Physics 207, Lecture 19, Nov. 8

- Agenda: Chapter 14, Finish, Chapter 15, Start
  - Ch. 14: Fluid flow
  - Ch. 15: Oscillatory motion
  - Linear oscillator
  - Simple pendulum
  - Physical pendulum
  - Torsional pendulum

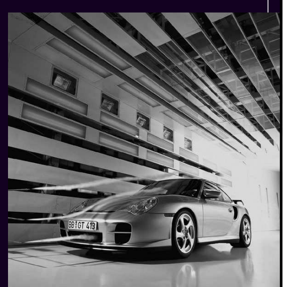
Assignments:

- Problem Set 7 due Nov. 14, Tuesday 11:59 PM
- For Monday, Finish Chapter 15, Start Chapter 16

Fluids in Motion

- Up to now we have described fluids in terms of their static properties:
  - Density ρ
  - Pressure *p*
- To describe fluid motion, we need something that can describe flow:
  - Velocity ***v***
- There are different kinds of fluid flow of varying complexity
  - non-steady / steady
  - compressible / incompressible
  - rotational / irrotational
  - viscous / ideal

Types of Fluid Flow



- Laminar flow
  - Each particle of the fluid follows a smooth path
  - The paths of the different particles never cross each other
  - The path taken by the particles is called a *streamline*
- Turbulent flow
  - An irregular flow characterized by small whirlpool like regions
  - Turbulent flow occurs when the particles go above some critical speed

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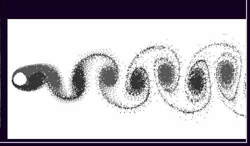
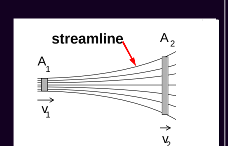
Onset of Turbulent Flow

The SeaWifS satellite image of a von Karman vortex around Guadalupe Island, August 20, 1999



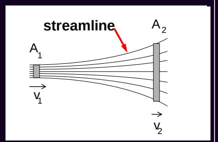
Ideal Fluids

- Fluid dynamics is very complicated in general (turbulence, vortices, etc.)
- Consider the simplest case first: <u>the Ideal Fluid</u>
  - No “viscosity” - no flow resistance (no internal friction)
  - Incompressible - density constant in space and time
- Simplest situation: consider ideal fluid moving with *steady flow* - velocity at each point in the flow is constant in time
- In this case, fluid moves on *streamlines*



Ideal Fluids

- Streamlines do not meet or cross
- Velocity vector is tangent to streamline
- Volume of fluid follows a tube of flow bounded by streamlines
- Streamline density is proportional to velocity
- Flow obeys **continuity equation**



Volume flow rate Q = A·v is constant along flow tube.

$\boxed{A_1 v_1 = A_2 v_2}$

Follows from mass conservation if flow is incompressible.

Lecture 19 *Exercise 1*
Continuity

- A housing contractor saves some money by reducing the size of a pipe from 1" diameter to 1/2" diameter at some point in your house.

❖ Assuming the water moving in the pipe is an ideal fluid, relative to its speed in the 1" diameter pipe, how fast is the water going in the 1/2" pipe?

(A) 2 v_1 (B) 4 v_1 (C) 1/2 v_1 (D) 1/4 v_1

Conservation of Energy for Ideal Fluid

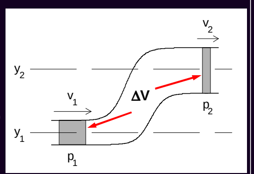
- Recall the standard work-energy relation $W = \Delta K = K_f - K_i$
  - ❖ Apply the principle to a section of flowing fluid with volume ΔV and mass $\Delta m = \rho\,\Delta V$ (here W is work done on fluid)
  - ❖ Net work by pressure difference over Δx ($\Delta x_1 = v_1\,\Delta t$)
  - ❖ Focus first on $W = F\,\Delta x$

$W = F_1\,\Delta x_1 - F_2\,\Delta x_2$

$= (F_1/A_1)\,(A_1\Delta x_1) - (F_2/A_2)\,(A_2\,\Delta x_2)$

$= P_1\,\Delta V_1 - P_2\,\Delta V_2$

and $\Delta V_1 = \Delta V_2 = \Delta V$ (incompressible)



