Physics 207, Lecture 24, Nov. 27

- Agenda: Mid-Term 3 Review
  - Elastic Properties of Matter, Moduli
  - Pressure, Work, Archimedes' Principle, Fluid flow, Bernoulli
  - Oscillatory motion, Linear oscillator, Pendulums
  - Energy, Damping, Resonance
  - Transverse Waves, Pulses, Reflection, Transmission, Power
  - Longitudinal Waves (Sound), Plane waves, Spherical waves
  - Loudness, Doppler effect

Assignments:

- Problem Set 9 due Tuesday, Dec. 5, 11:59 PM

Ch. 18: 9, 17, 21, 39, 53a, Ch. 19: 2, 12, 15, 31, 43, 57

- Mid-term 3, Tuesday, Nov. 28, Chapters 14-17, 90 minutes, 7:15-8:45 PM in rooms 105 and 113 Psychology. McBurney students will go to room 5130 Chamberlin (Grades on Monday)
- Wednesday, Chapter 19 (Temperature, then Heat & Thermodynamics)

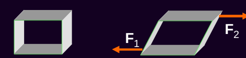
Some definitions

- Elastic properties of solids :
  - Young's modulus: measures the resistance of a solid to a change in its length.

$$Y \equiv \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{\Delta L / L_0}$$

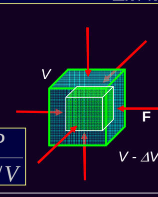
  - Shear modulus: measures the resistance to motion of the planes of a solid sliding past each other.

$$S \equiv \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x / h}$$



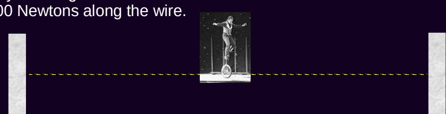
  - Bulk modulus: measures the resistance of solids or liquids to changes in their volume.

$$B \equiv -\frac{\Delta P}{\Delta V / V}$$



Example: Statics with Young's Modulus

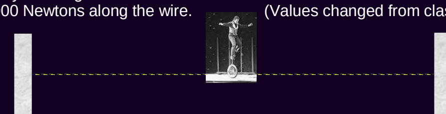
- A small person is riding a unicycle and is halfway between two posts 200 m apart. The guide wire was originally 200 m long, weighs 1.0 kg and has cross sectional area of 2 cm². Under the weight of the unicycle it sags down 1.0 m at the center and there is a tension of 5000 Newtons along the wire.



(a) What is the Young's Modulus of the wire (to two significant figures)?
(b) How long does it take a transverse wave (a pulse) to propagate from the support to the unicycle (Treat wire as a simple string)?
(c) If the pulse is now said to be a perfectly sinusoidal wave and has a frequency of 100 Hz, what is the angular frequency?
(d) At what transverse amplitude of the wave, in the vertical direction, will the wire's maximum acceleration just reach 10 m/s

Example: Statics with Young's Modulus

- A small person is riding a unicycle and is halfway between two posts 200 m apart. The guide wire was originally 200 m long, weighs 1.0 kg and has cross sectional area of 2 cm². Under the weight of the unicycle it sags down 1.0 m at the center and there is a tension of 5000 Newtons along the wire. (Values changed from class).



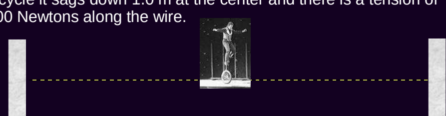
(a) What is the Young's Modulus of the wire (to two significant figures)?

$\Delta L = [(100^2 + 1.0^2)^{1/2} - 100]$ m $\cong 5 \times 10^{-3}$ m

$$Y = \frac{F/A}{\Delta L / L_0} = \frac{5000\,\text{N}/\,2\times10^{-4}\ \text{m}^2}{5.0\times10^{-5}} = 5.0\times10^{11}\ \text{N/m}^2$$

Example: Statics with Young's Modulus

- A small person is riding a unicycle and is halfway between two posts 200 m apart. The guide wire was originally 200 m long, weighs 1.0 kg and has cross sectional area of 2 cm² . Under the weight of the unicycle it sags down 1.0 m at the center and there is a tension of 5000 Newtons along the wire.



