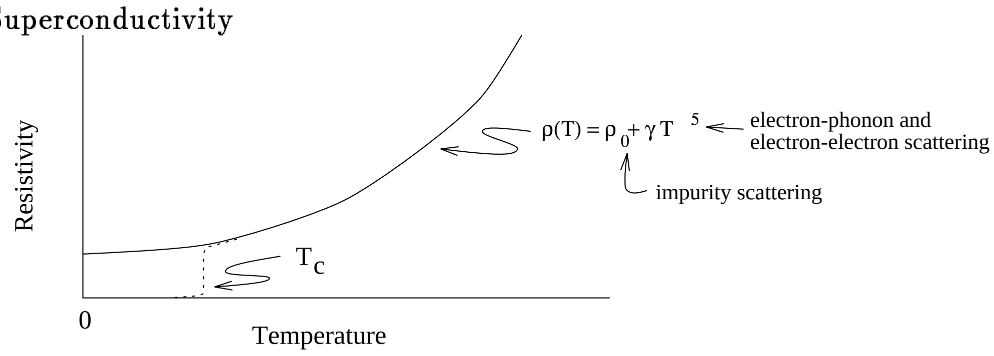
**Physics 0551 Lecture #33**

Title: Superconductivity




Superconductors are unusual for a variety of reasons. The first observations were made in 1911 by Kamerlingh Onnes (Denmark) on the low temperature behavior of Hg (mercury) and he observed a complete absence of $\rho$ in the vicinity of 4°K.

Note: There is nothing in the "classical" Drude' theory of metals which can account for this behavior.

There are many systematics which accompany the occurrence of superconductivity

No monovalent Elements

$\left.\begin{array}{c}\text{No ferromagnetic elements}\\ \text{No Rare-earths}\end{array}\right\}$ i.e., spin is *not* compatible with superconductivity (conventional)

Normal BCS (Bardeen-Cooper-Schrieffer) (1957)

$10^{-2}$K $< T_c <$ 23°K or so $Nb_3Ge$ 23.2°K

High $T_c$ superconductors are at odds with conventional wisdom.

Oxides 35°K $La_{1.8}\, Sr_{.2}CuO_4$ Bednoorz and Müller

95°K $(YBa_2)\, Cu_3O_7$

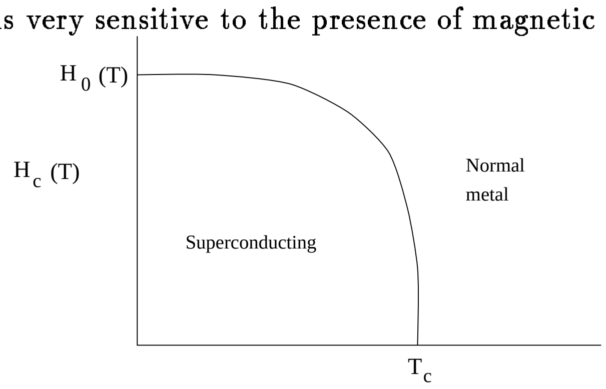
125°K $Th$- .... -$Cu$)

Note: Anomalous behavior in $\rho$ is often due to structural transformation

Organic superconductor (BEDT-TTF)$_2Cu(NCS)_2$ 10.4°K

$T_c$ is very sensitive to applied magnetic fields.

$T_c$ is very sensitive to the presence of magnetic impurities.



Second test for superconductivity is the Meissner/Ochsenfeld effect.

There is complete expulsion of applied magnetic fields.

