PHYS 551 Lecture #19

Title: Properties of a Fermi-Dirac Gas of Electrons

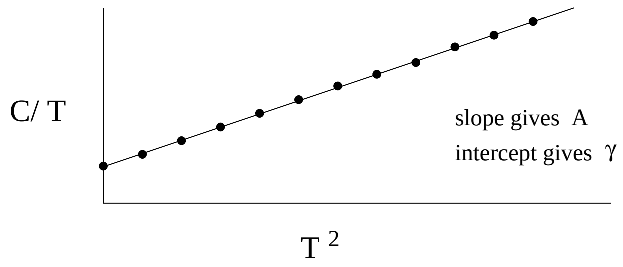
From the Sommerfeld calculation we now have the complete form of the heat capacity for a crystalline metallic solid

$$C_V = \underbrace{\gamma T}_{\text{electrons}} + \underbrace{AT^3}_{\text{ions}} - \underbrace{BT^3}_{\text{electrons (negligible)}}$$

Question: Why do the ions behave as if they interacted with only short range forces? (A metal does not have ionic long-range forces.)

Answer: Screening of the ion potential due to electron-gas.

At $T \approx T_{\text{room}}$, the net contribution of the electrons to the heat capacity is negligible.



γ is a measure of $\mathcal{D}(\varepsilon_F)$ $(T = 0)$! Observe that, in this case, bulk properties can be used to give information concerning microscopic details.

ELECTRICAL CONDUCTIVITY:

What happens in the presence of a DC electric field? $\quad \vec{p} = \hbar\vec{k}$

$\delta\vec{k} = -e\vec{E}/\hbar\ dt$ (Motion of $\vec{E}$ opposite to that $\vec{k}$) $\quad \frac{d\vec{p}}{dt} = -e\vec{E}$

$\delta\vec{k}$ is the change in the wave-vector of the electrons. $\quad d\vec{p} = -e\vec{E}\tau$

Again (as was the case for the simple classical gas), if there are no scattering processes, this should happen indefinitely. Recall, the characteristic scattering time from the Drude model.

Thus $dt \rightarrow \tau$ and assume it is the same for all e's

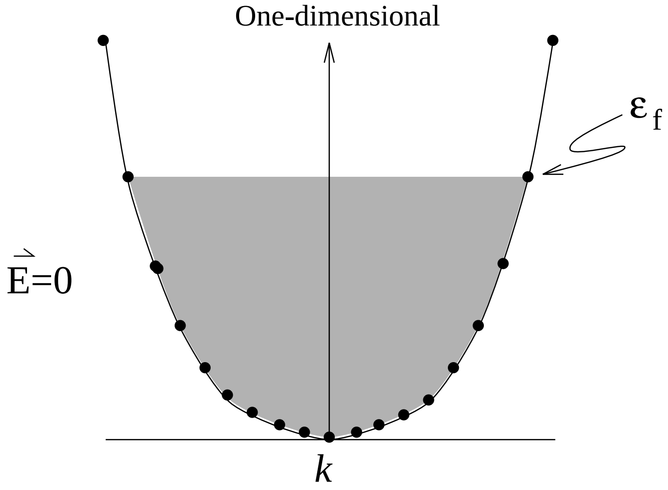
$\delta\vec{p} = -e\vec{E}\tau$

$\delta\vec{v} = -e\vec{E}\tau/m$

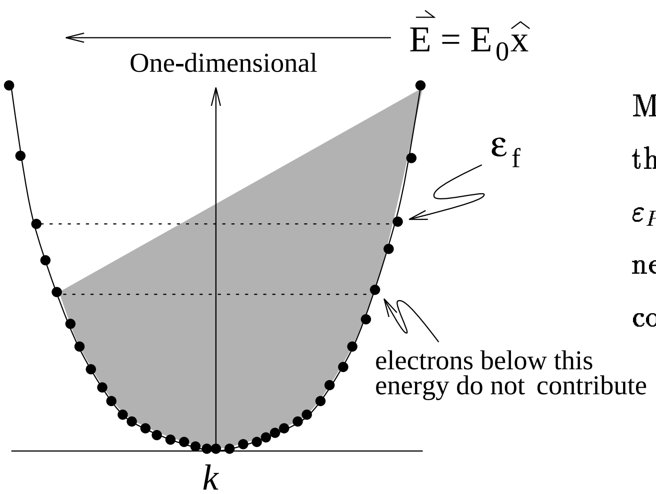
$\vec{j} = -ne\vec{v} = ne^2\vec{E}\tau/m = \sigma\vec{E}$

$\sigma = ne^2\tau/m$ (Same as Drude mode)

How can we visualize this?