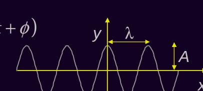
(b) How long does it take a transverse wave to propagate from the support to the unicycle (Note: treat wire as a simple string)?

time = distance / velocity = 100 m / $(T/\mu)^{1/2}$ = 100 / $(5000 / (1.0/200))^{1/2}$ s
t = 100 / $(1 \times 10^6)^{1/2}$ sec = 100 / 1 x 10^3 sec = 0.10 seconds

Notice $T/\mu = 5000 \times 200 = 10^6$ m²/s² & $Y/\rho = 5.0 \times 10^{11} \times (200 \times 2 \times 10^{-6}) = 2 \times 10^8$ m²/s²

Math Summary

- The formula $y(x,t) = A\cos(kx - \omega t + \phi)$ describes a harmonic wave of amplitude A moving in the $+x$ direction.



- Each point on the wave oscillates in the y direction with simple harmonic motion of angular frequency ω.
  - The wavelength of the wave is $\lambda = \frac{2\pi}{k}$
  - The speed of the wave is $v = \frac{\omega}{k}$
  - The quantity k is often called "wave number".

Velocity and Acceleration

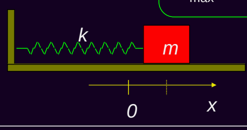
Position: $x(t) = A \cos(\omega t + \phi)$
Velocity: $v(t) = -\omega A \sin(\omega t + \phi)$
Acceleration: $a(t) = -\omega^2 A \cos(\omega t + \phi)$

by taking derivatives, since:

$$v(t) = \frac{dx(t)}{dt}$$

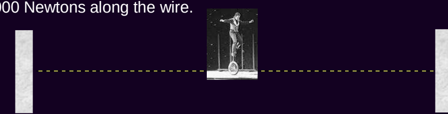
$$a(t) = \frac{dv(t)}{dt}$$

$x_{max} = A$
$v_{max} = \omega A$
$a_{max} = \omega^2 A$



Example: Statics with Young's Modulus

- A small person is riding a unicycle and is halfway between two posts 200 m apart. The guide wire was originally 200 m long, weighs 1.0 kg and has cross sectional area of 2 mm. Under the weight of the unicycle it sags down 0.01 m at the center and there is a tension of 5000 Newtons along the wire.



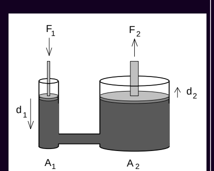
(c) If the pulse is said to be a perfectly sinusoidal wave and has a frequencyof 100 Hz, what is the angular frequecy? → 628 rad/s
(d) At what transverse amplitude of the wave, in the vertical direction, will the wire's maximum acceleration just reach 10 m/s²

$a(t) = -\omega^2 A \cos(\omega t + \phi)$

$a_{max} = \omega^2 A = (100 \times 2\pi)^2 A = 10 \text{ m/s}^2$

Pascal's Principle

- Consider the system shown:
  - A downward force F_1 is applied to the piston of area A_1.
  - This force is transmitted through the liquid to create an upward force F_2.
  - Pascal's Principle says that increased pressure from F_1 (F_1/A_1) is transmitted throughout the liquid.



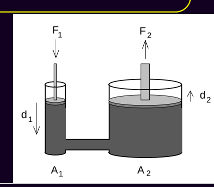
- $F_2 > F_1$: Is there conservation of energy?

Fluids: Pascal's Principle

- Pressure depends on depth: $\Delta p = \rho g \Delta y$
- Pascal's Principle addresses how a change in pressure is transmitted through a fluid.

Any change in the pressure applied to an enclosed fluid is transmitted to every portion of the fluid and to the walls of the containing vessel.

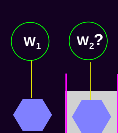
dW = **F • dx**
Here dW = F/A (A dx) or W = P dV
$F_1/A_1 \; A_1 d_1 = P A_1 d_1 = W$
$F_2/A_2 \; A_2 d_2 = P A_2 d_2 = W$
so $A_1 d_1 = A_2 d_2$



Archimedes' Principle

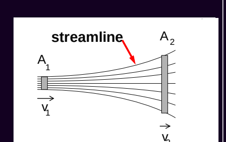
- The buoyant force is equal to the weight of the liquid that is displaced.
- If the buoyant force is larger than the weight of the object, it will float; otherwise it will sink.

- Since the pressure at the bottom of the object is greater than that at the top of the object, the water exerts a net upward force, <u>the buoyant force</u>, on the object.



