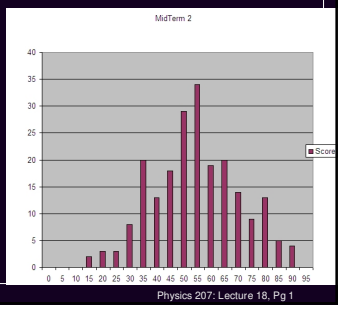
Physics 207, Lecture 18, Nov. 6

- MidTerm 2
  - Mean 58.4 (64.6)
  - Median 58
  - St. Dev. 16 (19)
  - High 94
  - Low 19

Nominal curve: (conservative)
80-100 A
62-79 B or A/B
34-61 C or B/C
29-33 marginal
19-28 D



Physics 207, Lecture 18, Nov. 6

- Agenda: Chapter 14, Fluids
  - Pressure, Work
  - Pascal's Principle
  - Archimedes' Principle
  - Fluid flow

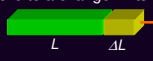
Assignments:
- Problem Set 7 due Nov. 14, Tuesday 11:59 PM
  Note: Ch. 14: 2,8,20,30,52a,54 (look at 21)
  Ch. 15: 11,19,36,41,49 Honors: Ch. 14: 58
- For Wednesday, Read Chapter 15

Fluids (Chapter 14)

- At ordinary temperature, matter exists in one of three states
  - Solid - has a shape and forms a surface
  - Liquid - has no shape but forms a surface
  - Gas - has no shape and forms no surface
- What do we mean by "fluids"?
  - Fluids are "substances that flow"…. "substances that take the shape of the container"
  - Atoms and molecules are free to move.
  - No long range correlation between positions.

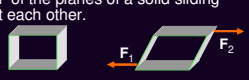
Some definitions

- Elastic properties of solids :
  - Young's modulus: measures the resistance of a solid to a change in its length.

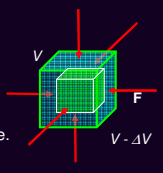


elasticity in length

  - Shear modulus: measures the resistance to motion of the planes of a solid sliding past each other.



elasticity of shape (ex. pushing a book)



  - Bulk modulus: measures the resistance of solids or liquids to changes in their volume.

volume elasticity

Fluids

- What parameters do we use to describe fluids?
  - Density

$$\rho = \frac{m}{V}$$

units : $kg/m^3 = 10^{-3}\ g/cm^3$

ρ(water) = 1.000×10^3 kg/m³ = 1.000 g/cm³
ρ(ice) = 0.917×10^3 kg/m³ = 0.917 g/cm³
ρ(air) = 1.29 kg/m³ = 1.29×10^{-3} g/cm³
ρ(Hg) = 13.6×10^3 kg/m³ = 13.6 g/cm³

Fluids

- What parameters do we use to describe fluids?
  - Pressure

$$p = \frac{F}{A}$$

units :
1 N/m² = 1 Pa (Pascal)
1 bar = 10^5 Pa
1 mbar = 10^2 Pa
1 torr = 133.3 Pa

1 atm = 1.013×10^5 Pa
= 1013 mbar
= 760 Torr
= 14.7 lb/ in² (=PSI)

- Any force exerted by a fluid is perpendicular to a surface of contact, and is proportional to the area of that surface.
  - Force (a vector) in a fluid can be expressed in terms of pressure (a scalar) as:

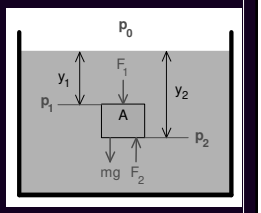
$$\vec{F} = pA\hat{n}$$

Pressure vs. Depth Incompressible Fluids (liquids)

- When the pressure is much less than the bulk modulus of the fluid, we treat the density as constant independent of pressure:

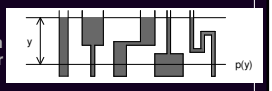
  incompressible fluid

- For an incompressible fluid, the density is the same everywhere, but the pressure is NOT!