B=0 inside superconducting regions of a superconductor

$\Longrightarrow$ Note that a superconductor is <u>NOT</u> a perfect conductor

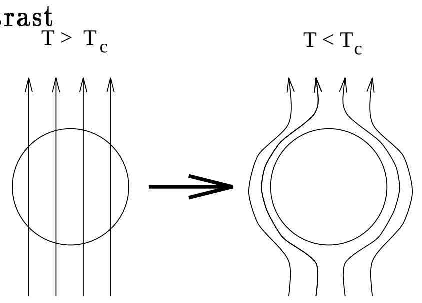
Ohm's Law $\vec{E} = \rho \vec{J}$

If $\rho = 0$, then $\vec{E} = 0$

But $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (cgs) $\quad \nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$ (SI)

If $\vec{E} = 0$ then $\frac{\partial \vec{B}}{\partial t} = 0$ $\quad \Rightarrow$ Magnetic flux cannot be expelled!

In contrast




i.e., superconductors are *perfect* diamagnets

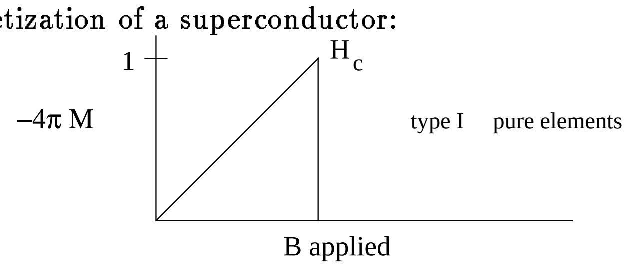
$$\vec{B} = \vec{H} + 4\pi \vec{M} \quad \text{where} \quad \chi \equiv \frac{\vec{M}}{\vec{H}} \quad \text{(cgs)}$$

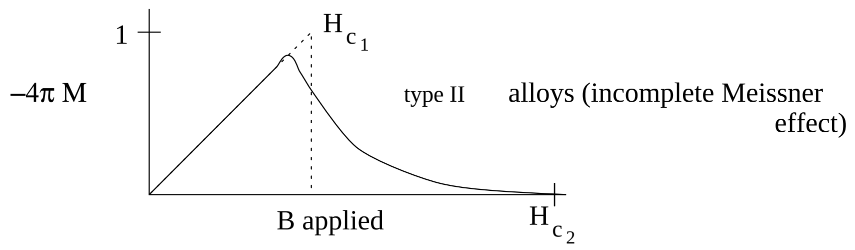
Since $\vec{B} = 0$, $\quad \chi = -\frac{1}{4\pi}$ $\quad$ (Not $-\infty$)

* Note since high $T_c$ samples are often multiphase

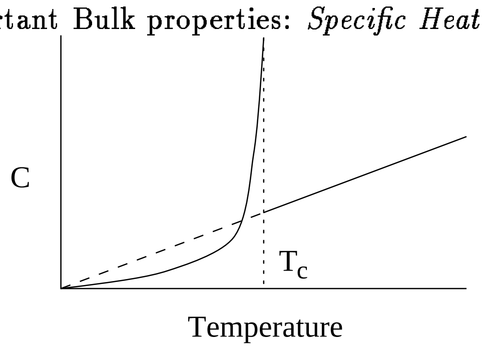
$$\chi_{\mathrm{HT}_c} < -\frac{1}{4\pi} \quad \text{or} \quad \% \text{ superconducting} = \frac{\chi_{\mathrm{HT}_c}}{-\frac{1}{4\pi}}$$

Magnetization of a superconductor:



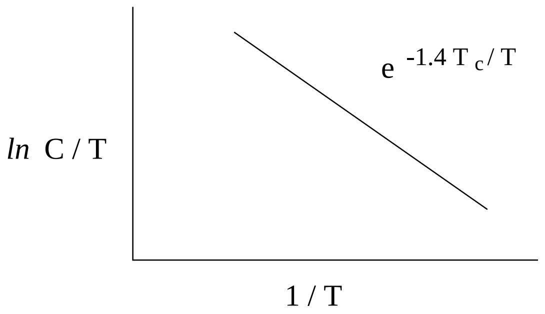



Other important Bulk properties: *Specific Heat*




$\alpha$ T $\rightarrow$ electron gas (No phonon contribution)

This measurement is difficult to do in high -temperature superconductors




exponential behavior implies a barrier which can be overcome by thermal activation (a GAP!)