Ideal Fluids

- Streamlines do not meet or cross
- Velocity vector is tangent to streamline
- Volume of fluid follows a tube of flow bounded by streamlines



- Streamline density is proportional to velocity
- Flow obeys **continuity equation**

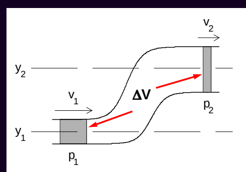
Volume flow rate $Q = A \cdot v$ is <u>constant</u> along flow tube.

$A_1 v_1 = A_2 v_2$

Follows from mass conservation if flow is incompressible.

Conservation of Energy for Ideal Fluid

- Recall the standard work-energy relation $W = \Delta K = K_f - K_i$
  - Apply the principle to a section of flowing fluid with volume ΔV and mass $\Delta m = \rho\, \Delta V$ (here W is work done on fluid)
  - Net work by pressure difference over Δx ($\Delta x_1 = v_1\, \Delta t$)

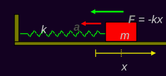


Bernoulli Equation → $P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = \text{constant}$

Physics 207: Lecture 24, Pg 13

Reviewing Simple Harmonic Oscillators

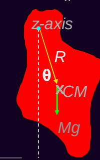
- Spring-mass system



$$\frac{d^2x}{dt^2} = -\omega^2 x \quad \text{where} \quad \omega = \sqrt{\frac{k}{m}}$$

→ $x(t) = A\cos(\omega t + \phi)$

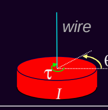
- Pendula



$$\frac{d^2\theta}{dt^2} = -\omega^2\theta$$

→ $\theta = \theta_0 \cos(\omega t + \phi)$

- General physical pendulum $\omega = \sqrt{\frac{MgR}{I}}$
- Torsion pendulum $\omega = \sqrt{\frac{\kappa}{I}}$



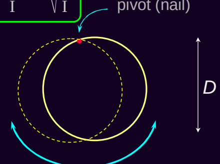
Physics 207: Lecture 24, Pg 14

Lecture 24, *Exercise*
Physical Pendulum

- A pendulum is made by hanging a thin hoola-hoop of diameter D on a small nail. What is the angular frequency of oscillation of the hoop for small displacements ?

($I_{CM} = mR^2$ for a hoop)

→ $\frac{d^2\theta}{dt^2} = -\omega^2\theta$ where $\omega = \sqrt{\frac{MgR}{I}} = \sqrt{\frac{\tau}{I}}$



(A) $\omega = \sqrt{\frac{g}{D}}$

(B) $\omega = \sqrt{\frac{2g}{D}}$

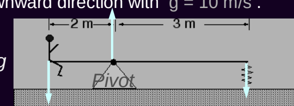
(C) $\omega = \sqrt{\frac{g}{2D}}$

Physics 207: Lecture 24, Pg 15

Sample Problem (Another physical pendulum)

- **PROBLEM:** A 30 kg child is sitting with his center of mass 2 m from the frictionless pivot of a massless see-saw as shown. The see-saw is initially horizontal and, at 3 m on the other side, there is a massless Hooke's Law spring (constant 120 N/m) attached so that it sits perfectly vertical (but slightly stretched). Gravity acts in the downward direction with g = 10 m/s .

$F_{child} = m_b g$



$F = k\Delta x$

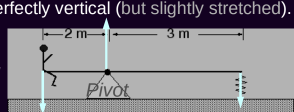
- Assuming everything is static and in perfect equilibrium. (a) What force, F, is provided by the spring?
  $\Sigma\tau = 0 = m_b g\,(2m) - F\,(3m)$ → $F = 600\text{ N} / 3 = 200\text{ N}$
- (b) Now the child briefly bounces the see-saw (with a small amplitude oscillation) and then moves with the see-saw. What is the angular frequency of the child?

Physics 207: Lecture 24, Pg 16

Sample Problem (Another physical pendulum)

- **PROBLEM:** A 30 kg child is sitting with his center of mass 2 m from the frictionless pivot of a massless see-saw as shown. The see-saw is initially horizontal and, at 3 m on the other side, there is a massless Hooke's Law spring (constant 120 N/m) attached so that it sits perfectly vertical (but slightly stretched).

$F_{child} = m_b g$



$F = k\Delta x$

$\Delta x = r\,\theta$

- $\Sigma\tau = 0 = m_b g\,(2m) - F\,(3m)$ → $F = 600\text{ N} / 3 = 200\text{ N}$
- (b) Now the child briefly bounces the see-saw (with a small amplitude oscillation) and then moves with the see-saw. What is the angular frequency of the child?

