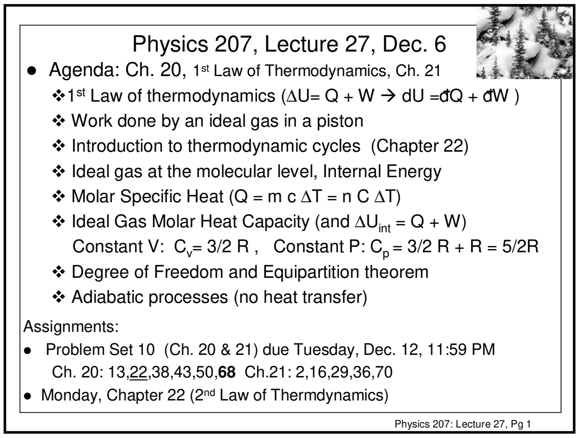
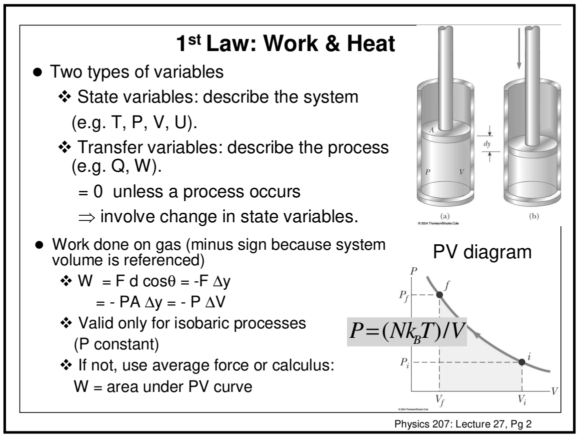


Physics 207, Lecture 27, Dec. 6

- Agenda: Ch. 20, 1st Law of Thermodynamics, Ch. 21
  - ❖ 1st Law of thermodynamics ($\Delta U = Q + W \rightarrow dU = đQ + đW$)
  - ❖ Work done by an ideal gas in a piston
  - ❖ Introduction to thermodynamic cycles (Chapter 22)
  - ❖ Ideal gas at the molecular level, Internal Energy
  - ❖ Molar Specific Heat ($Q = m\,c\,\Delta T = n\,C\,\Delta T$)
  - ❖ Ideal Gas Molar Heat Capacity (and $\Delta U_{int} = Q + W$)
    Constant V: $C_v = 3/2\,R$, Constant P: $C_p = 3/2\,R + R = 5/2R$
  - ❖ Degree of Freedom and Equipartition theorem
  - ❖ Adiabatic processes (no heat transfer)

Assignments:

- Problem Set 10 (Ch. 20 & 21) due Tuesday, Dec. 12, 11:59 PM
  Ch. 20: 13,22,38,43,50,**68** Ch.21: 2,16,29,36,70
- Monday, Chapter 22 (2nd Law of Thermdynamics)

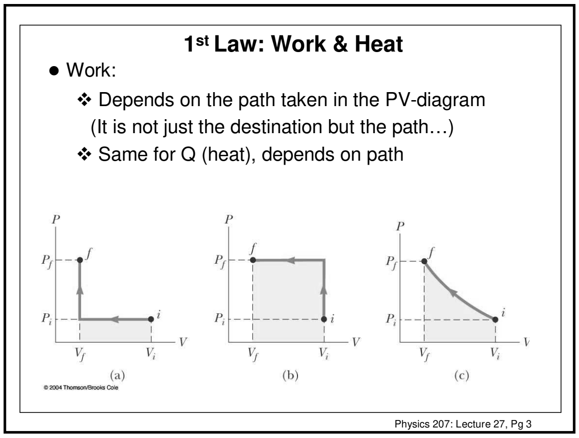


1st Law: Work & Heat

- Two types of variables
  - ❖ State variables: describe the system (e.g. T, P, V, U).
  - ❖ Transfer variables: describe the process (e.g. Q, W).
    = 0 unless a process occurs
    ⇒ involve change in state variables.
- Work done on gas (minus sign because system volume is referenced)
  - ❖ $W = F\,d\cos\theta = -F\,\Delta y$
    $= -PA\,\Delta y = -P\,\Delta V$
  - ❖ Valid only for isobaric processes (P constant)
  - ❖ If not, use average force or calculus: W = area under PV curve