$W = (P_1 - P_2)\,\Delta V$

Bernoulli Equation → $P_1 + ½\,\rho\,v_1^2 + \rho\,g\,y_1 = \text{constant}$

Conservation of Energy for Ideal Fluid

- Recall the standard work-energy relation $W = \Delta K = K_f - K_i$

$W = (P_1 - P_2)\,\Delta V$ and

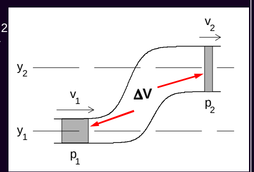
$W = ½\,\Delta m\,v_2^2 - ½\,\Delta m\,v_1^2$

$= ½\,(\rho\Delta V)\,v_2^2 - ½\,(\rho\Delta V)\,v_1^2$

$(P_1 - P_2) = ½\,\rho\,v_2^2 - ½\,\rho\,v_1^2$

$P_1 + ½\,\rho\,v_1^2 = P_2 + ½\,\rho\,v_2^2 = \text{constant}$

(in a horizontal pipe)



Bernoulli Equation → $P_1 + ½\,\rho\,v_1^2 + \rho\,g\,y_1 = \text{constant}$

Lecture 19 *Exercise 2*
Bernoulli's Principle

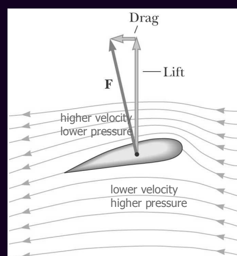
- A housing contractor saves some money by reducing the size of a pipe from 1" diameter to 1/2" diameter at some point in your house.

2) What is the pressure in the 1/2" pipe relative to the 1" pipe?

(A) smaller (B) same (C) larger

Applications of Fluid Dynamics

- Streamline flow around a moving airplane wing
- ***Lift*** is the upward force on the wing from the air
- ***Drag*** is the resistance
- The lift depends on the speed of the airplane, the area of the wing, its curvature, and the angle between the wing and the horizontal



Note: density of flow lines reflects velocity, not density. We are assuming an incompressible fluid.

Back of the envelope calculation

- **Boeing 747-400**
- Dimensions:
  - Length: 231 ft 10 inches
  - Wingspan: 211 ft 5 in
  - Height: 63 ft 8 in
- Weight:
  - Empty: 399, 000 lb
  - Max Takeoff (MTO): 800, 000 lb
  - Payload: 249, 122 lb cargo
- Performance:
  - Cruising Speed: 583 mph
  - Range: 7,230 nm
- $\rho\,(v_2^2 - v_1^2)/2 = P_1 - P_2 = \Delta P$

Let $v_2 = 220.0$ m/s $v_2 = 210$ m/s

So $\Delta P = 3 \times 10^3$ Pa $= 0.03$ atm

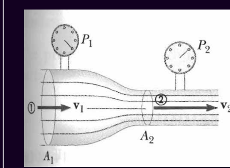
or 0.5 lbs/in^2

http://www.geocities.com/galemcraig/

Let an area of 200 ft x 15 ft produce lift or 4.5 x 10^5 in^2 or just 2.2 x 10^5 lbs → upshot

1. Downward deflection
2. Bernoulli (a small part)
3. Circulation theory

Venturi



Bernoulli's Eq.

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

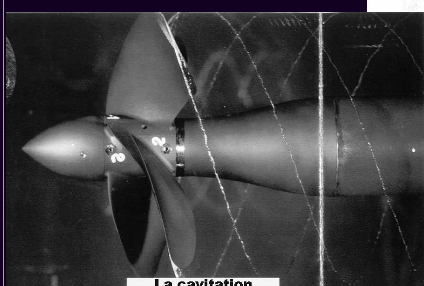
$$v_1 = \frac{A_2}{A_1} v_2$$

$$P_1 + \tfrac{1}{2}\rho\left(\frac{A_2}{A_1}\right)^2 v_2^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

$$v_2 = A_1\sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

Cavitation

Venturi result $v_2 = A_1\sqrt{\dfrac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$



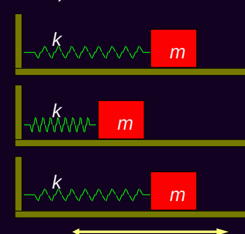
In the vicinity of high velocity fluids, the pressure can gets so low that the fluid vaporizes.