$I\,\alpha = I\, d^2\theta / dt^2 = -r\, k\Delta x \cong -r\, k\, r\, \theta$ → $\omega = (\tau/I)^{1/2} = (r^2 k / m r_b^2)^{1/2}$

$\omega = (9 \times 120 / 30 \times 4)^{1/2} = (4/4)^{1/2} = 3\text{ rad / s}$

Physics 207: Lecture 24, Pg 17

SHM: Velocity and Acceleration

Position: $x(t) = A\cos(\omega t + \phi)$

Velocity: $v(t) = -\omega A \sin(\omega t + \phi)$

Acceleration: $a(t) = -\omega^2 A \cos(\omega t + \phi)$

by taking derivatives, since:

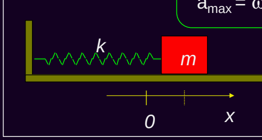
$$v(t) = \frac{dx(t)}{dt}$$

$$a(t) = \frac{dv(t)}{dt}$$

$x_{max} = A$

$v_{max} = \omega A$

$a_{max} = \omega^2 A$

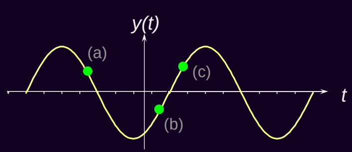


Physics 207: Lecture 24, Pg 18

Lecture 24, *Exercise* Simple Harmonic Motion

- A mass oscillates up & down on a spring. It's position as a function of time is shown below. At which of the points shown does the mass have <u>positive</u> velocity and <u>negative</u> acceleration ?

Remember: velocity is slope and acceleration is the curvature



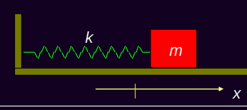
Example

- A mass $m = 2$ kg on a spring oscillates with amplitude $A = 10$ cm. At $t = 0$ its speed is at a maximum, and is $v=+2$ m/s
  - What is the angular frequency of oscillation ω ?
  - What is the spring constant k ?

General relationships $E = K + U =$ constant, $\omega = (k/m)^{1/2}$

So at maximum speed U=0 and $\frac{1}{2} mv^2 = E = \frac{1}{2} kA^2$

thus $k = mv^2/A^2 = 2 \times (2)^2/(0.1)^2 =$ 800 N/m, ω = 20 rad/sec



Lecture 24, *Example* Initial Conditions

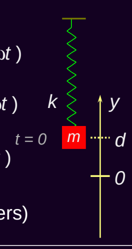
- A mass hanging from a vertical spring is lifted a distance d above equilibrium and released at $t = 0$. Which of the following describe its velocity and acceleration as a function of time (upwards is positive y direction):

(A) $v(t) = -v_{max} \sin(\omega t)$ $a(t) = -a_{max} \cos(\omega t)$

(B) $v(t) = v_{max} \sin(\omega t)$ $a(t) = a_{max} \cos(\omega t)$

(C) $v(t) = v_{max} \cos(\omega t)$ $a(t) = -a_{max} \cos(\omega t)$

(both v_{max} and a_{max} are positive numbers)



Lecture 24, *Exercise* Initial Conditions

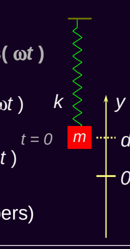
- A mass hanging from a vertical spring is lifted a distance d above equilibrium and released at $t = 0$. Which of the following describe its velocity and acceleration as a function of time (upwards is positive y direction):

(A) $v(t) = -v_{max} \sin(\omega t)$ $a(t) = -a_{max} \cos(\omega t)$

(B) $v(t) = v_{max} \sin(\omega t)$ $a(t) = a_{max} \cos(\omega t)$

(C) $v(t) = v_{max} \cos(\omega t)$ $a(t) = -a_{max} \cos(\omega t)$

(both v_{max} and a_{max} are positive numbers)



Energy of the Spring-Mass System

We know enough to discuss the mechanical energy of the oscillating mass on a spring.