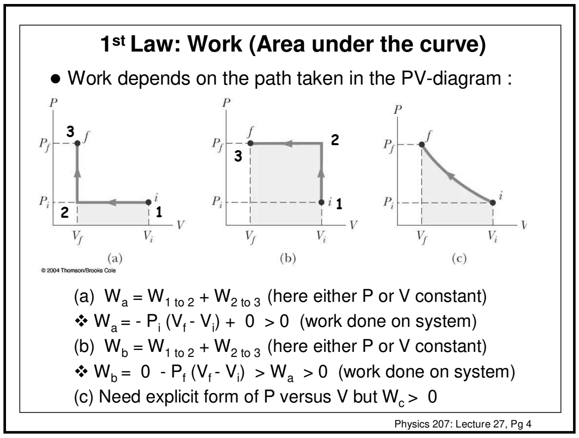
PV diagram

$P = (Nk_BT)/V$



1st Law: Work & Heat

- Work:
  - ❖ Depends on the path taken in the PV-diagram (It is not just the destination but the path…)
  - ❖ Same for Q (heat), depends on path



1st Law: Work (Area under the curve)

- Work depends on the path taken in the PV-diagram :

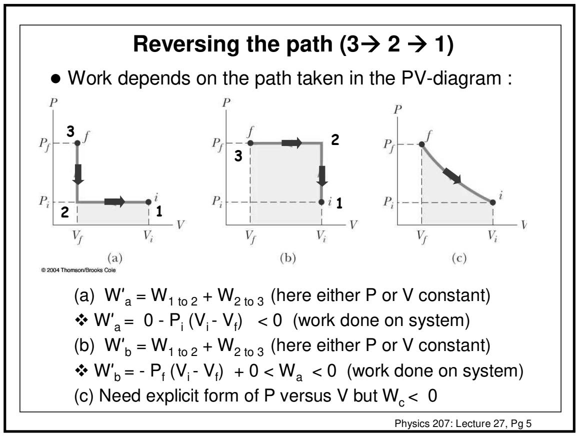
(a) $W_a = W_{1\text{ to }2} + W_{2\text{ to }3}$ (here either P or V constant)

❖ $W_a = -P_i\,(V_f - V_i) + 0 > 0$ (work done on system)

(b) $W_b = W_{1\text{ to }2} + W_{2\text{ to }3}$ (here either P or V constant)

❖ $W_b = 0 - P_f\,(V_f - V_i) > W_a > 0$ (work done on system)

(c) Need explicit form of P versus V but $W_c > 0$



Reversing the path (3→ 2 → 1)

- Work depends on the path taken in the PV-diagram :

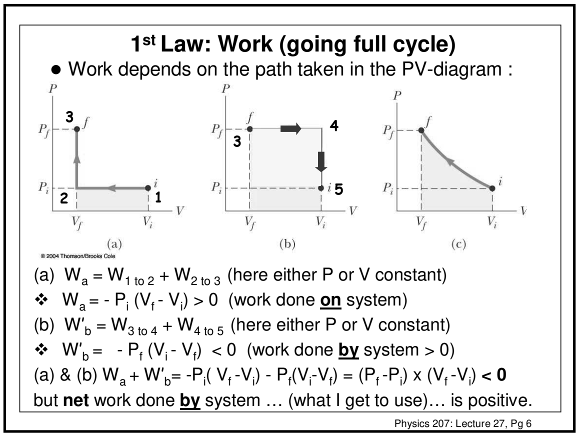
(a) $W'_a = W_{1\text{ to }2} + W_{2\text{ to }3}$ (here either P or V constant)

❖ $W'_a = 0 - P_i\,(V_i - V_f) < 0$ (work done on system)

(b) $W'_b = W_{1\text{ to }2} + W_{2\text{ to }3}$ (here either P or V constant)

❖ $W'_b = -P_f\,(V_i - V_f) + 0 < W_a < 0$ (work done on system)

(c) Need explicit form of P versus V but $W_c < 0$



1st Law: Work (going full cycle)

- Work depends on the path taken in the PV-diagram :

