Physics 551 Lecture #16

Title: Drude' Theory

Historical Perspective:

The theory of conduction clearly pre-dates quantum mechanics and the advent of the Free Electron Gas Model by Sommerfeld. Well before electrons themselves had been discovered, Ohm had empirically determined the electrical properties of metals.

1897 - Thomson discovered the electron

1900 - Drude' proposed that the electronic behavior of metals followed from the classical kinetic theory of gases.

1. A conducting solid consisted of "rigid" (localized) positively charged ions and a classical gas of electrons.
2. Between collisions, electrons moved ballistically and were non-interacting.
   Drude's model is still used today as a zeroeth order approximation.
   Successes: Ohm's law and the Wiedemann and Franz ratio
   WF ratio = Thermal Conductivity/Electrical Conductivity= k/σ

Ohm's law $\vec{E} = \rho\vec{j}$ $\quad (V = IR)$

$$\begin{aligned}
\vec{j} &= \text{current flux} \quad \text{amps/cm}^2 \\
\rho &= \text{resistivity} \quad \Omega\ \text{cm} \\
\vec{E} &= \text{Electric field V/cm}
\end{aligned}$$

For metals $\rho \sim 10^{-5}$ to $10^{-6}\Omega$ cm at 300°K.

If we apply an electric field to this gas of electron

Force on an electron $\rightarrow \ -e\vec{E} = m\vec{a}$

$$\begin{aligned}
\vec{v} &= -\frac{e\vec{E}t}{m} \qquad \vec{j} = -ne\vec{v} \qquad n = \text{density of free electrons} = \#e\text{'s/cm}^3 \\
\vec{j} &= \frac{ne^2\vec{E}t}{m}
\end{aligned}$$

According to this expression the conductivity should increase without limit. However, there are scattering processes

$$\begin{aligned} m\dot{\vec{v}} &= -e\vec{E} - \frac{m\vec{v}}{\tau} \text{ (drag force)} \\ \tau &= \text{characteristic time between collisions} \end{aligned}$$

At steady state (SS) $\dot{\vec{v}} = 0$

$$\begin{aligned} e\vec{E} &= \frac{-m\vec{v}}{\tau} \qquad \vec{v}_{SS} = \frac{-e\vec{E}\tau}{m} \\ \vec{j} &= \frac{+ne^2\vec{E}\tau}{m} = -ne\vec{v} \end{aligned}$$

so

$$\begin{aligned} \sigma &= \frac{1}{\rho} = ne^2\tau/m \qquad \text{Ohm's law} \\ \vec{j} &= \sigma\vec{E} \qquad \sigma = \frac{ne^2\tau}{m} \qquad \vec{j} = -ne\vec{v} \\ & \tau \text{ is on the order of } 10^{-14} \text{ sec} \end{aligned}$$

Hall effect and Magnetoresistance:

$$\vec{j} = -ne\vec{v} = -ne\vec{p}/m$$

Given $\vec{p}$, find $\vec{p}(t+dt)$ some time later and recall the characteristic relaxation time between collisions. If there is an electric field and time dt, the probability of a collision is dt/τ, and for a non-collision is $1 - dt/\tau$. These e's will simply evolve under the influence of the applied electric field.

$$\vec{p}(t+dt) = (1 - dt/\tau)[\vec{p}(t) + \vec{f}(t)dt]$$

$\vec{f}$(t) is the force, we can ignore those e's undergoing a collision
For the fraction undergoing a collision

$$\vec{p}(t+dt) = (dt/\tau)[\vec{f}(t)dt \text{ is the maximum.}$$

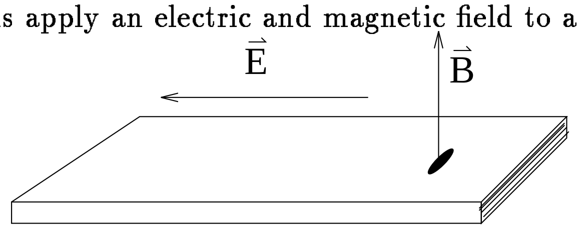
This is $\propto \mathcal{O}(dt)^2$, if dt is small this contribution is negligible.

