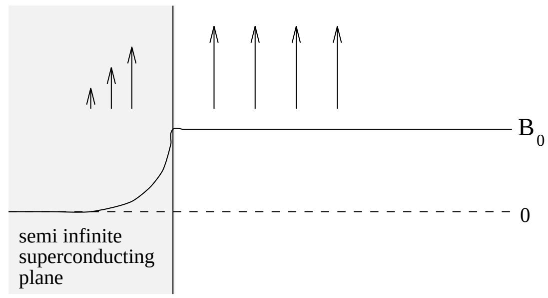

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Physics 0551 Lecture #33

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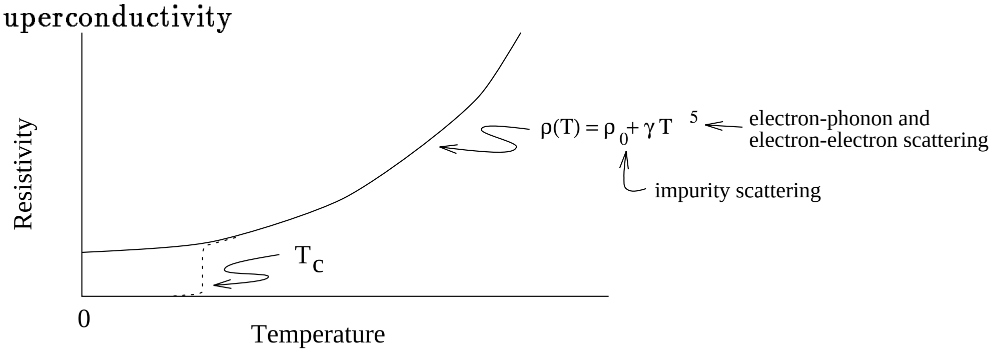

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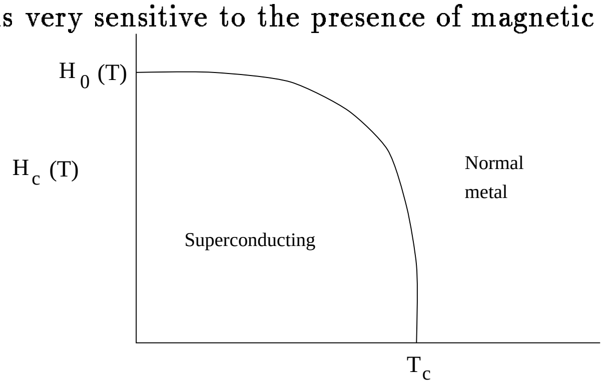

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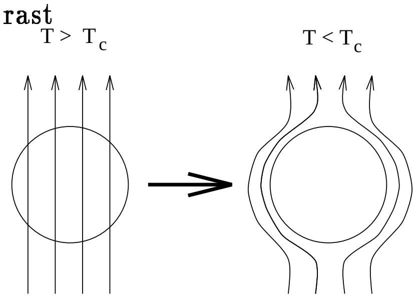

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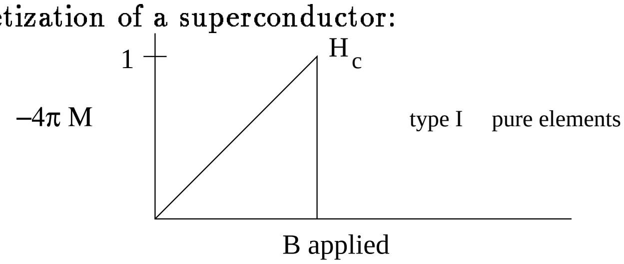

