Physics 551   Lecture #31

Title: Ferromagnetism

1. Some materials exhibit large spontaneous magnetization $\vec{M}$ below some temperature T_c when the applied field $|\vec{H}| = 0$

   $|\vec{M}| \neq 0$ for $T < T_c$ even if $|\vec{H}| = 0$

   These include the elements Fe, Co, Ni, Gd, Dy and many oxides in particular.

2. These materials are paramagnetic for $T > T_c$ and the temperature dependence of the magnetic susceptibility is $\chi = \frac{C}{T-T_c}$ where $T_c \equiv$ Curie temperature $\Longrightarrow$ Curie-Weiss behavior.

   For ordinary "isotropic" paramagnets

$$\chi = \frac{M}{H} = \frac{C}{T} = N\ J(J+1)g^2\mu_B^2/k_B T$$

   To explain this behavior we assume the existence of an internal effective field

$$\vec{H} = \vec{H}_{\text{applied}} + \lambda\vec{M}$$

   where $\lambda\vec{M}$ results from the molecular field due to the average of all the other constituents. This is referred to as the mean field or the exchange field. The strength of this exchange field is related by

$$g\mu_B H_{\text{exchange}} \sim k_B T_c. \qquad T_c \sim 500K$$

   so $\qquad H_{\text{exchange}} \sim 10^7$ guass.

   We might be tempted to claim that this field is the simple dipole field but for a magnetic dipole

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\left[\frac{3(\vec{\mu}\cdot\vec{r}) - \vec{\mu}r^2}{r^5}\right]$$

$$|\vec{B}(\vec{r})|_{\text{Max}} = \frac{\mu_0}{4\pi}\frac{2|\mu|}{r^3} \qquad\qquad \vec{\mu}\cdot\vec{r} = |\vec{\mu}|\ |\vec{r}|$$

   Let $|\vec{\mu}| = \mu_B \quad r^3 = 10^{-30}\text{m} \quad \mu_0 = 4\pi\times 10^{-7}$

   $B_{\text{max}} \sim 10^{-1}$ T $\sim 10^3$ Gauss.

   If H_{exchange} were due to magnetic dipoles, $T_c \sim 1^\circ K$ or less.

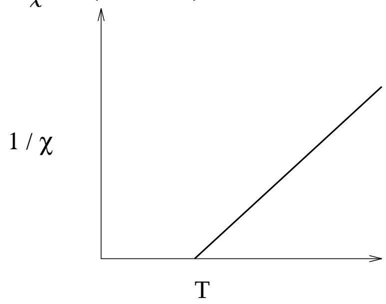
Back to $\vec{H} = \vec{H}_{\text{applied}} + \lambda\vec{M}$

$$\begin{aligned} \chi = \frac{M}{H\text{app} + \lambda M} &= \frac{C}{T} \qquad M = \frac{C}{T}(H + \lambda M) \\ M(1 - \lambda C/T) &= \frac{CH}{T} \end{aligned}$$

$$\chi = \frac{M}{H} = \frac{C}{T(1 - \lambda C/T)} = \frac{C}{T - \lambda C} = \frac{C}{T - T_c} \qquad T_c \equiv \lambda C$$

Recall $\quad C = NJ(J+1)g^2\mu_B^2/k_B$

So plotting $\quad \frac{1}{\chi} = (T - T_c)/C$ gives a linear relationship:



Now let us examine the region $T < T_c$, since the magnetization is large, the Curie law breaks down and the approximation $(\mu B/kT \to 0)$ is invalid. Thus,

$$\vec{M} = N\vec{\mu} \, \tanh \, \frac{\vec{\mu} \cdot \vec{H}}{k_B T}$$

but $\vec{H} = \lambda\vec{M}$ (let $H_{\text{app}} = 0$)