(a) $W_a = W_{1\text{ to }2} + W_{2\text{ to }3}$ (here either P or V constant)

❖ $W_a = -P_i\,(V_f - V_i) > 0$ (work done **on** system)

(b) $W'_b = W_{3\text{ to }4} + W_{4\text{ to }5}$ (here either P or V constant)

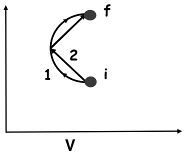
❖ $W'_b = -P_f\,(V_i - V_f) < 0$ (work done **by** system > 0)

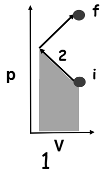
(a) & (b) $W_a + W'_b = -P_i(V_f - V_i) - P_f(V_i - V_f) = (P_f - P_i) \times (V_f - V_i)$ **< 0**

but **net** work done **by** system … (what I get to use)… is positive.

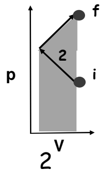
Lecture 27: *Exercise 1* (prelude) Work done by system

- Consider the path 2 connecting points **i** and f on the pV diagram.

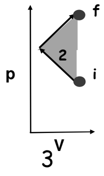




1



2

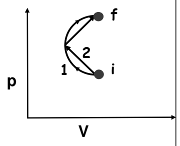


3

1. W_1 on system > 0, by system < 0 (if ideal gas, $PV=Nk_BT$)
2. W_2 on system < 0, by system > 0
3. W_1+W_2 on system < 0 , by system > 0 (area of triangle)

Lecture 27: *Exercise 1* Work done by system

- Consider the two paths, 1 and 2, connecting points **i** and f on the pV diagram.
  - ❖ The magnitude of the work, $|W_2|$, done by the system in going from **i** to **f** along path 2 is



(A) $|W_2| > |W_1|$ (B) $|W_2| = |W_1|$ (A) $|W_2| < |W_1|$

Work (W) and heat (Q) both depend on the path taken in the PV-diagram!

First Law of Thermodynamics with heat (Q) and/or work (W)

- First Law of Thermodynamics

$$\boxed{\Delta U = Q + W}$$

W → work done "on" the system
Q → heat flow "in" (+) or "out" (-)
ΔU → variation of internal energy

- ❖ ΔU is independent of path in PV-diagram
- ❖ Depends only on state of the system (P,V,T, …)

- Isolated system is defined as one with
  - ❖ No interaction with surroundings
  - ❖ Q = W = 0 ⇒ ΔU = 0.
  - ❖ $U_f = U_i$: internal energy remains constant.

Other Applications

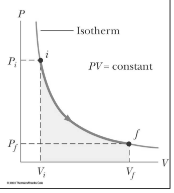
- Cyclic process:
  - ❖ Process that starts and ends at the same state ($P_i=P_f$, $T_i=T_f$ and $V_i=V_f$)
  - ❖ Must have ΔU = 0 ⇒ Q = -W .
- Adiabatic process:
  - ❖ No energy transferred through heat ⇒ Q = 0.
  - ❖ So, ΔU = W .
  - ❖ Important for
    - ➢ Expansion of gas in combustion engines
    - ➢ Liquifaction of gases in cooling systems, etc.
- Isobaric process: (P is constant)
  - ❖ Work (on system) is:

$$W = -\int_{V_1}^{V_2} P\,dV = -P(V_2 - V_1)$$

Other Applications (continued)

- Isovolumetric process:
  - ❖ Constant volume ⇒ W =0.
  - ❖ So ΔU = Q ⇒ all heat is transferred into internal energy
    - ➢ e.g. heating a "can" (and no work done).
- Isothermal process:
  - ❖ T is constant
  - ❖ If ideal gas: PV=nRT, we find P= nRT/V.
  - ❖ Work (on system) becomes :

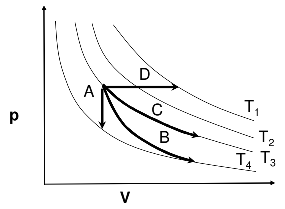
$$W = -\int_{V_1}^{V_2} P\,dV = -nRT\int_{V_1}^{V_2}\frac{dV}{V} = nRT\ln\left(\frac{V_1}{V_2}\right)$$