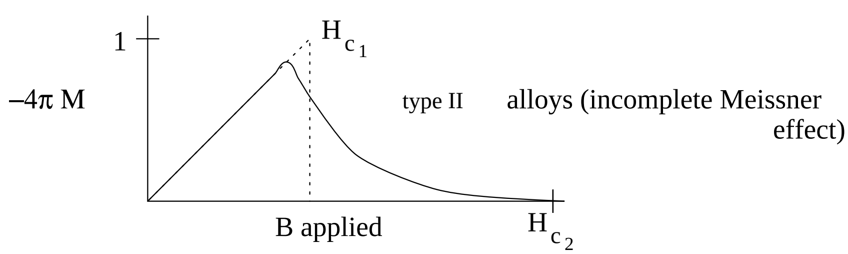

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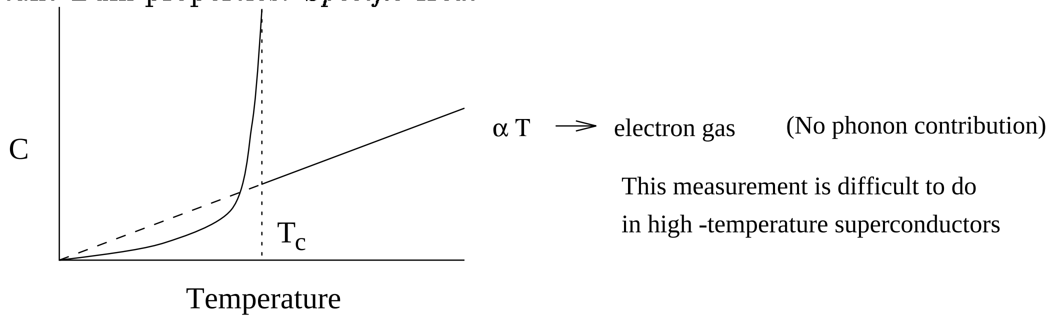

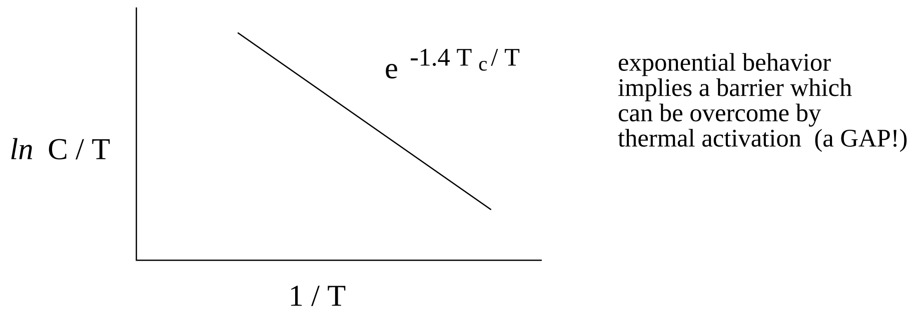

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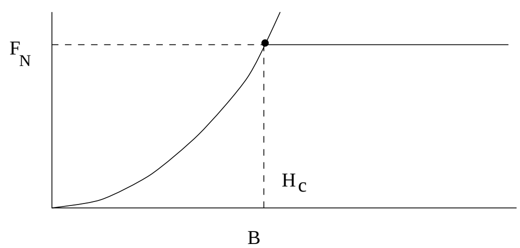

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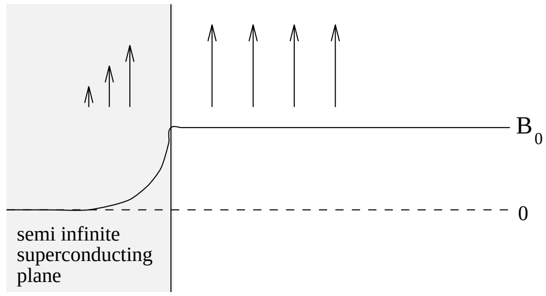