Chapter 15
Simple Harmonic Motion (SHM)

- We know that if we stretch a spring with a mass on the end and let it go the mass will oscillate back and forth (if there is no friction).
- This oscillation is called

  Simple Harmonic Motion

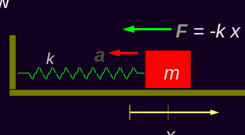
  and if you understand a sine or cosine is straightforward to understand.



SHM Dynamics

- At any given instant we know that $F = ma$ must be true.
- But in this case $F = -k\,x$

  and $ma = m\dfrac{d^2x}{dt^2}$



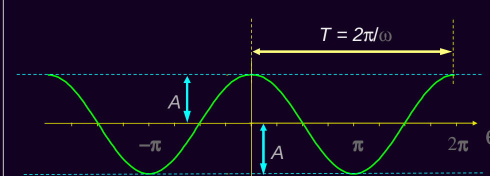
- So: $-k\,x = ma = m\dfrac{d^2x}{dt^2}$

$$\boxed{\frac{d^2x}{dt^2} = -\frac{k}{m}x}$$ → a differential equation for $x(t)$!

Simple approach, guess a solution and see if it works!

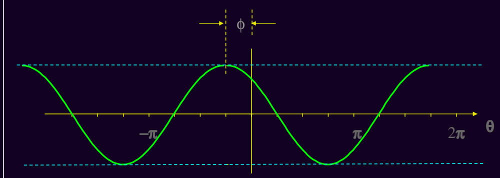
SHM Solution...

- Either $\cos(\omega t)$ or $\sin(\omega t)$ can work
- Below is a drawing of $A\cos(\omega t)$
- where A = amplitude of oscillation



SHM Solution...

- What to do if we need the sine solution?
- Notice A cos($\omega t + \phi$) = A [cos(ωt) cos(ϕ) - sin(ωt) sin(ϕ)
  = [A cos(ϕ)] cos(ωt) - [A sin(ϕ)] sin(ωt)
  = A′ cos(ωt) + A″ sin(ωt) (sine and cosine)
- Drawing of A cos($\omega t + \phi$)



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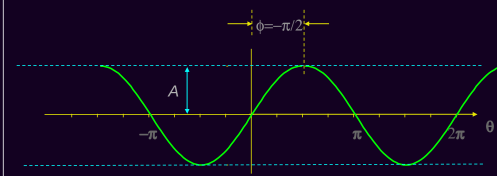
SHM Solution...

- Drawing of A cos ($\omega t - \pi/2$)



= A sin(ωt)

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What about Vertical Springs?

- For a vertical spring, if y is measured from the equilibrium position

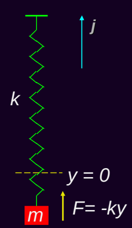
$$U = \frac{1}{2} k y^2$$

- Recall: force of the spring is the negative derivative of this function:

$$F = -\frac{dU}{dy} = -ky$$

- This will be just like the horizontal case:

$$-ky = ma = m\frac{d^2 y}{dt^2}$$



Which has solution $y(t)$ = A cos($\omega t + \phi$) where $\omega = \sqrt{\frac{k}{m}}$

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Velocity and Acceleration

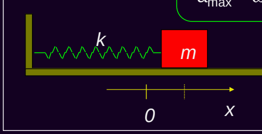
Position: $x(t) = A\cos(\omega t + \phi)$
Velocity: $v(t) = -\omega A \sin(\omega t + \phi)$
Acceleration: $a(t) = -\omega^2 A \cos(\omega t + \phi)$

by taking derivatives, since:

$$v(t) = \frac{dx(t)}{dt}$$

$$a(t) = \frac{dv(t)}{dt}$$

$x_{max} = A$
$v_{max} = \omega A$
$a_{max} = \omega^2 A$



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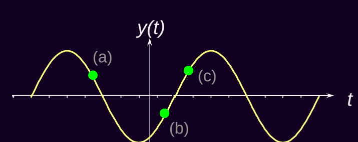
Lecture 19, *Exercise 3*
Simple Harmonic Motion

- A mass oscillates up & down on a spring. It's position as a function of time is shown below. At which of the points shown does the mass have positive velocity and negative acceleration ?