The entropy of the S.C. state is smaller than in the normal state.

Isotope effect (factor of 1/2 is approximate)

$$T_c \propto \frac{1}{M^{1/2}} \quad \Longrightarrow \text{ implies a } \textit{phonon} \text{ mechanism}$$

*Thermodynamics of a superconductor:*

S.C. $\leftrightarrow$ Normal Transition is a reversible process, hence a thermodynamic treatment is possible

$2^{nd}$ Order Transition $\rightarrow$ NO LATENT HEAT

$$\begin{aligned} d(U-TS) &= -\vec{M}\cdot d\vec{H} \\ dU &= TdS - \vec{M}\cdot d\vec{H} \\ dU &= dQ - dW \end{aligned}$$

How much energy is stored?

$$\vec{M}\cdot d\vec{H}$$

$$dU = TdS - \vec{M}\cdot d\vec{B}_a \qquad B_a \Longrightarrow \text{ magnetic field applied}$$

$$\text{but } M = -\frac{1}{4\pi}B_a$$

We want the change in the Free energy $(F)$ at constant temperature

$$dF = d(U-TS) = -\vec{M}\cdot d\vec{B}_a$$

$$\text{but } \vec{M} = \chi\vec{H} = -\frac{1}{4\pi}\,\vec{B}_a$$

$$dF = \frac{1}{4\pi}\,\vec{B}_a \cdot d\vec{B}_a \qquad F_{superconducting}$$

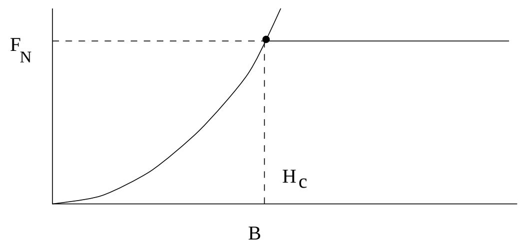
$$0 \text{ to } B_a \text{ is } B_{applied}$$

$$F_s(B_a) - F_s(0) = B_a^2/8\pi$$

for the normal $F_N(B_a) = F_N(0)$ ignore, $\chi_N \approx 0$!
At $H_c$ (critical field)

$$F_N(H_c) = F_S(H_c) \qquad \text{No latent heat}$$

therefore $F_N(0) = F_N(H_c) = F_S(0) + B_{H_c}^2/8\pi$
so $F_N(0) - F_s(0) = B_{H_c}^2/8\pi$




$F_s(0) < F_N(0)$ for $T < T_c$!
Since $|\vec{B}| = 0$ inside a superconductor, how does it behave microscopically? (i.e., what happens at the surface?)

London equation:

First consider a "perfect" conductor, and a momentum $\vec{E}$-field

$$\frac{d\vec{v}}{dt} = -\frac{e\vec{E}}{m} \qquad \frac{d\vec{j}}{dt} = \frac{ne^2\vec{E}}{m_e}$$

Faraday's law:

$$\begin{aligned}
\nabla \times \vec{E} &= -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \\
\nabla \times \vec{E} &= \frac{m_e}{ne^2}\,\nabla \times \frac{d\vec{j}}{dt} \Longrightarrow \nabla \times \frac{d\vec{j}}{dt} = -\frac{ne^2}{mc}\frac{\partial \vec{B}}{\partial t} \\
& \frac{\partial}{\partial t}\left(\nabla \times \vec{j} + \frac{ne^2}{mc}\vec{B}\right) = 0 \qquad (1) \\
& \text{and} \quad \nabla \times \vec{B} = \frac{4\pi}{c}\vec{j} \qquad (2)
\end{aligned}$$