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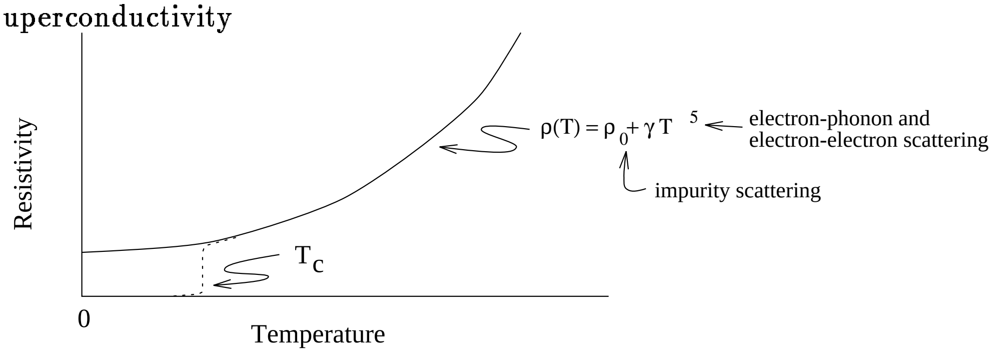
$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \left(\frac{\partial B_z}{\partial y} - \frac{\partial By}{\partial z}\right) & \left(\frac{\partial B_x}{\partial z} - \frac{\partial Bz}{\partial x}\right) & \left(\frac{\partial B_y}{\partial x} - \frac{\partial Bx}{\partial y}\right) \end{vmatrix} = \begin{matrix} \left(\frac{\partial^2 B_y}{\partial x \partial y} - \frac{\partial^2 Bx}{\partial y^2}\right) - \left(\frac{\partial^2 B_x}{\partial z^2} - \frac{\partial^2 B_z}{\partial x \partial z}\right) & \hat{i} + \\ \left(\frac{\partial^2 B_z}{\partial y \partial z} - \frac{\partial^2 Bx}{\partial z^2}\right) - \left(\frac{\partial^2 B_y}{\partial x^2} - \frac{\partial^2 B_x}{\partial y \partial x}\right) & \hat{j} + \\ (\qquad\qquad) - (\qquad\qquad) & \hat{k} \end{matrix}$$

$$\begin{aligned} \left(\frac{\partial^2}{\partial x^2} - \nabla^2\right) B_x + \frac{\partial^2 B_y}{\partial x \partial y} + \frac{\partial B_z}{\partial x \partial z} &= -\nabla^2 B_y + \frac{\partial}{\partial x}\left(\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}\right) \\ \left(\frac{\partial^2}{\partial y^2} - \nabla^2\right) B_y + ... &= -\nabla^2 B_y + \frac{\partial}{\partial y}(\nabla \cdot \vec{B}) \\ \left(\frac{\partial^2}{\partial z^2} - \nabla^2\right) B_z + ... &= -\nabla^2 B_z + \frac{\partial}{\partial z}(\nabla \cdot \vec{B}) \end{aligned}$$

$$-\nabla^2\vec{B} + \nabla(\nabla \cdot B) = \nabla \times (\nabla \times \vec{B}) \quad \text{and} \quad \nabla \cdot \vec{B} = 0$$

Physics 0551 Lecture #33

Title: Superconductivity



Superconductors are unusual for a variety of reasons. The first observations were made in 1911 by Kamerlingh Onnes (Denmark) on the low temperature behavior of Hg (mercury) and he observed a complete absence of ρ in the vicinity of 4°K.

Note: There is nothing in the "classical" Drude' theory of metals which can account for this behavior.

There are many systematics which accompany the occurrence of superconductivity

No monovalent Elements
No ferromagnetic elements
No Rare-earths
} i.e., spin is *not* compatible with superconductivity (conventional)

Normal BCS (Bardeen-Cooper-Schrieffer) (1957)

10^{-2}K $< T_c <$ 23°K or so Nb_3Ge 23.2°K

High T_c superconductors are at odds with conventional wisdom.

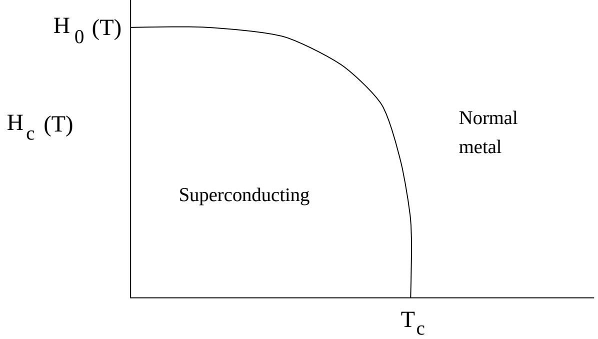
Oxides	35°K	$La_{1.8}\,Sr_{.2}CuO_4$	Bednoorz and Müller
	95°K	$(YBa_2)\,Cu_3O_7$	
	125°K	Th- .... -Cu)	

Note: Anomalous behavior in ρ is often due to structural transformation

Organic superconductor (BEDT-TTF)$_2Cu(NCS)_2$ 10.4°K

T_c is very sensitive to applied magnetic fields.

T_c is very sensitive to the presence of magnetic impurities.



Second test for superconductivity is the Meissner/Ochsenfeld effect.

There is complete expulsion of applied magnetic fields.