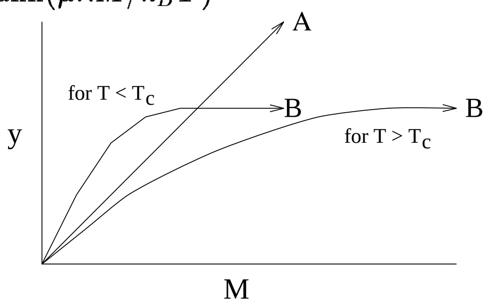
So $\quad \vec{M} = N\vec{\mu} \, \tanh(\vec{\mu} \cdot \lambda\vec{M}/k_B T) = y$

This gives $\vec{M}$ versus T for $H = 0$.

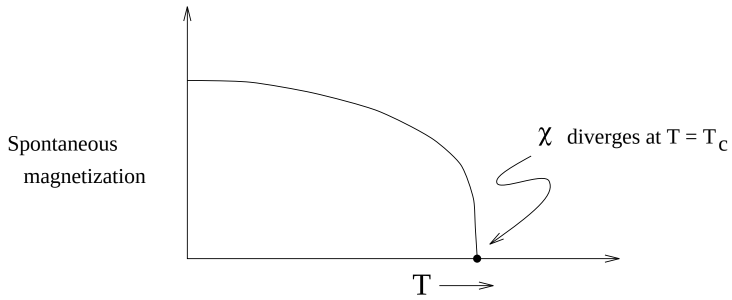
Assume $\vec{\mu} \parallel \vec{M}$.

(A) $y = \vec{M}$

(B) $y = N\mu \, \tanh(\mu\lambda M/k_B T)$



so we can solve for M versus T $\ (T < T_c)$



So where does this *EXCHANGE* field come from?
To show this we need to refer back to the H_2 molecule. Recall that the electron wave function has both spatial and spin components.
SPATIAL:

$$\begin{aligned}\psi_{\text{symmetric}} &= \phi_A(1)\phi_B(2) + \phi_A(2)\phi_B(1)\\ \psi_{\text{anti-symmetric}} &= \phi_A(1)\phi_B(2) - \phi_A(2)\phi_B(1)\end{aligned}$$

$$\begin{aligned}A, B &\implies \text{corresponds to each hydrogen atom}\\ 1,2 &\implies \text{corresponds to each of the electrons}\end{aligned}$$

We assume that the H_2 molecular solution can be approximated by a product of the two atomic solutions with one electron on one atom and one electron on the other atom.

The total electron wave-function Φ (which includes spatial and spin components) *must* be antisymmetric under exchange of electrons. Two electrons cannot occupy the same state - PAULI exclusion principle

$$\begin{array}{lllllll}\Phi & = & \psi_{\text{symmetric}}\times & |\uparrow\downarrow\rangle & s=0 & m_s=0 & \text{(singlet state)}\\ \text{or} & & & & & & \\ \Phi & = & \psi_{\text{antisymmetric}}\times & |\uparrow\uparrow\rangle & & m_s=1 & \\ & & & |\uparrow\downarrow+\downarrow\uparrow\rangle & s=1 & m_s=0 & \text{(triplet state)}\\ & & & |\downarrow\downarrow\rangle & & m_s=-1 & \end{array}$$

Recall $\langle U\rangle = \frac{\int \psi^* U \psi}{\int \psi^* \psi}$
In the case of our H_2 molecule

$$U = \underbrace{\frac{e^2}{|\vec{r}_a - \vec{r}_b|}}_{\text{ion-ion}} + \underbrace{\frac{e^2}{|\vec{r}_1 - \vec{r}_2|}}_{\text{electron-electron}} \overbrace{- \frac{e^2}{|\vec{r}_1 - \vec{r}_a|} - \frac{e^2}{|\vec{r}_2 - \vec{r}_b|} \underbrace{- \frac{e^2}{|\vec{r}_1 - \vec{r}_b|} - \frac{e^2}{|\vec{r}_2 - \vec{r}_a|}}_{\text{these are very weak}}}^{\text{electron-ion}}$$