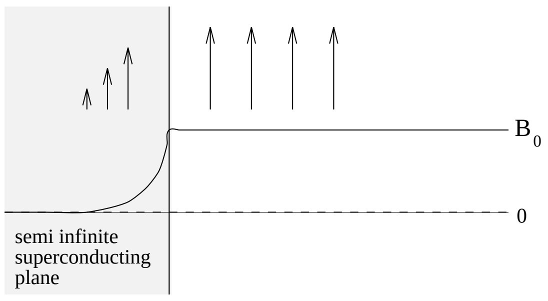
* These two equations relate $\vec{B}$ and $\vec{j}$ for a perfect conductor, they also allow time independent solution of $\vec{B}$ and $\vec{j}$

(1) $\nabla\times\vec{j}+\frac{ne^2}{mc}\vec{B} = const. =??? \to 0$ (is what London said for a superconductor)

$\nabla\times$ both sides of (2)

$\nabla\times(\nabla\times\vec{B}) = \frac{4\pi}{c}\nabla\times\vec{j}$

$-\nabla^2\vec{B} = \frac{4\pi}{c}\left(-\frac{ne^2}{mc}\vec{B}\right)$




$\nabla^2\vec{B} - 4\pi\frac{ne^2}{mc^2}\vec{B} = 0$ $\qquad B_x(\hat{z}) = B_0 e^{-x/\Lambda}$ $\qquad$ If $B_a = B_0\hat{z}$

$\Lambda \equiv \left(\frac{mc^2}{4\pi ne^2}\right)^{1/2}$ $\qquad$ London penetration depth $\sim$ 100-1000Å

NOTES:

$\nabla\times(\nabla\times\vec{B}) \implies -\nabla^2\vec{B}+\nabla(\nabla\cdot B)$

$$\nabla\times\vec{B} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix} = \left(\frac{\partial B_z}{\partial y}-\frac{\partial B_y}{\partial z}\right)\hat{i} + \left(\frac{\partial B_x}{\partial z}-\frac{\partial B_z}{\partial x}\right)\hat{j} + \left(\frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}\right)\hat{k}$$

$\nabla\times(\nabla\times\vec{B}) \implies$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \left(\frac{\partial B_z}{\partial y}-\frac{\partial By}{\partial z}\right) & \left(\frac{\partial B_x}{\partial z}-\frac{\partial Bz}{\partial x}\right) & \left(\frac{\partial B_y}{\partial x}-\frac{\partial Bx}{\partial y}\right) \end{vmatrix} = \begin{array}{ll} \left(\frac{\partial^2 B_y}{\partial x\partial y}-\frac{\partial^2 Bx}{\partial y^2}\right)-\left(\frac{\partial^2 B_x}{\partial z^2}-\frac{\partial^2 B_z}{\partial x\partial z}\right) & \hat{i}+ \\ \left(\frac{\partial^2 B_z}{\partial y\partial z}-\frac{\partial^2 Bx}{\partial z^2}\right)-\left(\frac{\partial^2 B_y}{\partial x^2}-\frac{\partial^2 B_x}{\partial y\partial x}\right) & \hat{j}+ \\ (\qquad\qquad)-(\qquad\qquad) & \hat{k} \end{array}$$

$$\begin{aligned} \left(\frac{\partial^2}{\partial x^2}-\nabla^2\right)B_x+\frac{\partial^2 B_y}{\partial x\partial y}+\frac{\partial B_z}{\partial x\partial z} &= -\nabla^2 B_y+\frac{\partial}{\partial x}\left(\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+\frac{\partial B_z}{\partial z}\right) \\ \left(\frac{\partial^2}{\partial y^2}-\nabla^2\right)B_y+\ldots\ldots &= -\nabla^2 B_y+\frac{\partial}{\partial y}(\nabla\cdot\vec{B}) \\ \left(\frac{\partial^2}{\partial z^2}-\nabla^2\right)B_z+\ldots\ldots &= -\nabla^2 B_z+\frac{\partial}{\partial z}(\nabla\cdot\vec{B}) \end{aligned}$$

$$-\nabla^2\vec{B}+\nabla\left(\nabla\cdot B\right) = \nabla\times\left(\nabla\times\vec{B}\right) \quad \text{and} \quad \nabla\cdot\vec{B}=0$$