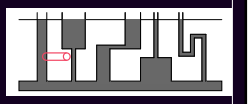


Pressure vs. Depth

- **For a uniform fluid in an open container pressure same at a given depth independent of the container**



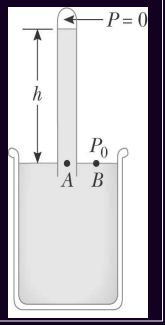
- **Fluid level is the same everywhere in a connected container, assuming no surface forces**
- **Why is this so? Why does the pressure below the surface depend only on depth if it is in equilibrium?**



- **Imagine a tube that would connect two regions at the same depth.**
- **If the pressures were different, fluid would flow in the tube!**
- **However, if fluid did flow, then the system was NOT in equilibrium since no equilibrium system will spontaneously leave equilibrium.**

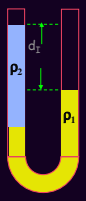
Pressure Measurements: Barometer

- Invented by Torricelli
- A long closed tube is filled with mercury and inverted in a dish of mercury
  - ❖ The closed end is nearly a vacuum
- Measures atmospheric pressure as One 1 atm = 0.760 m (of Hg)



Lecture 18, *Exercise 1* Pressure

- *What happens with two fluids??*
- Consider a U tube containing liquids of density ρ_1 and ρ_2 as shown:
  - ❖ Compare the densities of the liquids:



(A) $\rho_1 < \rho_2$ (B) $\rho_1 = \rho_2$ (C) $\rho_1 > \rho_2$

Pascal's Principle

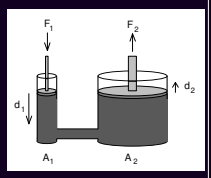
- So far we have discovered (using Newton's Laws):
  - ❖ Pressure depends on depth: $\Delta p = \rho g \Delta y$
- Pascal's Principle addresses how a change in pressure is transmitted through a fluid.

> Any change in the pressure applied to an enclosed fluid is transmitted to every portion of the fluid and to the walls of the containing vessel.

- Pascal's Principle explains the working of hydraulic lifts
  - ❖ i.e., the application of a small force at one place can result in the creation of a large force in another.
  - ❖ Will this "hydraulic lever" violate conservation of energy?
    - ➢ No

Pascal's Principle

- Consider the system shown:
  - ❖ A downward force F_1 is applied to the piston of area A_1.
  - ❖ This force is transmitted through the liquid to create an upward force F_2.
  - ❖ Pascal's Principle says that increased pressure from F_1 (F_1/A_1) is transmitted throughout the liquid.



- $F_2 > F_1$: Is there conservation of energy?

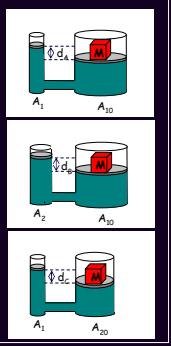
Lecture 18, *Exercise 2*
Hydraulics

- Consider the systems shown on right.
  - ❖ In each case, a block of mass M is placed on the piston of the large cylinder, resulting in a difference d_i in the liquid levels.
  - ❖ If $A_2 = 2\ A_1$, compare d_A and d_B.

(A) $d_A = (1/2)\ d_B$ (B) $d_A = d_B$ (C) $d_A = 2d_B$

If $A_{10} = 2\ A_{20}$, compare d_A and d_C.

(A) $d_A = (1/2)\ d_C$ (B) $d_A = d_C$ (C) $d_A = 2d_C$



Lecture 18, *Exercise 2*
Hydraulics

- Consider the systems shown on right.
  - ❖ If $A_2 = 2\ A_1$, compare d_A and d_B.

$Mg = \rho\ d_A\ A_1$ and $Mg = \rho\ d_B\ A_2$

$d_A\ A_1 = d_B\ A_2$

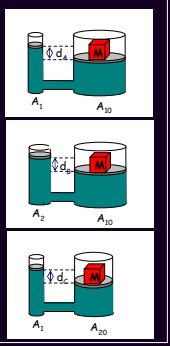
$d_A = 2\ d_B$

(A) $d_A = (1/2)\ d_B$ (B) $d_A = d_B$ (C) **$d_A = 2d_B$**

If $A_{10} = 2\ A_{20}$, compare d_A and d_C.

$Mg = \rho\ d_A\ A_1$ and $Mg = \rho\ d_c\ A_1$

(A) $d_A = (1/2)\ d_C$ (B) **$d_A = d_C$** (C) $d_A = 2d_C$



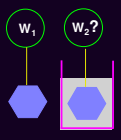
Archimedes' Principle

- Suppose we weigh an object in air (1) and in water (2).
  - ❖ How do these weights compare?