Remember,

$x(t) = A \cos(\omega t + \phi)$
$v(t) = -\omega A \sin(\omega t + \phi)$
$a(t) = -\omega^2 A \cos(\omega t + \phi)$

Kinetic energy is always

$K = \frac{1}{2} mv^2$

$K = \frac{1}{2} m [-\omega A \sin(\omega t + \phi)]^2$

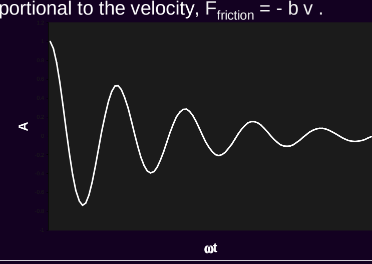
And the potential energy of a spring is,

$U = \frac{1}{2} k x^2$

$U = \frac{1}{2} k [A \cos(\omega t + \phi)]^2$

What about Friction?

- Friction causes the oscillations to get smaller over time
- This is known as DAMPING.
- As a model, we assume that the force due to friction is proportional to the velocity, $F_{friction} = -bv$.



What about Friction?

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2} \quad \Rightarrow \quad \frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

We can guess at a new solution.

$$x = A\exp\left(-\tfrac{bt}{2m}\right)\cos(\omega t + \phi)$$ and now $\omega_0^2 \equiv k/m$

With,

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} = \sqrt{\omega_o^2 - \left(\frac{b}{2m}\right)^2}$$

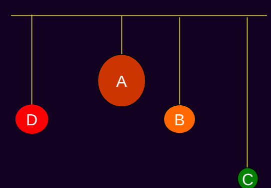
What about Friction?

$$x(t) = A\exp\left(-\tfrac{bt}{2m}\right)\cos(\omega t + \phi) \quad \text{if} \quad \omega_o > b/2m$$

What does this function look like?

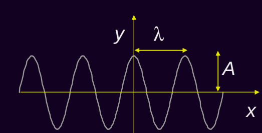
Lecture 24, *Exercise* Resonant Motion

- Consider the following set of pendulums all attached to the same string



If I start bob D swinging which of the others will have the largest swing amplitude ?
(A) (B) (C)

And then traveling waves on a string



- General harmonic waves

$$y(x,t) = A\cos(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda} \quad \omega = 2\pi f = \frac{2\pi}{T}$$

$$v = \lambda f = \frac{\omega}{k}$$

- Waves on a string

$$v = \sqrt{\frac{F}{\mu}}$$ (F: tension, μ: mass / length)

$$\bar{P} = \frac{1}{2}\mu v \omega^2 A^2$$

$$\frac{d\bar{E}}{dx} = \frac{1}{2}\mu\omega^2 A^2$$

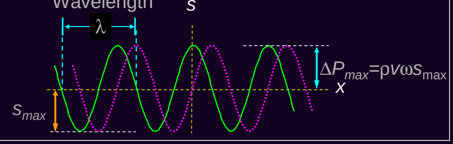
Sound Wave Properties

- Displacement: The maximum relative displacement s of a point on the wave. Displacement is longitudinal.
- Maximum displacement has minimum velocity

$$s(x,t) = s_{max}\cos[(2\pi/\lambda)\,x - \omega t)]$$

$$ds/dt = \omega s_{max}\sin[(2\pi/\lambda)\,x - \omega t)]$$

Molecules "pile up" where the relative velocity is maximum (i.e., $ds/dt = \omega\, s_{max}$)



Example, Energy transferred by a string

- Two strings are held at the same tension and driven with the same amplitude and frequency. The only difference is that one is thicker and has a mass per unit length that is four times larger than the thinner one. Which string (and by how much) transfers the most power? (Circle the correct answer.)

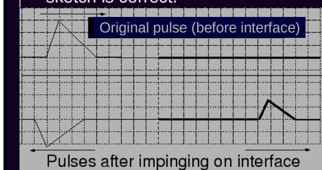
(A) the thicker string by a factor of 4.
(B) the thicker string by a factor of 2.
(C) they transfer an equivalent amount of energy.
(D) the thinner string by a factor of 2.
(E) the thinner string by a factor of 4.

$$\bar{P}_{thick} = \frac{1}{2}4\mu v\omega^2 A^2 = \frac{1}{2}4\mu\sqrt{\frac{T}{4\mu}}\omega^2 A^2 = 2\frac{1}{2}\mu\sqrt{\frac{T}{\mu}}\omega^2 A^2 = 2\bar{P}_{thin}$$

Example, pulses on a string

- A transverse pulse is initially traveling to the right on a string that is joined, on the right, to a thicker string of higher mass per unit length. The tension remains constant T throughout. Part of the pulse is reflected and part transmitted. The drawing to the right shows the before (at top) and after (bottom) the pulse traverses the interface. There are however a few mistakes in the bottom drawing.