- ❖ PV is constant.
- ❖ PV-diagram: isotherm

Lecture 27: *Exercise 2* Processes

- Identify the nature of paths A, B, C, and D
  - ❖ Isobaric, isothermal, isovolumetric, and adiabatic

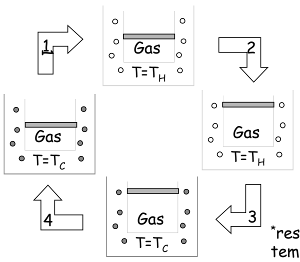


Heat Engines

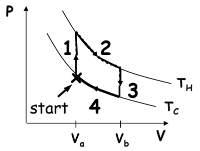
- We now try to do more than just raise the temperature of an object by adding heat. We want to add heat and get some work done!
- Heat engines:
  - ❖ Purpose: Convert heat into work using a cyclic process
  - ❖ Example: Cycle a piston of gas between hot and cold *reservoirs** (Stirling cycle)
    1) hold volume fixed, raise temperature by adding heat
    2) hold temperature fixed, do work by expansion
    3) hold volume fixed, lower temperature by draining heat
    4) hold temperature fixed, compress back to original V

Heat Engines

- Example: The Stirling cycle



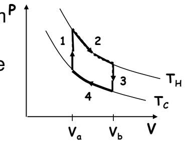
We can represent this cycle on a P-V diagram:



*reservoir: large body whose temperature does not change when it absorbs or gives up heat

Heat Engines

- Identify whether
  - ❖ Heat is ADDED or REMOVED from the gas
  - ❖ Positive work is done BY or ON the gas for each step of the Stirling cycle:



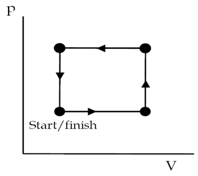
$\Delta U = Q + W$ (references system)

step	1	2	3	4
HEAT	$\boxed{\text{ADDED}}$ REMOVED	$\boxed{\text{ADDED}}$ REMOVED	ADDED $\boxed{\text{REMOVED}}$	ADDED $\boxed{\text{REMOVED}}$
Positive WORK	~~BY~~ ~~ON~~	$\boxed{\text{BY}}$ ON	~~BY~~ ~~ON~~	BY $\boxed{\text{ON}}$

Lecture 27: *Exercise 3*
Cyclic processes

- Identify A gas is taken through the complete cycle shown.

The net work done **on** the system was

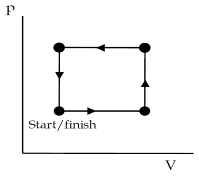


(A) positive (B) negative (C) zero

Lecture 27: *Exercise 3*
Cyclic processes (going in circles)

- Identify A gas is taken through the complete cycle shown.

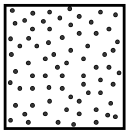
The net work done on the system (by the world) was



(A) positive (B) negative (C) zero

Work is done only on the horizontal paths, and the area under the third path segment is positive and larger than the area under the first path segment, which is negative. Hence the net work (on the system) is positive. (We, the world, are not gaining positive work.)

Ch. 21: Kinetic Theory of an Ideal Gas



- Microscopic model for a gas
  - ❖ Goal: relate T and P to motion of the molecules
- Assumptions for ideal gas:
  - ❖ Number of molecules N is large
  - ❖ They obey Newton's laws (but move randomly as a whole)
  - ❖ Short-range interactions during elastic collisions
  - ❖ Elastic collisions with walls (an impulse)
  - ❖ Pure substance: identical molecules
- This implies that temperature, for an ideal gas, is a direct measure of average kinetic energy of a molecule

$$PV = Nk_BT$$

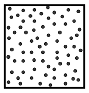
$$PV = \frac{2}{3}N\left(\frac{1}{2}m\overline{v^2}\right)$$

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_BT$$

Lecture 27, Exercise *3*

- Consider a fixed volume of ideal gas. When N or T is doubled the pressure increases by a factor of 2.