$W_1 < W_2$ $W_1 = W_2$ $W_1 > W_2$

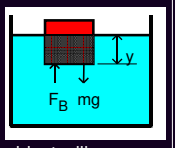
- ❖ Why?

Since the pressure at the bottom of the object is greater than that at the top of the object, the water exerts a net upward force, the buoyant force, on the object.



Sink or Float?

- The buoyant force is equal to the weight of the liquid that is displaced.
- If the buoyant force is larger than the weight of the object, it will float; otherwise it will sink.



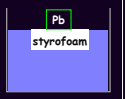
- We can calculate how much of a floating object will be submerged in the liquid:
  - ❖ Object is in equilibrium ➡ $F_B = mg$

➡ $\rho_{liquid} \cdot g \cdot V_{liquid} = \rho_{object} \cdot g \cdot V_{object}$

➡ $\dfrac{V_{liquid}}{V_{object}} = \dfrac{\rho_{object}}{\rho_{liquid}}$

Lecture 18, *Exercise 3*
Buoyancy

- A lead weight is fastened to a large styrofoam block and the combination floats on water with the water level with the top of the styrofoam block as shown.



- ❖ If you turn the styrofoam + Pb upside-down, What happens?

(A) It sinks (B) (C) (D)

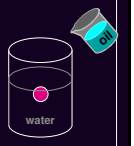
Active Figure

Lecture 18, *Exercise 4*
More Buoyancy

- Two cups are filled to the same level with water. One of the two cups has plastic balls floating in it.
  - ❖ Which cup weighs more?

(A) Cup I (B) Cup II (C) the same (D) can't tell

Lecture 18, *Exercise 5*
Even More Buoyancy



- A plastic ball floats in a cup of water with half of its volume submerged. Next some oil ($\rho_{oil} < \rho_{ball} < \rho_{water}$) is slowly added to the container until it just covers the ball.
  - Relative to the water level, the ball will:

Hint 1: What is the bouyant force of the part in the oil as compared to the air?

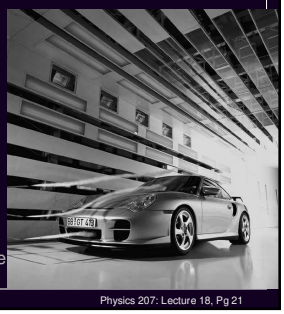
(A) move up (B) move down (C) stay in same place

Fluids in Motion

- Up to now we have described fluids in terms of their static properties:
  - Density ρ
  - Pressure *p*
- To describe fluid motion, we need something that can describe flow:
  - Velocity ***v***
- There are different kinds of fluid flow of varying complexity
  - non-steady / steady
  - compressible / incompressible
  - rotational / irrotational
  - viscous / ideal

Types of Fluid Flow

- Laminar flow
  - Each particle of the fluid follows a smooth path
  - The paths of the different particles never cross each other
  - The path taken by the particles is called a *streamline*
- Turbulent flow
  - An irregular flow characterized by small whirlpool like regions
  - Turbulent flow occurs when the particles go above some critical speed



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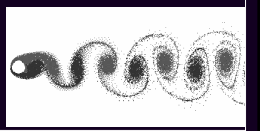
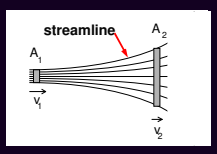
Onset of Turbulent Flow

The SeaWifS satellite image of a von Karman vortex around Guadalupe Island, August 20, 1999



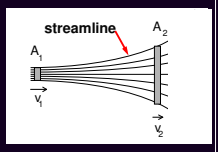
Ideal Fluids

- Fluid dynamics is very complicated in general (turbulence, vortices, etc.)
- Consider the simplest case first: the Ideal Fluid
  - No "viscosity" - no flow resistance (no internal friction)
  - Incompressible - density constant in space and time
- Simplest situation: consider ideal fluid moving with *steady flow* - velocity at each point in the flow is constant in time
- In this case, fluid moves on *streamlines*



Ideal Fluids

- Streamlines do not meet or cross
- Velocity vector is tangent to streamline
- Volume of fluid follows a tube of flow bounded by streamlines
- Streamline density is proportional to velocity
- Flow obeys **continuity equation**



Volume flow rate $Q = A \cdot v$ is constant along flow tube.

$$A_1 v_1 = A_2 v_2$$

Follows from mass conservation if flow is incompressible.