Identify two things wrong in the bottom sketch assuming the top sketch is correct.



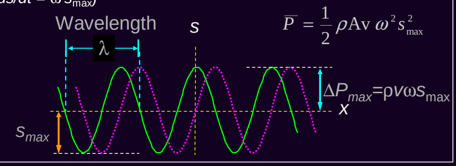
Sound Wave, A longitudinal wave

- Displacement: The maximum relative displacement s of a point on the wave. Displacement is longitudinal.
- Maximum displacement has minimum velocity

$$s(x,t) = s_{max} \cos[(2\pi/\lambda)\, x - \omega t)]$$

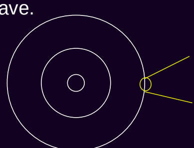
$$ds/dt = \omega s_{max} \sin[(2\pi/\lambda)\, x - \omega t)]$$

Molecules "pile up" where the relative velocity is maximum (i.e., $ds/dt = \omega\, s_{max}$)

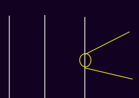


Waves, Wavefronts, and Rays

- If the power output of a source is constant, the total power of any wave front is constant.
- The Intensity at any point depends on the type of wave.



$$I = \frac{P_{av}}{A} = \frac{P_{av}}{4\pi R^2}$$



$$I = \frac{P_{av}}{A} = \frac{P_{av}}{\text{const}}$$

Intensity of sounds

- The amplitude of pressure wave depends on
  - Frequency ω of harmonic sound wave
  - Speed of sound v and density of medium ρ of medium
  - Displacement amplitude s_{max} of element of medium

$$\Delta P_{max} = \omega v \rho s_{max}$$

- Intensity of a sound wave is

$$I = \frac{\Delta P_{max}^2}{2\rho v}$$

  - Proportional to (amplitude)^2
  - This is a general result (not only for sound)
- Threshold of human hearing: $I_0 = 10^{-12}$ W/m^2

Sound Level, Example

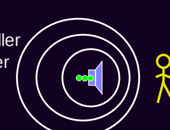
$$\beta = 10 \log_{10}\left(\frac{I}{I_0}\right)$$

- What is the sound level that corresponds to an intensity of 2.0 x 10^{-7} W/m^2 ?
- β = 10 log (2.0 x 10^{-7} W/m^2 / 1.0 x 10^{-12} W/m^2)
  = 10 log 2.0 x 10^5 = 53 dB
- Rule of thumb: An apparent "doubling" in the loudness is approximately equivalent to an increase of 10 dB.
- This factor is not linear with intensity

Doppler effect, moving sources/receivers

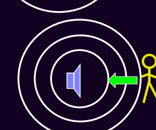
- If the source of sound is moving
  - Toward the observer ⇒ λ seems smaller
  - Away from observer ⇒ λ seems larger

$$f_{observer} = \left(\frac{v}{v \pm v_s}\right) f_{source}$$



- If the observer is moving
  - Toward the source ⇒ λ seems smaller
  - Away from source ⇒ λ seems larger

$$f_{observer} = \left(\frac{v \pm v_o}{v}\right) f_{source}$$



- If both are moving $f_{observer} = \left(\frac{v \pm v_o}{v \mp v_s}\right) f_{source}$

Doppler Example Audio

- Examples: police car, train, etc. (Recall: v is vector)

Doppler Example Visual

Physics 207, Lecture 24, Nov. 27

- Agenda: Mid-Term 3 Review
  - ❖ Elastic Properties of Matter, Moduli
  - ❖ Pressure, Work, Archimedes' Principle, Fluid flow, Bernoulli
  - ❖ Oscillatory motion, Linear oscillator, Pendulums
  - ❖ Energy, Damping, Resonance
  - ❖ Transverse Waves, Pulses, Reflection, Transmission, Power
  - ❖ Longitudinal Waves (Sound), Plane waves, Spherical waves
  - ❖ Loudness, Doppler effect

Assignments:

- Problem Set 9 due Tuesday, Dec. 5, 11:59 PM

Ch. 18: 3, 18, 30, 40, 58 Ch. 19: 2, 12, 15, 31, 43, 57

- Mid-term 3, Tuesday, Nov. 28, Chapters 14-17, 90 minutes, 7:15-8:45 PM in rooms 105 and 113 Psychology. (Grades on Monday)
- Wednesday, Chapter 19 (Temperature, Heat and Thermodynamics)