B=0 inside superconducting regions of a superconductor

$\Longrightarrow$ Note that a superconductor is NOT a perfect conductor

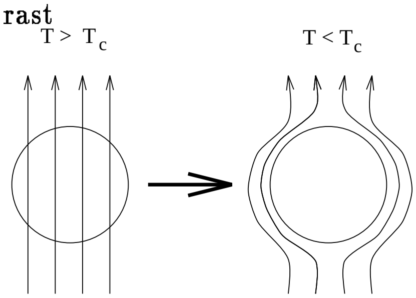
Ohm's Law $\vec{E} = \rho\vec{J}$

If $\rho = 0$, then $\vec{E} = 0$

But $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (cgs) $\quad \nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$ (SI)

If $\vec{E} = 0$ then $\frac{\partial \vec{B}}{\partial t} = 0$ $\quad \Rightarrow$ Magnetic flux cannot be expelled!

In contrast



i.e., superconductors are *perfect* diamagnets

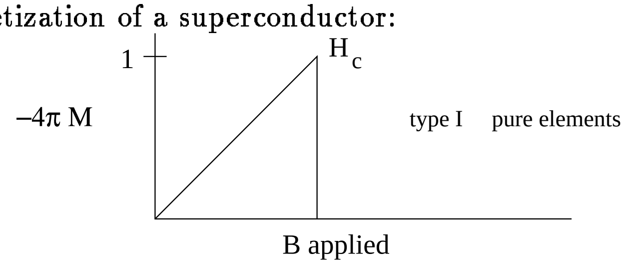
$$\vec{B} = \vec{H} + 4\pi\vec{M} \quad \text{where} \quad \chi \equiv \frac{\vec{M}}{\vec{H}} \quad \text{(cgs)}$$

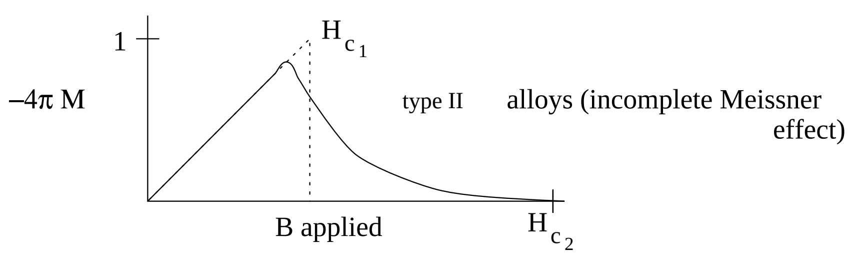
Since $\vec{B} = 0$, $\quad \chi = -\frac{1}{4\pi}$ $\quad$ (Not $-\infty$)

* Note since high T_c samples are often multiphase

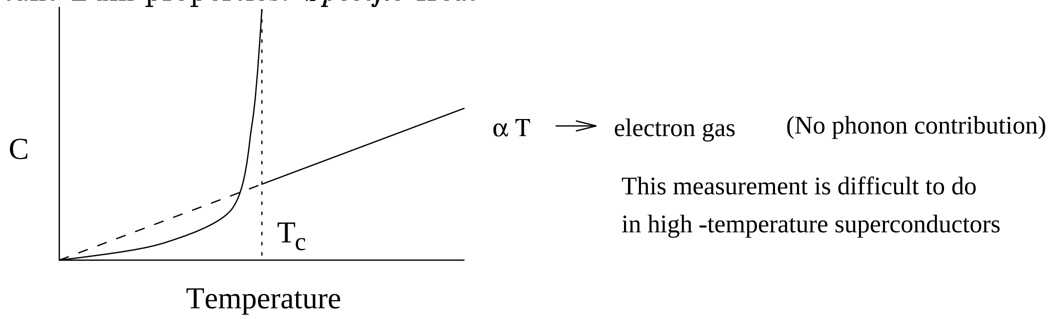
$$\chi_{\mathrm{HT}_c} < -\frac{1}{4\pi} \quad \text{or} \quad \%\ \text{superconducting} = \frac{\chi_{\mathrm{HT}_c}}{-\frac{1}{4\pi}}$$