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_B T \qquad PV = N k_B T$$



1. If T is doubled, what happens to the rate at which *a single molecule* in the gas has a wall bounce?

   (A) x1.4 **(B) x2** **(C) x4**

2. If N is doubled, what happens to the rate at which *a single molecule* in the gas has a wall bounce?

   (A) x1 **(B) x1.4** **(C) x2**

Kinetic Theory of an Ideal Gas

- Theorem of equipartition of energy (A key result of classical physics)

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2} = \frac{1}{3}\overline{v^2} \quad \Rightarrow \quad \frac{1}{2}m\overline{v_i^2} = \frac{1}{2}k_B T$$

  - ❖Each degree of freedom contributes $k_B T/2$ to the energy of a system (e.g., translation, rotation, or vibration)
- Total translational kinetic energy of a system of N ideal gas molecules

$$K_{\text{tot trans}} = N\left(\frac{1}{2}m\overline{v^2}\right) = \frac{3}{2}Nk_B T = \frac{3}{2}nRT$$

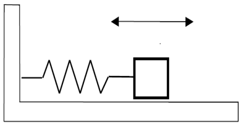
  - ❖ Internal energy of monoatomic gas: $U = K_{\text{ideal gas}} = K_{\text{tot trans}}$
- Root-mean-square speed: $v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\dfrac{3k_B T}{m}}$

Lecture 27, Exercise *4 & 5*

- A gas at temperature T is mixture of hydrogen and helium gas. Which atoms have more KE (on average)?

(A) H (B) He (C) Both have same KE

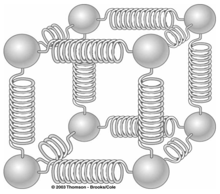
- How many degrees of freedom in a 1D simple harmonic oscillator?



(A) 1 (B) 2 (C) 3 (D) 4 (E) Some other number

Lecture 27, Exercise *6*

- An atom in a classical solid can be characterized by three independent harmonic oscillators, one for the x, y and z-directions?
- How many degrees of freedom are there?



(A) 1 (B) 2 (C) 3 (D) 4 (E) Some other number

Ideal Gas Molar Heat Capacities

- Definition of molar heat capacities (relates change in the internal energy to the temperature)

$$C \equiv \tfrac{1}{n}\lim_{\infty\to 0} Q/\Delta T = \tfrac{1}{n}\,\delta Q/\delta T$$

Ideal Gas Internal Energy

$$K_{\text{tot trans}} = U = \frac{3}{2}Nk_B T = \frac{3}{2}nRT$$

- There is only microscopic kinetic energy (i.e., no springs) in a monoatomic ideal gas (He, Ne, etc.)
  - ❖ At constant V, work W is 0 so ΔU = Q

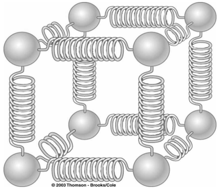
$$\boxed{C_V = \frac{3}{2}R}$$

  - ❖ At constant P : ΔU = Q + W = Q - P ΔV

$$PV = nRT \qquad \boxed{C_P = \frac{3}{2}R + R}$$

Lecture 27, Exercise *6*

- An atom in a classical solid can be characterized by three independent harmonic oscillators, one for the x, y and z-directions (U per atom = 3 RT) ?
- What is the classical molar heat capacity ($P\,\Delta V \cong 0$)?



(A) nR (B) 2nR (C) 3nR (D) 4nR (E) Some other number

Adiabatic Processes

- By definition a process in which no heat tranfer (Q) occurs

For an Ideal Gas:

$$PV^{\gamma} = \text{const} \qquad \gamma \equiv \frac{C_{\text{P}}}{C_{\text{V}}}$$

- Adiabatic process:
  - ❖ If ideal gas then PV^{γ} is constant
  - ❖ PV=nRT but **not** isothermal
  - ❖ Work (on system) becomes :

$$W = -\int_{V_1}^{V_2} P\,dV = -\int_{V_1}^{V_2} \frac{\text{const}}{V^{\gamma}} \frac{dV}{V} = \frac{\text{const}}{\gamma} (V_2^{-\gamma} - V_1^{-\gamma})$$

Recap, Lecture 27

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  - ❖ Work done by an ideal gas in a piston
  - ❖ Introduction to thermodynamic cycles (Chapter 22)
  - ❖ Ideal gas at the molecular level, Internal Energy
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