Remember: velocity is slope and acceleration is the curvature



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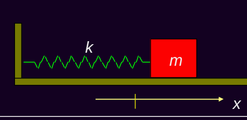
Example

- A mass $m = 2$ kg on a spring oscillates with amplitude A = 10 cm. At t = 0 its speed is at a maximum, and is v=+2 m/s
  - What is the angular frequency of oscillation ω ?
  - What is the spring constant k ?

General relationships E = K + U = constant, $\omega = (k/m)^{1/2}$

So at maximum speed U=0 and ½ mv² = E = ½ kA²

thus $k = mv^2/A^2$ = 2 x (2) $^2/(0.1)^2$ = 800 N/m, ω = 20 rad/sec



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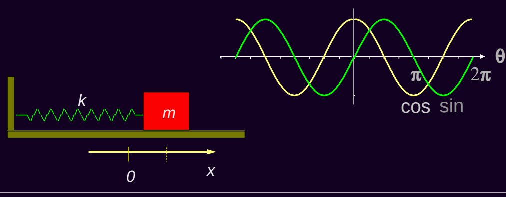
Initial Conditions

Use "initial conditions" to determine phase ϕ !



Lecture 19, *Example 4*
Initial Conditions

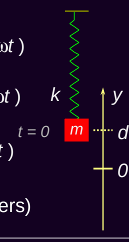
- A mass hanging from a vertical spring is lifted a distance d above equilibrium and released at $t = 0$. Which of the following describe its velocity and acceleration as a function of time (upwards is positive y direction):

(A) $v(t) = -v_{max}\sin(\omega t)$ $\quad a(t) = -a_{max}\cos(\omega t)$

(B) $v(t) = v_{max}\sin(\omega t)$ $\quad a(t) = a_{max}\cos(\omega t)$

(C) $v(t) = v_{max}\cos(\omega t)$ $\quad a(t) = -a_{max}\cos(\omega t)$

(both v_{max} and a_{max} are positive numbers)



Energy of the Spring-Mass System

We know enough to discuss the mechanical energy of the oscillating mass on a spring.

Remember,

$x(t) = A\cos(\omega t + \phi)$
$v(t) = -\omega A\sin(\omega t + \phi)$
$a(t) = -\omega^2 A\cos(\omega t + \phi)$

Kinetic energy is always

$K = \frac{1}{2} mv^2$

$K = \frac{1}{2} m[-\omega A\sin(\omega t + \phi)]^2$

And the potential energy of a spring is,

$U = \frac{1}{2} k x^2$

$U = \frac{1}{2} k[A\cos(\omega t + \phi)]^2$

Energy of the Spring-Mass System

Add to get E = K + U = constant.

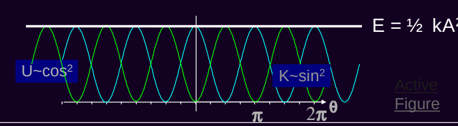
$\frac{1}{2} m(\omega A)^2\sin^2(\omega t + \phi) + 1/2\, k(A\cos(\omega t + \phi))^2$

Remember that $\omega = \sqrt{\frac{k}{m}} \Rightarrow \omega^2 = \frac{k}{m}$

so, $E = \frac{1}{2} k A^2\sin^2(\omega t + \phi) + \frac{1}{2} kA^2\cos^2(\omega t + \phi)$

$= \frac{1}{2} k A^2[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)]$

$= \frac{1}{2} k A^2$



Active Figure

SHM So Far

- The most general solution is $x = A\cos(\omega t + \phi)$
  where A = amplitude
  ω = (angular) frequency
  ϕ = phase constant
- For SHM without friction, $\omega = \sqrt{\frac{k}{m}}$
  - The frequency does *not* depend on the amplitude !
  - We will see that this is true of all simple harmonic motion!
- The oscillation occurs around the equilibrium point where the force is zero!
- Energy is a constant, it transfers between potential and kinetic.

The Simple Pendulum

- A pendulum is made by suspending a mass m at the end of a string of length L. Find the frequency of oscillation for small displacements.