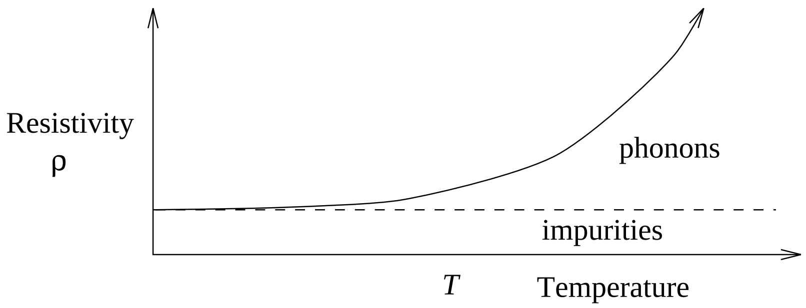
$T = 0$ and no electric field, half of the conduction electrons are travelling to the left, half to the right. NO NET CURRENT



More than half are moving to the right. Electrons far below ε_F balance out. Only electrons near the Fermi level effectively contribute to the net current.

What determines τ?

The electron-electron scattering, electron-phonon scattering, and the electron-impurity scattering.



$\frac{1}{\tau} = \frac{1}{\tau_L} + \frac{1}{\tau_i}$

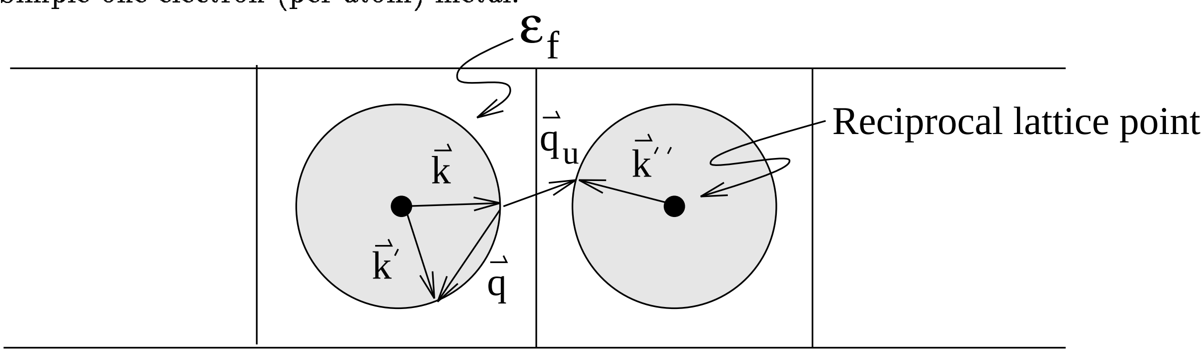
$\tau \rightarrow$ relaxation time

$i \rightarrow$ impurities

$L \rightarrow$ lattice

$\rho = \rho_L + \rho_i \rightarrow$ Matthiessen's rule

The lattice scattering requires that there be conservation of crystal momentum and conservation of energy.
Since the energy of a phonon is on the order of a few meV, only electrons near ε_f are effectively scattered by phonons.
Simple one electron (per atom) metal:



Recall that $\varepsilon_f \sim 2 - 10$ev and $\hbar\omega_{\text{phonon}} \sim 100$meV
Note: If $|\vec{k}| << |\vec{k}_f|$, then there are no states available which can satisfy conservation of energy.

$$\vec{k} + \vec{q} = \vec{k}' \qquad \frac{\hbar^2 k^2}{2m} + \hbar\omega_q = \frac{\hbar^2 k'^2}{2m} \qquad \text{Simple case}$$

$$\vec{k} + \vec{q}_u = \vec{k}'' + \vec{G} \qquad \frac{\hbar^2 k^2}{2m} + \hbar\omega_q = \frac{\hbar^2 k''^2}{2m} \qquad \text{Umklapp process}$$

Thermal conductivity:

Since the electrons form a "gas", the same formula found derived previously for a phonon gas is appropriate.

$$\kappa = \frac{1}{3} c v \ell$$

c is the heat capacity

v is the average velocity

ℓ is the mean free path

$$
\begin{aligned}
\ell &= v_F\tau \\
c &= \frac{1}{2}\pi^2 N k_B T/T_F \\
&= \frac{1}{2}\pi^2 N k_B^2 T/\varepsilon_F \\
&= \frac{1}{2}^2 N k_B^2 T/\frac{1}{2}m v_F^2 \\
&= \pi^2 N k_B^2 T/m v_F^2 \\
\kappa &= \frac{\pi^2}{3} N k_B^2 T/m v_F^2 v_F \cdot v_F\tau \\
&= \frac{\pi^2}{3} N k_B^2 T\tau/m
\end{aligned}
$$

On the whole, Sommerfeld's model is slightly more complex than Drude's.

Explains Pauli susceptibility (T independent magnetic susceptibility)

Explains heat capacity

Explains thermal conductivity

Does not explain Hall effect!

More importantly, it does not explain why some materials are metals, while others are insulators.

For this we have to study the motion of electrons in a periodic ion potential (i.e., band theory).