Notice that $U(\vec{r}_1, \vec{r}_2) = U(\vec{r}_2, \vec{r}_1)$

Evaluating the potential energy of both states gives:

$$\langle U \rangle_{\substack{S \\ T}} = \frac{\begin{array}{l}[\int \phi_A^*(1)\phi_B^*(2)\; U\; \phi_A(1)\phi_B(2) \\ \quad \pm \int \phi_A^*(1)\phi_B^*(2)\; U\; \phi_A(2)\phi_B(1) \\ \quad\quad \pm \int \phi_A^*(2)\phi_B^*(1)\; U\; \phi_A(1)\phi_B(2) \\ \quad\quad\quad + \int \phi_A^*(2)\phi_B^*(1)\; U\; \phi_A(2)\phi_B(1)]\end{array}}{\int \phi^*\phi \;\; (\equiv \frac{1}{N^2})} \qquad \frac{1}{N_s^2} \approx \frac{1}{N_T^2}$$

1 & 2 are exchanged with 2 & 1

$$\langle U \rangle_S - \langle U \rangle_T = 4N^2 \int \phi_A^*(1)\phi_B^*(2)\; U(r_1, r_2)\phi_A(2)\phi_B(1)$$

This integral is called the *EXCHANGE* integral.

Let $\langle U \rangle_{\mathrm{S}} - \langle U \rangle_{\mathrm{T}} = J_{\mathrm{exchange}}$ (exchange coupling constant)

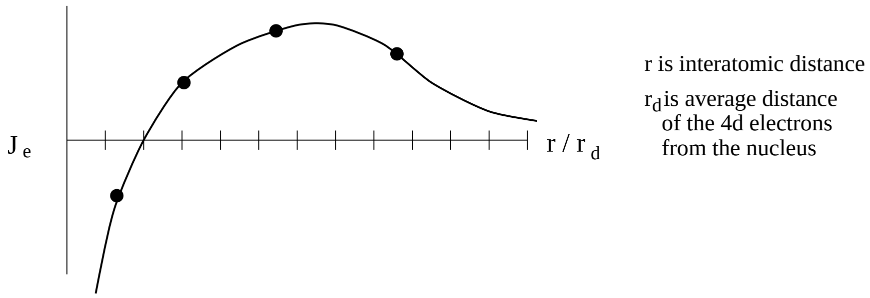
If $J > 0$, then $\langle U \rangle_{\mathrm{S}} \; > \; \langle U \rangle_{\mathrm{T}}$ Ferromagnetic

If $J < 0$, then $\langle U \rangle_{\mathrm{S}} \; < \; \langle U \rangle_{\mathrm{T}}$ Antiferromagnetic

Looking more closely at U

$$\frac{e^2}{|\vec{r}_a - \vec{r}_b|} \text{ and } \frac{e^2}{|\vec{r}_1 - \vec{r}_2|} \quad \rightarrow \quad \text{favor ferromagnetism}$$

$$\frac{-e^2}{|\vec{r}_1 - \vec{r}_a|} \text{ and } \frac{-e^2}{|\vec{r}_2 - \vec{r}_b|} \quad \rightarrow \quad \text{favor antiferromagnetism}$$



Note: this exchange energy is often written

$$E = -\frac{J_{\mathrm{e}}}{\hbar^2}\, \vec{s}_i \cdot \vec{s}_j \text{ or } -2J\vec{s}_i \cdot \vec{s}_j$$

Notice that the origin of this interaction is coulombic!

$$\begin{aligned} \vec{S}^2 = (\vec{s}_1 + \vec{s}_2)^2 &= \hbar^2 S(S+1) \qquad S = 0, 1 \\ s_1^2 + s_2^2 + 2\vec{s}_1 \cdot \vec{s}_2 &= 0,\; 2\hbar^2 \end{aligned}$$