Back to the fraction not undergoing a collision:

$$\vec{p}(t+dt) - \vec{p}(dt) = -\frac{dt}{\tau}\vec{p}(t) + \vec{f}(t)dt - \mathcal{O}(dt)^2$$

$$\frac{d\vec{p}}{dt} = -\frac{\vec{p}(t)}{\tau} + \vec{f}(t) \quad \text{is the equation of motion}$$

Now let us apply an electric and magnetic field to a metal slab



Forces	E-field	B-field
	$\vec{F} = -e\vec{E}$	$\vec{F} = -\frac{e}{c}(\vec{v} \times \vec{H})$

Thus the electrons flowing will be pushed to one side of the conductor. This will create an opposing $\vec{E}$-field.

Quantities: E_x/j_x

$\rho(H)$ = magnetoresistance $= \rho - \rho_0$

$R_H = E_y/j_x H \ \rightarrow$ Hall coefficient

Note that we can in principle extract the sign of the charge carriers if carriers are positive, opposite R_H

$$\frac{d\vec{p}}{dt} = -e(\vec{E} + \frac{\vec{p}}{mc} \times \vec{H}) - \frac{\vec{p}}{\tau}$$

$$\text{if steady state } \frac{d\vec{p}}{dt} = 0!$$

$$\hat{x}\text{-dir} \quad 0 = -eE_x - \frac{mv_y}{mc}eH - \frac{p_x}{\tau}$$

$$\hat{y}\text{-dir} \quad 0 = -eE_y + \frac{mv_x}{mc}eH - \frac{p_y}{\tau}$$

$$\text{if} \quad \omega_c \equiv \frac{eH}{mc} \quad \text{(Cyclotron frequency)}$$

$$0 = -eE_x - \omega_c p_y - p_x/\tau \qquad \text{Multiply both equations by } (-ne\tau/m)$$

$$
\begin{aligned}
0 &= -eE_y + \omega_c p_x - p_y/\tau \qquad \text{Recall } \sigma_0 = +ne^2\tau/m \\
0 &= \frac{\tau e^2 n}{m}E_x + \omega_c \tau n e v_y + n e v_x \qquad j_y\omega_c\tau + j_x = \sigma_0 E_x \\
0 &= \frac{\tau e^2 n}{m}E_y - \omega_c \tau n e v_x + n e v_y \qquad -j_x\omega_c\tau + j_y = \sigma_0 E_y
\end{aligned}
$$

Solving for $j_y = 0$, (no transverse current) gives

$$
\begin{aligned}
E_y &= -j_x\omega_c\tau/\sigma_0, \qquad E_x = j_x/\sigma_0 \text{ which is the same as the zero magnetic field case.} \\
E_y &= -\left(\frac{H}{nec}\right) j_x \\
R_H &= -\frac{1}{nec} \qquad \text{(independent of } H\text{)}
\end{aligned}
$$

For most metals this is not true in general.
Notice that there is no magnetoresistance!

We can in principle apply a varying AC electric field and determine the AC electrical conductivity.

$$
\begin{aligned}
\vec{E}(t) &= Re(\vec{E}(\omega)e^{-i\omega t}) \\
\sigma(\omega) &= \frac{\sigma_0}{1 - i\omega\tau}
\end{aligned}
$$

A time varying $\vec{E}$ field implies a time varying $\vec{H}$ field.

What about this magnetic field? It is too weak.

What about the spatial variation?

Let $\lambda_{photon} >$ mean free path $\rightarrow$ this implies that the electrons will scatter.

If we know the $\vec{j}$ (current density) we can solve Maxwell's equations

$$
\begin{aligned}
\nabla \cdot \vec{E} &= 0 \qquad \nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t} \\
\nabla \cdot \vec{H} &= 0 \qquad \nabla \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}
\end{aligned}
$$