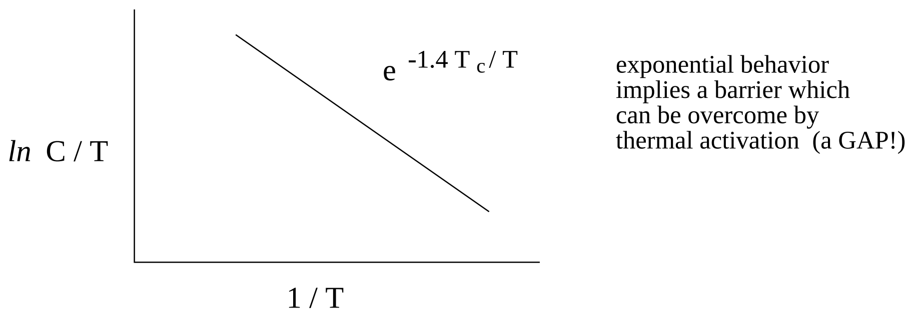
Magnetization of a superconductor:





Other important Bulk properties: *Specific Heat*





The entropy of the S.C. state is smaller than in the normal state.

Isotope effect (factor of 1/2 is approximate)

$T_c \propto \frac{1}{M^{1/2}}$ $\Longrightarrow$ implies a *phonon* mechanism

Thermodynamics of a superconductor:

S.C. $\leftrightarrow$ Normal Transition is a reversible process, hence a thermodynamic treatment is possible

2^{nd} Order Transition $\rightarrow$ NO LATENT HEAT

$$\begin{aligned} d(U - TS) &= -\vec{M} \cdot d\vec{H} \\ dU &= TdS - \vec{M} \cdot d\vec{H} \\ dU &= dQ - dW \end{aligned}$$

How much energy is stored?

$$\vec{M} \cdot d\vec{H}$$

$$dU = TdS - \vec{M} \cdot d\vec{B}_a \qquad B_a \Longrightarrow \text{ magnetic field applied}$$

$$\text{but } M = -\frac{1}{4\pi} B_a$$

We want the change in the Free energy (F) at constant temperature

$$dF = d(U - TS) = -\vec{M} \cdot d\vec{B}_a$$

$$\text{but } \vec{M} = \chi \vec{H} = -\frac{1}{4\pi}\, \vec{B}_a$$

$$dF = \frac{1}{4\pi}\,\vec{B}_a \cdot d\vec{B}_a \qquad F_{superconducting}$$

$$0 \text{ to } B_a \text{ is } B_{applied}$$

$$F_s(B_a) - F_s(0) = B_a^2/8\pi$$

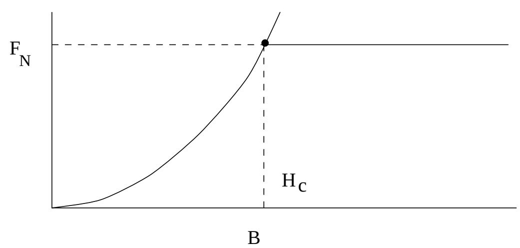
for the normal $F_N(B_a) = F_N(0)$ ignore, $\chi_N \approx 0$!

At H_c (critical field)

$$F_N(H_c) = F_S(H_c) \qquad \text{No latent heat}$$

therefore $F_N(0) = F_N(H_c) = F_S(0) + B_{H_c}^2/8\pi$

so $F_N(0) - F_s(0) = B_{H_c}^2/8\pi$



$F_s(0) < F_N(0)$ for $T < T_c$!

Since $|\vec{B}| = 0$ inside a superconductor, how does it behave microscopically? (i.e., what happens at the surface?)

London equation:

First consider a "perfect" conductor, and a momentum $\vec{E}$-field

$$\frac{d\vec{v}}{dt} = -\frac{e\vec{E}}{m} \qquad \frac{d\vec{j}}{dt} = \frac{ne^2\vec{E}}{m_e}$$

Faraday's law:

$$\begin{aligned}
\nabla \times \vec{E} &= -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \\
\nabla \times \vec{E} &= \frac{m_e}{ne^2}\,\nabla \times \frac{d\vec{j}}{dt} \Longrightarrow \nabla \times \frac{d\vec{j}}{dt} = -\frac{ne^2}{mc}\frac{\partial \vec{B}}{\partial t}
\end{aligned}$$