$\Sigma F_y = ma_c = T - mg\cos(\theta) = m\,v^2/L$

$\Sigma F_x = ma_x = -mg\sin(\theta)$

If θ small then $x \cong L\,\theta$ and $\sin(\theta) \cong \theta$

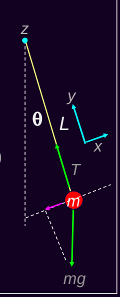
$dx/dt = L\; d\theta/dt$

$a_x = d^2x/dt^2 = L\, d^2\theta/dt^2$

so $a_x = -g\,\theta = L\, d^2\theta/dt^2 \rightarrow L\, d^2\theta/dt^2 - g\,\theta = 0$

and $\theta = \theta_0\cos(\omega t + \phi)$ or $\theta = \theta_0\sin(\omega t + \phi)$

with $\omega = (g/L)^{1/2}$



The Rod Pendulum

- A pendulum is made by suspending a thin rod of length L and mass M at one end. Find the frequency of oscillation for small displacements.

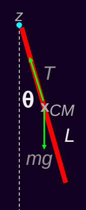
$\Sigma \tau_z = I\alpha = -|\mathbf{r} \times \mathbf{F}| = (L/2)\ mg \sin(\theta)$

(no torque from T)

$-[\ mL^2/12 + m\,(L/2)^2\ ]\ \ \alpha \cong\ L/2\ mg\ \theta$

$-1/3\ L\ d^2\theta/dt^2 = ½\ g\ \ \theta$

The rest is for homework…



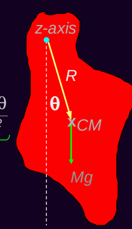
General Physical Pendulum

- Suppose we have some arbitrarily shaped solid of mass M hung on a fixed axis, that we know where the CM is located **and** what the moment of inertia I about the axis is.
- The torque about the rotation (z) axis for small θ is (sin $\theta \cong \theta$)

$\tau = -MgR\sin\theta \cong -MgR\theta \rightarrow \underbrace{-MgR\,\theta}_{\tau} = I\underbrace{\frac{d^2\theta}{dt^2}}_{\alpha}$

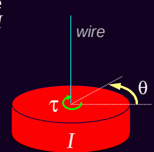
$\Rightarrow\ \frac{d^2\theta}{dt^2} = -\omega^2\theta$ where $\boxed{\omega = \sqrt{\frac{MgR}{I}}}$

$\Rightarrow\ \theta = \theta_0 \cos(\omega t + \phi)$



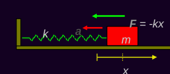
Torsion Pendulum

- Consider an object suspended by a wire attached at its CM. The wire defines the rotation axis, and the moment of inertia I about this axis is known.
- The wire acts like a "rotational spring".
  - ❖ When the object is rotated, the wire is twisted. This produces a torque that opposes the rotation.
  - ❖ In analogy with a spring, the torque produced is proportional to the displacement: $\tau = -\kappa\,\theta$ where κ is the torsional spring constant
  - ❖ $\omega = (\kappa/I)^{½}$



Reviewing Simple Harmonic Oscillators

- Spring-mass system

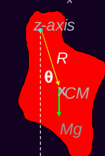


$\frac{d^2x}{dt^2} = -\omega^2 x$ where $\omega = \sqrt{\frac{k}{m}}$

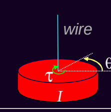
$\Rightarrow\ x(t) = A\cos(\ \omega t + \phi)$

- Pendula $\frac{d^2\theta}{dt^2} = -\omega^2\theta$

$\Rightarrow\ \theta = \theta_0 \cos(\ \omega t + \phi)$

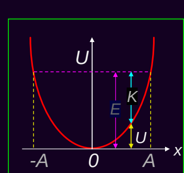


- ❖ General physical pendulum $\omega = \sqrt{\frac{MgR}{I}}$
- ❖ Torsion pendulum $\omega = \sqrt{\frac{\kappa}{I}}$



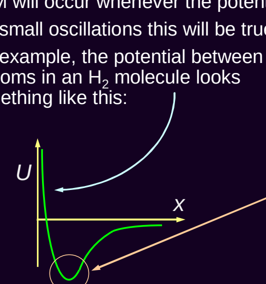
Energy in SHM

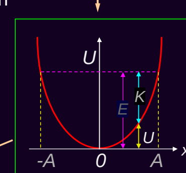
- For both the spring and the pendulum, we can derive the SHM solution using energy conservation.
- The total energy (K + U) of a system undergoing SMH will always be constant!
- This is not surprising since there are only conservative forces present, hence energy is conserved.



SHM and quadratic potentials

- SHM will occur whenever the potential is quadratic.
- For small oscillations this will be true:
- For example, the potential between H atoms in an H_2 molecule looks something like this:





Lecture 19, Recap

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