Lecture 18 *Exercise 6* Continuity

- A housing contractor saves some money by reducing the size of a pipe from 1" diameter to 1/2" diameter at some point in your house.

❖ Assuming the water moving in the pipe is an ideal fluid, relative to its speed in the 1" diameter pipe, how fast is the water going in the 1/2" pipe?

(A) 2 v_1 (B) 4 v_1 (C) 1/2 v_1 (D) 1/4 v_1

Lecture 18 *Exercise 6* Continuity

- A housing contractor saves some money by reducing the size of a pipe from 1" diameter to 1/2" diameter at some point in your house.

(A) 2 v_1 **(B) 4 v_1** (C) 1/2 v_1 (D) 1/4 v_1

❖ For equal volumes in equal times then ½ the diameter implies ¼ the area so the water has to flow four times as fast.

❖ But if the water is moving four times as fast the it has 16 times as much kinetic energy. Something must be doing work on the water (the pressure drops at the neck and we recast the work as $P\,\Delta V = (F/A)\,(A\Delta x) = F\,\Delta x$)

Conservation of Energy for Ideal Fluid

- Recall the standard work-energy relation $W = \Delta K = K_f - K_i$
  - ❖ Apply the principle to a section of flowing fluid with volume ΔV and mass $\Delta m = \rho\,\Delta V$ (here W is work done on fluid)
  - ❖ Net work by pressure difference over Δx ($\Delta x_1 = v_1\,\Delta t$)

$W = F_1\,\Delta x_1 - F_2\,\Delta x_2 = (F_1/A_1)\,(A_1\Delta x_1) - (F_2/A_2)\,(A_2\,\Delta x_2)$

$= P_1\,\Delta V_1 - P_2\,\Delta V_2$

and $\Delta V_1 = \Delta V_2 = \Delta V$ (incompressible)

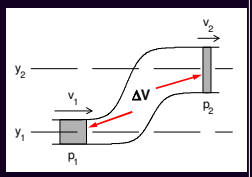
$W = (P_1 - P_2)\,\Delta V$ and

$W = \tfrac{1}{2}\,\Delta m\,v_2^2 - \tfrac{1}{2}\,\Delta m\,v_1^2$

$= \tfrac{1}{2}\,(\rho\Delta V)\,v_2^2 - \tfrac{1}{2}\,(\rho\Delta V)\,v_1^2$

$(P_1 - P_2) = \tfrac{1}{2}\,\rho\,v_2^2 - \tfrac{1}{2}\,\rho\,v_1^2$

$P_1 + \tfrac{1}{2}\,\rho\,v_1^2 = P_2 + \tfrac{1}{2}\,\rho\,v_2^2 = \text{const.}$



Bernoulli Equation → $P_1 + \tfrac{1}{2}\,\rho\,v_1^2 + \rho\,g\,y_1 = \text{constant}$

Lecture 18 *Exercise 7* Bernoulli's Principle

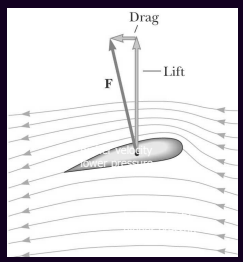
- A housing contractor saves some money by reducing the size of a pipe from 1" diameter to 1/2" diameter at some point in your house.

2) What is the pressure in the 1/2" pipe relative to the 1" pipe?

(A) smaller (B) same (C) larger

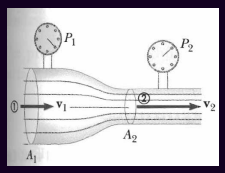
Applications of Fluid Dynamics

- Streamline flow around a moving airplane wing
- ***Lift*** is the upward force on the wing from the air
- ***Drag*** is the resistance
- The lift depends on the speed of the airplane, the area of the wing, its curvature, and the angle between the wing and the horizontal



Note: density of flow lines reflects velocity, not density. We are assuming an incompressible fluid.

Venturi



Bernoulli's Eq.

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

$$v_1 = \frac{A_2}{A_1} v_2$$

$$P_1 + \tfrac{1}{2}\rho\left(\frac{A_2}{A_1}\right)^2 v_2^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

$$v_2 = A_1\sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

Cavitation

Venturi result $v_2 = A_1\sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$

In the vicinity of high velocity fluids, the pressure can gets so low that the fluid vaporizes.

Lecture 18, Recap

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  - Archimedes' Principle
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