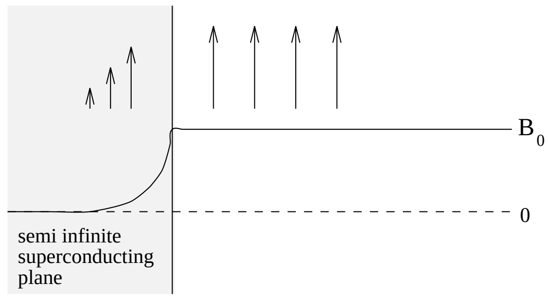
$$\frac{\partial}{\partial t}\left(\nabla \times \vec{j} + \frac{ne^2}{mc}\vec{B}\right) = 0 \qquad (1)$$

$$\text{and} \qquad \nabla \times \vec{B} = \frac{4\pi}{c}\vec{j} \qquad (2)$$

* These two equations relate $\vec{B}$ and $\vec{j}$ for a perfect conductor, they also allow time independent solution of $\vec{B}$ and $\vec{j}$

(1) $\nabla \times \vec{j} + \frac{ne^2}{mc}\vec{B} = const. = ??? \rightarrow 0$ (is what London said for a superconductor)

$\nabla\times$ both sides of (2)



$$\nabla \times (\nabla \times \vec{B}) = \frac{4\pi}{c}\nabla \times \vec{j}$$

$$-\nabla^2 \vec{B} = \frac{4\pi}{c}\left(-\frac{ne^2}{mc}\vec{B}\right)$$

$$\nabla^2 \vec{B} - 4\pi \frac{ne^2}{mc^2}\vec{B} = 0 \qquad B_x(\hat{z}) = B_0 e^{-x/\Lambda} \qquad \text{If } B_a = B_0\hat{z}$$

$$\Lambda \equiv \left(\frac{mc^2}{4\pi ne^2}\right)^{1/2}$$

London penetration depth $\sim$ 100-1000Å

NOTES:

$$\nabla \times (\nabla \times \vec{B}) \implies -\nabla^2\vec{B} + \nabla(\nabla \cdot B)$$

$$\nabla \times \vec{B} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix} = \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right)\hat{i} + \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right)\hat{j} + \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)\hat{k}$$

$$\nabla \times (\nabla \times \vec{B}) \Longrightarrow$$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \left(\frac{\partial B_z}{\partial y} - \frac{\partial By}{\partial z}\right) & \left(\frac{\partial B_x}{\partial z} - \frac{\partial Bz}{\partial x}\right) & \left(\frac{\partial B_y}{\partial x} - \frac{\partial Bx}{\partial y}\right) \end{vmatrix} = \begin{matrix} \left(\frac{\partial^2 B_y}{\partial x \partial y} - \frac{\partial^2 Bx}{\partial y^2}\right) - \left(\frac{\partial^2 B_x}{\partial z^2} - \frac{\partial^2 B_z}{\partial x \partial z}\right) & \hat{i} + \\ \left(\frac{\partial^2 B_z}{\partial y \partial z} - \frac{\partial^2 Bx}{\partial z^2}\right) - \left(\frac{\partial^2 B_y}{\partial x^2} - \frac{\partial^2 B_x}{\partial y \partial x}\right) & \hat{j} + \\ (\qquad\qquad) - (\qquad\qquad) & \hat{k} \end{matrix}$$

$$\begin{aligned} \left(\frac{\partial^2}{\partial x^2} - \nabla^2\right) B_x + \frac{\partial^2 B_y}{\partial x \partial y} + \frac{\partial B_z}{\partial x \partial z} &= -\nabla^2 B_y + \frac{\partial}{\partial x}\left(\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}\right) \\ \left(\frac{\partial^2}{\partial y^2} - \nabla^2\right) B_y + \ldots\ldots &= -\nabla^2 B_y + \frac{\partial}{\partial y}(\nabla \cdot \vec{B}) \\ \left(\frac{\partial^2}{\partial z^2} - \nabla^2\right) B_z + \ldots\ldots &= -\nabla^2 B_z + \frac{\partial}{\partial z}(\nabla \cdot \vec{B}) \end{aligned}$$

$$-\nabla^2\vec{B} + \nabla\left(\nabla \cdot B\right) = \nabla \times \left(\nabla \times \vec{B}\right) \quad \text{and} \quad \nabla \cdot \vec{B} = 0$$